

User guide to the RALEF-2D code

M.M. Basko

*KIAM, Moscow and GSI, Darmstadt**

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*Electronic address: mmbasko@gmail.com; URL: <http://www.basko.net>

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1. HOW TO RUN THE RALEF CODE

1.1. File names and directory structure

Assume that the principal project directory for simulating a particular problem, from which the RALEF executable is to be executed, has a name `p/`. Then, in a standard configuration, the structure of this directory would be

```
p/code/
p/input/
p/output/
p/in2d
```

(1.1)

Here

- the subdirectory `p/code/` contains all the FORTRAN source-code files `*.f`;
- the subdirectory `p/input/` contains all the files `*.ii` with the input information that might be needed during the code run (like the GLT-table files `tab-glteos17.f17` and `tab-gltrcrad.f17`, the file `xspmonbc.ii` with a tabular form of the incident x-ray spectrum, the files `lastpro.ii` and `lasspro.ii` with the tabular temporal and spatial laser profiles, etc.);
- the subdirectory `p/output/` is where all the output files — except for the dump files `dp2d****` — are written; if the user does not create this subdirectory beforehand, the RALEF code should, normally, create it in the process of execution; however, the latter is not guaranteed for any compiler under any operating system;
- the `in2d` is the principal input file, where the values of various parameters from the `namelist/input/` are assigned.

Here and below, to distinguish the alone-standing file names from other types of variables, we surround them by single quotation marks like `' '`.

Note that `in2d` is the only input file which has no extension, and which must be placed into the principal project directory `p/`. All the other files with the numeric-data input information must have either the extension `.ii` or the extension `.f17`, and must be placed into the subdirectory `input/`, which must (!) be created because it appears explicitly in the corresponding FORTRAN statements. And only if no input information from files other than `in2d` is required for the current job, the subdirectory `input/` may be omitted. In contrast, the subdirectory `code/` does not appear explicitly in the FORTRAN statements, which means that the user is free either to change its name or to omit it altogether.

Once the RALEF executable is executed from the project directory `p/` (by, say, the UNIX command

```
nohup ./code/a.out >> aa.dat &
```

when `code/` is the compilation directory), all the output files will be written into the subdirectory `p/output/`, and will all have either the extension `.dat` or the extension `.vtk`. In contrast, the dump files `dp2d****` are all written into the project directory `p/`.

1.2. Compilation and execution

RALEF-2D is a parallelized code, with a hybrid MPI/OpenMP parallelization scheme. Hence, it can be run in two different modes: an MPI mode, and a non-MPI mode. In

addition, any of these two modes can be combined with the OpenMP compiler option. Thus, the code in the non-MPI mode, compiled without OpenMP, will be executed sequentially; the code in the non-MPI mode, compiled with the OpenMP option, will be executed in the OpenMP-parallelized mode.

For the compilation and execution of the RALEF code, go through the following steps.

1. In the MPI mode:

- (a) uncomment all the MPI-declarations in the FORTRAN file 'f01_main_ccMPI.f' by deleting all combinations of the 4 symbols !MPI at the start of every code line where this combination is encountered; change the file name to the standard one 'f01_main.f';
- (b) gather the 13 FORTRAN source-code files

```
f00_comod.f
f01_main.f      (MPI-declarations are active)
f02_init.f
f03_bound.f
f04_hydro.f
f05_remap.f
f06_eos.f
f07_lnktbls.f
f08_util.f
f09_rad.f
f10_taskinpt.f
f11_taskdepo.f
f12_taskout.f
```

into the source-code subdirectory `code/`.

In the non-MPI mode: gather the 13 FORTRAN source-code files

```
f00_comod.f
f01_main_ccMPI.f  (MPI-declarations are deactivated)
f02_init.f
f03_bound.f
f04_hydro.f
f05_remap.f
f06_eos.f
f07_lnktbls.f
f08_util.f
f09_rad.f
f10_taskinpt.f
f11_taskdepo.f
f12_taskout.f
```

into the source-code subdirectory `code/`,

or

gather the 14 FORTRAN source-code files

```
f00_comod.f
f01_main.f      (MPI-declarations are active)
```

```

f02_init.f
f03_bound.f
f04_hydro.f
f05_remap.f
f06_eos.f
f07_lnktbls.f
f08_util.f
f09_rad.f
f10_taskinpt.f
f11_taskdepo.f
f12_taskout.f
f13_mpi-dummy.f

```

into the source-code subdirectory `code/`.

2. If no tabular EOS or opacities are used, the file ‘f07_lnktbls.f’ may be replaced with the dummy file ‘f07_lnktbls-dummy.f’.
3. By using an appropriate compiler,
 - (a) compile the three files ‘f00_comod.f’, ‘f08_util.f’ and ‘f13_mpi-dummy.f’ (when present), which contain various modules;
 - (b) compile and link the rest of the code.

