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Particle Penetration and Radiation Effects

General Aspects and Stopping of Swift Point Charges

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Peter Sigmund

Particle Penetration and Radiation Effects

General Aspects
and Stopping of Swift Point Charges

With 131 Figures

Springer

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Preface

This book has emerged from lectures given to physics students at the University of Copenhagen and the University of Southern Denmark in Odense. Part of the material was first compiled in connection with lectures at international summer schools in Predeal (Romania), Split (Croatia), Porto Vecchio (Corsica), Haifa (Israel), Viana do Castelo (Portugal) and Alicante (Spain).

The book addresses mainly two different groups of readers. Chapters 1–5 of this volume serve as the main text of a standard course for graduate students in physics. Here I have tried to focus on general-physics aspects which should be of interest even to someone who never gets in touch with a charged-particle beam. General principles are introduced ab initio, and important physical arguments and mathematical derivations are presented in extenso. Consultation of the original literature should only rarely be necessary during the first reading, but pertinent references have been supplied generously on all major items for the reader who wants to dig deeper.

The second group of prospective readers is scientists, engineers and medical doctors who apply charged-particle beams, in particular ion beams, in a wide variety of contexts and who need an introduction to the underlying physical principles in order to understand and quantify some of their activities. Many of those readers are not physicists and have typically only a minor background – if any at all – in collision theory. Therefore also this readership may appreciate a reasonably self-contained introduction into the field. On the other hand, those readers, as well as students specializing in the field, need to go further. Chapters 6–9 are intended to guide them up to the level of current research in the area.

I shall assume the reader to be familiar with classical dynamics of one- and two-particle systems and elements of electromagnetic theory. Special relativity will enter in numerous instances, but the reader uninterested in relativistic beam energies will be able to jump over those passages without loss of essential information for what follows. Quantal concepts enter throughout the book, but mainly in the common Schrödinger picture. Yet quantal collision theory, which is frequently omitted from introductory courses in quantum theory, will be introduced ab initio. Some experience with elements of probability theory will be beneficial. A generous amount of supporting material that is – more or less – standard in a physics curriculum but not necessarily familiar to a reader with a different background has been collected in an appendix.

The main text contains passages which a novice in the field might prefer to jump over in a first reading. I have tried to mark such passages by a star ($\star$), even though there might be an occasional reference to such passages in unstarred sections.

Problems added to all chapters are intended to serve four purposes,

- to provide order-of-magnitude estimates of important experimental parameters,
- to illustrate the significance of a general result by adopting specific input,
- to train the student in casting a physics question into a form tractable by quantitative calculation, and
- to complete formal derivations quoted in the text where appropriate. This type of problem is dominating in the second half of the volume.

Some help has been provided whenever judged necessary, although I do not by any means want to discourage the reader from trying to take a different and, possibly, more efficient route.

A first draft of this book was written at IBM Watson Research Laboratory many years ago. That version focused on energy loss of ion beams and did not get finished because I noticed a lack of knowledge on key features of the theory. I could, in principle, have published a status report, but instead I initiated research programs addressing topics that I felt needed attention. This process continued over the years, interrupted by an extended period of administrative duties but compensated by a sabbatical at Argonne National Laboratory, short visiting appointments at Université Paris-Sud, École Polytechnique and the University of Pretoria, and a month at San Cataldo monastery. While the style of the book is still very much like that of the first draft, I find that the subject matter, or at least my personal grasp of it, has developed to the point that this book can serve a useful purpose.

This first volume presents general concepts and, moreover, focuses on the stopping of swift point charges. The first part provides an overview of the field and its application areas, a chapter on elementary penetration theory inspired by the 'Danish school' of Bohr and Lindhard, and elements of classical and quantal scattering theory as far as needed in context. Chapters 4 and 5 outline classical and quantal theory of electronic stopping of swift point charges. Taken together, Chapters 1–5 provide ample material for a semester course in particle penetration. Of the remaining four chapters, only a fraction is likely to be found suitable for presentation in a university course, but I hope that they will be useful to those who want to specialize in the field.

A second volume is planned to extend the treatment to heavy ions, including molecule and cluster ions, stopping at lower velocities, as well as multiple scattering, ion ranges and channeling. Associated radiation effects are planned to be treated in a third volume.

My own introduction to the field has been inspired indirectly by the late Niels Bohr whom I never got a chance to meet or listen to, and more directly by his close collaborator, the late Jens Lindhard whose lectures and papers as well as numerous discussions greatly stimulated my approach to the field.

This book would never have come along without the excellent working conditions provided by the University of Southern Denmark, as well as my hosts at various sabbatical stays, most notably J.F. Ziegler at IBM, D.S. Gemmell and H.G. Berry at Argonne, Y. Lebeyec at Orsay, Y. Quéré and Annie Dunlop at Palaiseau, and E. Friedland and J.B. Malherbe in Pretoria.

Numerous colleagues, junior and senior ones, have contributed to develop my knowledge of and insight into the field over the years. In connection with the present volume I like to mention in particular H.H. Andersen, J.U. Andersen, N.R. Arista, G. Basbas, A. Belkaçem, F. Besenbacher, E. Bonderup, H. Esbensen, D.J. Fu, L.G. Glazov, A. Gras-Martí, U. Haagerup, P. Hvelplund, M. Inokuti, J. Jensen, K. Johannessen, H. Knudsen, E. Merzbacher, H.H. Mikkelsen, E.H. Mortensen, S.P. Møller, J. Oddershede, H. Paul, R.H. Ritchie, A. Schinner, A.H. Sørensen, A. Tofterup, S.M. Tougaard, K.B. Winterbon, and J.F. Ziegler.

Drafts of the first five chapters have been read by numerous students who returned useful corrections, comments and questions. Lev Glazov and Nestor Arista have spent much time, care and thought in reading the whole volume and have provided invaluable feedback. Nevertheless, the blame for any serious omissions and errors is the author's and not theirs.

Financial support has been received from the Danish Natural Science Research Council (FNU) which has enabled the author to travel, to conduct an extensive visitors' program, and to support research students and postdoctoral fellows. Support has also been received from NORDITA, the Carlsberg Foundation, the Ib Henriksen Foundation, the NATO Collaborative Research Programme, the IBM World Trade Program and the Institution San Cataldo. It was a particular honor to spend a year at Argonne National Laboratory as an Argonne Fellow.

Odense, October 2005 *Peter Sigmund*

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