UNIX EXAMPLES:

```

ifort -c -openmp f00_comod.f f08_util.f f13_mpi-dummy.f
ifort -openmp *.f

```

```

mpif90 -c -openmp f00_comod.f f08_util.f
mpif90 -openmp *.f

```

4. Gather all the needed numeric-data files with the input information for the RALEF code (like ‘table-glt-eos.ii’, ‘table-glt-tcrad.ii’, ‘lastpro.ii’, ‘xspmonbc.ii’, etc.) into the subdirectory `p/input/`.
5. Run the code by executing the executable from the project directory `p/`, which contains the principal input file ‘in2d’.

UNIX EXAMPLE with 12 MPI-tasks:

```

mpirun -np 12 ./code/a.out > aa.dat &

```

1.3. Assigning the problem parameters in the input file ‘in2d’

The values of the principal run-control parameters are collected into the `namelist/input/`, and are assigned by editing the text file ‘in2d’. The first line of this file before the operator `$INPUT` contains the job title. An example of the ‘in2d’ file for simulation of an empty cylindrical hohlraum is given below.

```

Au cyl.hohlraum: case 01: empty, 1 nu-group
&input ! Begin NAMELIST "input"
nrestart=0

```

```
! Units of measurement (in terms of CGS units):
unilngth=1.d-1
unitime=1.d-8
unimass=1.d-3
unitemp=1.60217733d-9

! Geometry and mesh:
igeom =3 ! r-theta polar mesh with iradial=0
iradial=0
nblks=1
nprts(1,1)=3 ! 3 parts along theta
ncell(1,1,1)=54,24,180
ncell(1,2,1)=40
xxl(1,1,1)=24.d0, 78.d0, 102.d0, 336.d0 ! degrees
xxl(1,2,1)=0.35d0, .365d0 ! radial dimensions

! Material properties:
matnum(1,1)=1,1,1
proprty(1,1)=7.d0 ! EOS type
proprty(2,1)=.1d0 ! rho00, used for wt(i) in REZONE
proprty(4,1)=1.2d0 ! strong shock parameter
proprty(5,1)=1.d0 ! m-number in GLT tables
proprty(27,1)=196.97d0, 79.d0 ! A and Z for analytical formulae
proprty(30,1)=7.d0 ! conduction model #
proprty(31,1)=1.d0 ! m-number in GLT tables for conduction
proprty(37,1)=-.5d0 ! no flux limit on conduction
proprty(40,1)=7.d0, 1.d0 ! opacity
proprty(46,1)=3.d0 ! laser absorption

! Initial and boundary conditions:
rho0(1,1)=19.d0, 1.d-4, 19.d0
pr0(1,1)=1.d-7, 1.d-7, 1.d-7
pbc(1,1,1)=1.d-7, 1.d-7, 1.d-7
pbc(1,2,1)=1.d-7, 1.d-7, 1.d-7
pbc(1,3,1)=1.d-7
pbc(1,4,1)=1.d-7

! Radiation, thermal treatment:
itemp=4
nfreqsets=2
nfreqs=1,200
kradSn=3
ifshadow=1
iflasdep=1
iradfb=0
iradb=0
TEMPFLR=3.d-5
TEMPSNS=2.d-3
EPSOSSI=.2d0
EPS1SSI=.1d0
```

```

! Runtime, printout control:
twfin=.2d0
ncycmax=1
ntty=100
twmovi=0.0d0, 1.d7, .05d0
twprnt=0.d0, 1.d35, .05d0
twfilm=0.d0, 1.d1, 1.3d2, 2.d35
twspctr=0.d0, .1d9, .05d9
twdump=1.d2, 1.d35, 1.d2

! ALE, rezone control:
iorder=1
alecoef=.99d0
dt0=1.d-6
dtmin=1.d-60
dtmax=1.d-4
dtfac=0.5

nadvskp=1
wtampl=6.d0
nsmooth=4
itrezn=5

ibcrezn(1,1)=2,1,1,1
crezsm=.4d0
prezsm=1.5d0
arezsm=.03d0
nrezbay=12
arezbay=.6d0
crezbay=.4d0

/ ! End NAMELIST "input"

```

The meaning of all the parameters that can be set via this namelist is explained in the documentation file '00glossar_RALEF.txt'.

1.4. Mesh construction

Typically, the user is supposed to select one of the preprogrammed mesh types from the RALEF mesh library by assigning the corresponding value of the `igeom` parameter in the namelist `input` (file 'in2d'). All possible values of the `igeom` parameter are listed in section 9 below, together with a detailed description of the corresponding mesh and its other control parameters that are to be specified in the namelist `input`.

If the user does not find the needed type of mesh in the current version of the RALEF mesh library, he (she) is supposed to assign a new value `igeom = xxxx` and write the corresponding mesh construction routines subroutine `MSHPxxxx` and subroutine `MSHBxxxx(XV)` by analogy with those already available in file 'f02_init.f' — thus enriching the RALEF mesh library.

1.5. Equation of state and opacities

1. Analytical equation of state and/or opacities

If one of the preprogrammed analytical models is used for the equation of state, or the thermal conduction coefficient, or the radiation absorption coefficient, then the parameters of the corresponding analytical model are set up in the input file ‘in2d’. These parameters are all contained in the principal material-property array `property(1:100,1:30)`; the meaning of all relevant elements of this array is explained in section 11 below.

2. GLT-tabular equation of state and/or opacities

If either for the equation of state, or for the thermal conduction coefficient, or for the radiation absorption coefficients a tabular option in the form of the general logarithmic tables (GLT) is chosen by setting `property(1,imat) = 7.0` [or `property(30,imat) = 7.0`, or `property(40,imat) = 7.0`], then the files ‘tab-glteos.f17’ and/or ‘tab-gltcrad.f17’ with corresponding tables must be provided. This is accomplished by running separate code packages GLT-EOS and GLT-TCR: the GLT-EOS package (version 2017) generates the file ‘tab-glteos.f17’, which contains the GLT database for the tabular EOS; the GLT-TCR package generates the file ‘tab-gltcrad.ii’, which contains the GLT database for the tabular thermal conduction coefficient and/or the tabular Rosseland, Planckian and spectral opacities. In this way tabular opacities and/or thermal conductivity can be combined with any analytical EOS and vice versa.

2. HOW TO GENERATE THE GLT TABLES

2.1. Table for GLT-EOS17

The file ‘tab-glteos.f17’, comprising all the data for the tabular equation of state GLT-EOS17 (version of the year 2017), is created by running the program GTESFILL17 from file ‘tabeos17.f’ in the separate GLT-EOS code package. This is accomplished by going through the following steps:

1. Edit the code blocks in Steps 1–3 of the program GTESFILL17, marked as


```
!@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@for_user_to_edit:
                                     . . .
!@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@end_user-edit.
```

where all the key GLT-table parameters, specifying the number and type of materials used, the type of the source EOS, the 2D $\{\ln \rho_i\} \otimes \{\ln T_j\}$ table grid, etc., are assigned.

2. Gather the 7 FORTRAN files

```
‘f00_comod.f’,
‘f07_lnktmls.f’,
‘tabeos17.f’,
‘sourceos17_Basko.f’,
‘sourceos17_Novik.f’,
‘sourceos17_NVK_2.f90’,
‘sourceos17_FEOS.f’,
```

needed to run the GLT-EOS package, into the code directory, compile and link.

3. Run the executable, created in the previous step.

For the FEOS source-EOS model, a static object library ‘libfeos.a’ (UNIX) or ‘FEOS16.lib’ (Windows) must be added at the linkage stage, and the file ‘FEOS_TF-Table_1197.dat’ must be placed into the execution directory.

If the FEOS model is not actually used and/or ‘libfeos.a’ is not available, replace the file ‘sourceos17_FEOS16.f’ with the ‘FEOS_dummy.f’ file at the compilation stage.

The meaning of the user-defined parameters of the GLT-EOS table that are to be assigned in Step 3 of the program GTESFILL17 is as follows:

`nmatgtes` = total number of materials in the GLT-EOS table;

`srcEOStag(m)` = a *character* tag, defining which source EOS must be used for material # m (the sequential material number in the GLT-EOS table); allowed values are predefined *character* variables `eostag_FEOS`, `eostag_BASKO`, `eostag_NOVIK`, `eostag_NVK_2`;

`matnamgtes(m)` = a *character(128)* variable, containing the user-defined name for material # m ;

`imatSES(m)` = an *integer(4)* SESAME-number of material; must coincide with the corresponding SESAME material number `iSES_FEOSdb` in the module `EOS_FEOS_dbase`;

`Maxwell(m)` = a *logical(1)* flag; if equal to `.true.`, the GLT-EOS table contains the equilibrium EQ-EOS for material # m ; otherwise the metastable MS-EOS is tabulated;

`zmolgtes(m)` = a *real(8)* variable, equal to the atomic (“molecular”) number of material # m ; must coincide with the atomic number of this material in the source EOS (if assigned independently);

`amolgtes(m)` = a *real(8)* variable, equal to the atomic (“molecular”) mass of material # m ; must not coincide with the atomic mass of this material in the source EOS (if assigned independently);

`rho1ww(m)` = the left (lower) boundary of the density grid $\{\rho_i\}$ (i.e. ρ_1) for material # m ;

`rhoN1ww(m)` = the right (upper) boundary of the density grid $\{\rho_i\}$ (i.e. $\rho_{N_{\rho}+1}$) for material # m ;

`rho_cen(m)` = the “central” point of the density grid $\{\rho_i\}$ for material # m , towards which the grid is condensed — i.e. near $\rho = \text{rho_cen}(m)$ the increment $\ln \rho_{i+1} - \ln \rho_i$ is minimal; if the user assigns `rho_cen(m) < 0`, than it is set equal to the critical density ρ_{cp} ; needed by the subroutine `GENGRI1CP(...)`, when the latter is employed to generate a centered-progressive grid along the $\ln \rho$ coordinate;

`drln_min(m)` = the minimum increment $\ln \rho_{i+1} - \ln \rho_i$ of the density grid $\{\rho_i\}$ for material # m ; needed by the subroutine `GENGRI1CP(...)`, when the latter is employed to generate a centered-progressive grid along the $\ln \rho$ coordinate;

`drln_max(m)` = the maximum increment $\ln \rho_{i+1} - \ln \rho_i$ of the density grid $\{\rho_i\}$ for material # m ; needed by the subroutine `GENGRI1CP(...)`, when the latter is employed to generate a centered-progressive grid along the $\ln \rho$ coordinate;

$\text{rhox1ww}(m)$ = the left (lower) boundary of the “cold-curve” grid $\{\rho_{x,i}\}$ (i.e. $\rho_{x,1}$) for material # m ;

$\text{rhoxN1ww}(m)$ = the right (upper) boundary of the “cold-curve” grid $\{\rho_{x,i}\}$ (i.e. $\rho_{x,N_{\rho x}+1}$) for material # m ;

$\text{T1ww}(m)$ = the lower boundary of the temperature grid $\{T_i\}$ (i.e. T_1) for material # m ;

$\text{TN1ww}(m)$ = the upper boundary of the temperature grid $\{T_i\}$ (i.e. T_{N_T+1}) for material # m ;

$\text{T_cen}(m)$ = same as $\text{rho_cen}(m)$ but for the temperature grid $\{T_i\}$ for material # m ;

$\text{Txww}(m)$ = the “cold-curve” temperature for material # m ;

$\text{dTln_min}(m)$ = same as $\text{drln_min}(m)$ but for the temperature grid $\{T_i\}$ for material # m ;

$\text{dTln_max}(m)$ = same as $\text{drln_max}(m)$ but for the temperature grid $\{T_i\}$ for material # m .

In addition, the user is supposed to update the elements **hd(2)** and **hd(3)** of the **header** to the file ‘**tab-glteos.f17**’ in order to provide a unique identification (like certain specific table characteristics, its date of creation, etc.) to the generated GLT-EOS table. Note that the full list of all arrays and variables, written into the file ‘**tab-glteos.f17**’, is provided by the same **header**.

2.2. Table for GLT-TCRAD17

3. HOW TO SET UP LASER DRIVE

3.1. General steps

1. Principal control parameters

To activate the laser energy deposition, go through the following steps:

1. in the namelist/input/, file ‘in2d’:

- (a) assign `iflasdep=1` to activate the laser deposition model # 1 — a non-refractive model along short characteristics with the X transport mode for all laser beams, or
assign `iflasdep=2` to activate the laser deposition model # 2 — a reflective/refractive model along short characteristics in the RH mode of laser transport, and long characteristics in the H mode of transport, or
assign `iflasdep=3` to activate the laser deposition model # 3 — a universal model along long characteristics in either X, H, or RH mode of laser transport for every individual laser beam;
- (b) set the logical flag `ifshdwlas=.false.` if the search for shadows, cast by protruding parts of external boundaries, is not needed; presently (as of 2015-10-02), `ifshdwlas=.true.` works only in model # 1;

- (c) assign `proppty(46,imat) = 3` to use the Kramers formula for the inverse-bremsstrahlung absorption coefficient (or the corresponding formula for the collision frequency — when the plasma dielectric constant is needed) of the laser light, or `proppty(46,imat) = 5` to use the semi-empirical Basko model (matching Kramers at low-densities and high temperatures) either for the laser absorption coefficient, or for the complex dielectric permittivity; consult Table III for additional material properties that may need to be assigned;

2. in the subroutine LASINPT, file ‘f10_taskinpt.f’:

- Step 1: assign the values of the following parameters, common for all beams:
 - `nblas` = total number of individual laser beams used,
 - `ntprlas_ii` = total number of different *temporal* laser power profiles that are to be loaded from the external file ‘lastpro.ii’,
 - `nsprlas_ii` = total number of different *spatial* laser power profiles that are to be loaded from the external file ‘lasspro.ii’;
- Step 2: assign the values of the following beam-individual laser deposition parameters for every individual laser beam `iblas = 1, 2, ..., nblas`:
 - `Omeblasx(iblas)` = x_1 -component of the propagation vector $\vec{\Omega}_{las}$ along the axis of the incident laser beam `iblas` (laser-axis vector);
 - `Omeblasy(iblas)` = x_2 -component of the propagation vector $\vec{\Omega}_{las}$ along the axis of the incident laser beam `iblas`;
 - `xfocblas(iblas)` = x_1 -coordinate of the focal point on the axis of the incident laser beam `iblas`;
 - `yfocblas(iblas)` = x_2 -coordinate of the focal point on the axis of the incident laser beam `iblas`;
 - `dvpblas(iblas)` = distance from the focal point $(x, y) = (xfocblas(iblas), yfocblas(iblas))$ to the “view” plane downstream along the laser-axis vector $\vec{\Omega}_{las}$;
 - `dRlblas(iblas)` = the Rayleigh length of the laser beam `iblas`;
 - `freqblas(iblas)` = frequency of the incident laser beam `iblas`;
 - `spolfrac(iblas)` = fraction of s-polarization in the laser deposition models # 2 and # 3;
 - `tblasdelay(iblas)` = time offset for the temporal profile of the laser beam `iblas` programmed in function FLASTPRO or read from file ‘input/lastpro.ii’;
 - `itprcaslas(iblas)` = case (option) number for the temporal profile of the laser beam `iblas`, programmed in function FLASTPRO;
 - `itprlas_ii(iblas)` = sequential number of the temporal profile from file ‘lastpro.ii’ to be used with the laser beam `iblas` when `itprcaslas(iblas) = 0`;
 - `tFWHMblas(iblas)` = the FWHM of the temporal pulse profile of the laser beam `iblas`;
 - `isprcaslas(iblas)` = the case (option) number for the spatial profile of the laser beam `iblas`, programmed in the function FLASSPRO;
 - `isprlas_ii(iblas)` = sequential number of the spatial profile from file ‘lasspro.ii’ to be used with the laser beam `iblas` when `isprcaslas(iblas) = 0`;

`rfocblas(iblas)` = effective focal spot radius (as measured in the “view” plane) of the incident laser beam `iblas`;
`sapeblas(1:2,iblas)` = aperture of the incident laser beam `iblas` as measured in the “view” plane from right to left across the beam axis when looking down the beam propagation direction $\vec{\Omega}_{las}$;
`Fblas00(iblas)` = reference value of the intensity of the incident laser beam `iblas` — normally, per unit surface area perpendicular to the beam axis (but per unit length perpendicular to the axis of a conical laser beam);
`axrhlas(iblas)` = parameter defining the transport mode of the beam `iblas`: `axrhlas` = -1.0 → X mode, `axrhlas` = 0.0 → H mode, $|\text{axrhlas}| > \text{dfloor}$ → RH mode; if `axrhlas` < 0.0, the wave-optics deposition rate along the incident, reflected and evanescent rays is calculated without the rescaling procedure, i.e. fully includes the oscillating interference term; the default value is `axrhlas` = 0.9;
`ala3GausRH(iblas)` = ray curvature threshold for transition from Gaussian to RH algorithm in model # 3;
`nlongrays(iblas)` = number of long rays in laser beam `iblas` in model # 3, or number of entry points per cell edge in laser beam `iblas` in model # 2;
`atlasfloor(iblas)` = attenuation floor, below which tracing of individual rays (j-beamlets) is stopped in laser beam `iblas` in models # 2 and # 3;
`rlasavrmx(iblas)` = radius of artificial smoothing of the laser deposition rate of beam `iblas` near the rotation axis and global reflective boundaries in models # 2 and # 3;
`ovcrkblas(iblas)` = multiplier (limiting factor) for the Kramers absorption coefficient near and beyond the critical surface in models # 1 and # 2; default value `ovcrkblas` = 10.0;

3. in the subroutine `TASKINPT`, file ‘`f10_taskinpt.f`’:

if an output for angular distribution of the reflected laser light is required, assign the value of `nlasoutet` (= the desired number of angular groups) and specify the direction of the reference axis (`Omlasrefx`, `Omlasrefy`) at Step 6 of subroutine `TASKINPT`; it will be written out into the ‘`elasoutet.dat`’ file through the `twfilm` output channel.

4. in file ‘`f10_taskinpt.f`’:

when needed, modify or program anew the case-specific temporal and/or spatial laser profiles in functions `FLASTPRO` and/or `FLASSPRO`.

Issues to pay attention to:

- one and the same value of `iflasdep` must be used for all the laser beams throughout the current job; even if the start of laser irradiation is delayed in time (i.e. when `tblasdelay(iblas) > 0`), assign the chosen value of `iflasdep` at the start of the job;
- the values of all the laser parameters, initially assigned at 2.–Step 2, can be changed in the course of job execution, i.e. either at successive job restarts or according to the pre-programmed variation in time in the subroutine `RUNCTRL`, file ‘`f10_taskinpt.f`’.

2. Secondary control parameters

The principal parameter, which controls transition from the 2D geometric-optics approximation to the 1D wave-optics approximation in our hybrid model of laser deposition, is $\alpha_{gop} = \text{axrhlas}(\text{iblas})$. However, the actual condition for such transition in the deposition model #3 consists of three checking levels and contains three additional secondary control parameters $\alpha_{g0} = \text{alfG0}$, $\beta_{wo} = \text{betwo}$, and $\Delta_0 = \text{cos0marg}$, defined in the subroutine LA3DEPR as the “`real(8), parameter`” variables. Namely, while tracing every individual elementary laser ray, the algorithm is switched over from the 2D geometric optics to the 1D wave optics after the following three necessary conditions are fulfilled

$$(i) \quad \bar{n}_e \equiv \frac{n_e}{n_{cr}} > \min(\alpha_{gop}, \alpha_{g0}), \quad (3.1)$$

$$(ii) \quad \bar{n}_e + \beta_{wo}\lambda |\nabla \bar{n}_e| > \alpha_{gop}, \quad (3.2)$$

$$(iii) \quad \cos \theta_0 > \Delta_0, \quad \text{and vector } \vec{g} \text{ points into the entered cell,} \quad (3.3)$$

where λ is the laser wavelength, θ_0 is the angle between the ray direction and the gradient vector \vec{g} .

The main condition is (ii) with the default value of $\beta_{wo} = 1$. Condition (i) plays a subordinate role: with $\alpha_{g0} \ll 1$ (the default value is $\alpha_{g0} = 0.01$) it serves to prevent too early transitions to the wave optics that might occur according to (ii) in a strongly underdense plasma with sharp local changes of \bar{n}_e within the limits $\bar{n}_e \ll 1$ (implying large values of gradient $|\nabla \bar{n}_e|$ over very short distances). Condition (iii) is needed for the 1D wave-optics treatment to be physically and mathematically consistent; the default value of Δ_0 is 0.05.

3.2. How to use laser profiles provided in the form of numerical tables

1. Numeric temporal profiles

To use a temporal laser-power profile $p_t(t)$, provided as a numeric table, go through the following steps:

1. prepare the file ‘`input/lastpro.ii`’ with the corresponding numeric input data;
2. assign a positive non-zero value to the variable `ntprlas_ii` in the subroutine `LASINPT`, file ‘`f10_taskinpt.f`’, which must be equal to the total number of different numeric t-profiles that are to be read out from file ‘`input/lastpro.ii`’; the actual number of numeric profiles, used in the current job, may be smaller than `ntprlas_ii`;
3. for every laser beam `iblas`, which should use a numeric temporal profile, assign `itprcaslas(iblas) = 0` (profile option # 0) and the corresponding value of `itprlas_ii(iblas) =` the sequential number of the numeric profile (from the total of `ntprlas_ii` numeric profiles read out from file) that is to be used with the beam `iblas`;

The input data file ‘`input/lastpro.ii`’ must have the following structure. Every of the `ntprlas_ii` profiles must be specified as two columns of numbers (the column of times t and the column of corresponding powers p_t), recognized by the `G` format descriptor and preceded by a header of comment lines. The header may consist of an arbitrary number of record lines beginning with the sign `#`. No empty lines are allowed in the header. The first numeric profile may have no header. The two-column numeric sections of file ‘`input/lastpro.ii`’ are allowed to be interspersed by empty record lines.

To synchronize the units of measurement in the ‘input/lastpro.ii’ file and the RALEF code, the header before any numeric profile should (but must not) contain one or two comment lines of the form

```
#col-1_unit  u1
```

```
#col-2_unit  u2
```

where u_1 [u_2] is a number (recognized by the G format descriptor), giving the unit (in terms of the CGS units; time – in seconds) used in the first [second] numeric column of the file ‘input/lastpro.ii’.

When assigning the units for different profiles in file ‘input/lastpro.ii’, the following rules are to be observed:

- if unit u_j ($j = 1$ or 2) is specified for none of the read-out profiles, it is set equal to 1 (i.e. to the corresponding CGS unit) for all the profiles;
- if unit u_j is specified for the first, and only for the first profile in file ‘input/lastpro.ii’, this same unit is used for the corresponding column in all the other profiles from this file;
- if unit u_j is specified for only some, but not all, profiles (or for only one profile which is not the first), it is set equal to 1 (i.e. to the corresponding CGS unit) for all the profiles where it was not explicitly specified, and a warning is printed out.

Example of an ‘input/lastpro.ii’ file with two t-profiles where time is measured in nanoseconds:

```
# Example of file with two t-profiles
#col-1_unit 1.d-9
#t-profile #1:
0.d0 0.d0
1.e-1 1.e0
1.e0 1.e0

# t-profile #2:
1.2e-12 1.e-12
1.e0 5.e0

2.e0 3.e0
3.e0 3.e0
```

The time values in the first numeric column of file ‘input/lastpro.ii’ must be monotonically increasing. If the first time t_1 is negative, the whole time sequence is shifted forward by $-t_1$, so that all times become non-negative. Finally, all the times are rescaled to the currently used RALEF time unit.

The power values p_t may be given in arbitrary units. Having been read out, they are normalized to the peak value of $p_{t,max} = 1$. Thus normalized powers are integrated over time (already in the RALEF units) to provide the effective fluence value

$$\text{tplsefflas.ii(itprlas.ii(iblas))} = E_{lb,eff} = \int_0^{\infty} p_t(t) dt,$$

which is needed to establish a relation between the peak laser intensity Fblas00(iblas) and the total pulse energy of the laser beam iblas .

2. Numeric spatial profiles

Application of numeric spatial laser profiles $p_s(s)$ is organized in the same manner as that of the numeric temporal profiles:

1. prepare the file 'input/lasspro.ii' with the corresponding numeric input data;
2. assign a positive non-zero value to the variable `nsprlas_ii` in the subroutine `LASINPT`, file 'f10_taskinpt.f', which must be equal to the total number of different numeric s-profiles that are to be read out from file 'input/lasspro.ii'; the actual number of numeric profiles used in the current job may be smaller than `nsprlas_ii`;
3. for every laser beam `iblas`, which is supposed to use a numeric spatial profile, assign `isprcaslas(iblas) = 0` (profile option # 0) and the corresponding value of `isprlas_ii(iblas) =` the sequential number of the numeric profile (from the total of `nsprlas_ii` numeric profiles read out from file) that is to be used with the beam `iblas`.

The input data file 'input/lasspro.ii' must have the same structure as the file 'input/lastpro.ii'. To synchronize the length units in the 'input/lasspro.ii' file and the RALEF code, the header before any numeric profile should (but must not) contain a comment line of the form

```
#col-1_unit  ul
```

where u_l is a number (recognized by the `G` format descriptor), giving the length unit (in cm) used in the current column. When specifying the units, the same rules, described in the previous subsection, should be observed. In particular, it is sufficient to specify the units for the first profile only to make them applicable to all the remaining profiles as well. If the length unit is not specified, it is assumed to be 1 cm.

Example of an 'input/lasspro.ii' file with two s-profiles where the length is measured in microns:

```
# Example of file with two s-profiles
#col-1_unit 1.d-4
#s-profile #1:
-125 1.18E-34
-120 0.003979966
0.e0 1.e0
125 1.18E-34

# s-profile #2:
-125 1.18E-34
-120 0.003979966
0.e0 5.e0

20.e0 3.e0
125.e0 0.e0
```

The power values p_s may be given in arbitrary units. Having been read out, they are normalized to the peak value of $p_{s,max} = 1$. Thus normalized powers are integrated in space

(already in the RALEF units) to provide the effective focal area value

$$\text{Sfefflas_ii}(\text{isprlas_ii}(\text{iblas})) = S_{lb,eff} = \begin{cases} \int_{-\infty}^{+\infty} p_s(s) ds, & \text{iradial} = 0, \\ \pi \int_0^{\infty} p_s(s) ds^2, & \text{iradial} = 1, 2, \end{cases}$$

which is needed to establish a relation between the peak laser intensity $F_{las00}(\text{iblas})$ and the total pulse energy of the laser beam iblas . In the rz -geometry (for $\text{iradial} = 1, 2$) only the values of $p_s(s)$ at $s \geq 0$ are used.

3.3. How to emulate a cylindrical laser beam with a conical one in the (r, z) -geometry

In the (r, z) -geometry cylindrical laser beams propagate strictly along the rotation axis Z , i.e. have $\Omega_{las,R} = 0$. As a matter of convenience, for such beams we use the flux density F_{00} itself (which has the dimensionality of $[\text{erg cm}^{-2} \text{s}^{-1}]$) to specify the incident laser intensity in subroutine `LASINPT` rather than the modified flux $\tilde{F}_{las} = RF_{las}$ with the dimensionality of $[\text{erg cm}^{-1} \text{s}^{-1}]$. The radial dependence of the laser beam intensity is prescribed by a dimensionless spatial-profile function $p_s(s)$, where s is the distance to laser beam axis.

In some cases it may be desirable to tilt such a cylindrical beam by a physically negligible angle (say, by an angle of the order of $\lesssim 10^{-8}$) with respect to the rotation axis — i.e. to emulate a physically cylindrical laser beam with a mathematically conical one. Indeed, this can be done by setting

$$\Omega_{las,Z} = \pm 1, \quad \Omega_{las,R} = -10^{-8}, -10^{-9}, \dots \quad (3.4)$$

(so that $|\Omega_{las,Z}^2 + \Omega_{las,R}^2 - 1| < 10^{-16}$) and by simultaneously modifying the radial profile $p_s(s)$ as

$$p_s(s) \rightarrow sp_s(s). \quad (3.5)$$

Then the original value of the incident flux F_{00} $[\text{erg cm}^{-2} \text{s}^{-1}]$ may be left unchanged.

4. HOW TO SET UP VARIOUS NUMERICAL DIAGNOSTICS AND DATA OUTPUT

4.1. Angular distribution of emission in selected spectral band

The selected spectral band

$$\text{k1nusel} \leq \text{kfreq} \leq \text{k2nusel} \quad (4.1)$$

is set up by specifying (in file ‘`in2d`’) the values of parameters `k1nusel` and `k2nusel` from the range $1 \leq \text{k1nusel} \leq \text{k2nusel} \leq \text{nfreqs}(1)$ for the main frequency set # 1; here `kfreq` is the index for frequency groups from this set.

To enact the printout of angular distribution of the thermal radiation, coming out of the simulated region in this spectral band, one has

1. to assign some positive value to the integer parameter `nnuseltet` ≥ 1 at Step 7 of subroutine `TASKINPT` in file ‘`f10_taskinpt.f`’, and

2. to set up the direction (`Omnustetx,Omnustety`) of the reference axis, with respect to which the diagnostic (polar) angle θ should be measured, at the same Step 7 of subroutine `TASKINPT`.

The angular distribution of the emitted radiation, integrated over the full physical boundary of the simulated region, is written into the file ‘`enusoutet.dat`’ at times, specified by the `twfilm` array.

Once it is greater than zero, the specific value of `nnuseltet`, assigned by the user, is not relevant: it is always adjusted to the current order `kradSn` of the S_n quadrature,

$$\text{nnuseltet} = \begin{cases} 4 \times \text{kradSn}, & \text{iradial} = 0, \\ 2 \times \text{kradSn}, & \text{iradial} = 1, 2. \end{cases} \quad (4.2)$$

The value of `nnuseltet` will be automatically changed at code restart whenever the value of `kradSn` is changed. In such a case accumulation of the emitted energy in the array `enuseltet(:)` is interrupted, and the subsequent dump files ‘`dp2dxxxx`’ become incompatible with the previous ones.

The user-provided components (`Omnustetx,Omnustety`) of the vector along the reference axis must not necessarily be normalized: the normalization is automatically done by the code. For `iradial` = 1,2 the reference axis must be colinear with the rotation axis; in this case the emitted radiation is collected from the entire 4π of the solid angle, i.e. with participation of all the S_n directions. For `iradial` = 0 the reference axis must be colinear either with the x -axis or with the y -axis; in this case the emitted radiation is collected only from the equatorial (with respect to the z -axis) band of the S_n directions.

5. GRID NOTATION

5.1. General quantities

The entire computational mesh is assumed to be comprised of `nblks` ≥ 1 individual *blocks*. Each block `iblk`=1,2, ... has a topologically rectangular mesh with `n1(iblk)` *physical cells* along mesh direction 1, and `n2(iblk)` *physical cells* along mesh direction 2; see Fig. 5.1. Clearly, each block `iblk` has `n1(iblk) + 1` *physical vertices* along mesh direction 1, and `n2(iblk) + 1` *physical vertices* along mesh direction 2. To these are added 4 belts of *ghost vertices* and *ghost cells* along the 4 edges of each rectangular block. Together with the ghost cells and vertices, a block `iblk` requires a memory of

$$\text{msz}(\text{iblk}) = [\text{n1}(\text{iblk}) + 3][\text{n2}(\text{iblk}) + 3] \quad (5.1)$$

locations for any scalar quantity.

The global index for a scalar quantity in block `iblk` is

$$I = j \cdot [\text{n1}(\text{iblk}) + 3] + i + 1 + \text{mob}(\text{iblk}), \quad (5.2)$$

where $i = 0, 1, \dots, \text{n1}(\text{iblk}) + 2$ is the local index along mesh direction 1, and $j = 0, 1, \dots, \text{n2}(\text{iblk}) + 2$ is the local index along mesh direction 2 for a cell (i, j) in block `iblk`,

$$\text{mob}(\text{iblk}) = \sum_{k=1}^{\text{iblk}-1} \text{msz}(k). \quad (5.3)$$

The global indices for the first and second components of a two-component vector quantity are

$$I_x = j \cdot [n1(iblk) + 3] + i + 1 + 2 \cdot mob(iblk) = I + mob(iblk), \quad (5.4)$$

$$I_y = I_x + msz(iblk). \quad (5.5)$$

If the global index I for a scalar quantity is known, then the local indices i and j can be restored as

$$j = [I - mob(iblk) - 1] / [n1(iblk) + 3], \quad (5.6)$$

$$i = I - mob(iblk) - 1 - j \cdot [n1(iblk) + 3]. \quad (5.7)$$

The total number `msoccp` of memory locations required by a scalar quantity is given by

$$msoccp = mob(nblks) + msz(nblks), \quad (5.8)$$

where `nblks` is the total number of blocks.

In block-local subroutines, a block-local single index

$$i = j \cdot [n1(iblk) + 3] + i + 1 = I - mob(iblk) \quad (5.9)$$

is used to order scalar quantities. Analogously, the block-local indices for two components of a vector field are

$$i_x = j \cdot [n1(iblk) + 3] + i + 1 = i, \quad (5.10)$$

$$i_y = i_x + msz(iblk). \quad (5.11)$$

Each block has 4 edges `ib=1,2,3,4` and 4 corners `ic=1,2,3,4`. The numbering convention for the block edges `ib` is shown in Fig. 5.1: if index i runs from left to right, and index j runs from bottom to top, then `ib=1` is the bottom edge, `ib=2` is the top edge, `ib=3` is the left edge, and `ib=4` is the right edge of the block. The numbering convention for the block corners `ic` is as follows: `ic=ib`, where `ib` is the number of the bottom edge after the block is rotated such that the corner `ic` becomes a lower-left corner of the block; see Fig. 5.2.

Notice that, while we have a single row of ghost vertices along each edge of a block, we have one row of ghost cells along edges `ib = 1` and `3`, and two rows of ghost cells along edges `ib = 2` and `4`; see Fig. 5.1. Hence, in certain cases we have to distinguish between the *ghost cells of the 1-st row* and the *ghost cells of the 2-nd row*.

5.2. Offsets for neighbors of a vertex and/or a cell

5.3. Offsets for do-loops along block edges

To perform do-loops along block edges, arrays `i1bc(ib,iblk)`, `i2bc(ib,iblk)`, and `i3bc(ib,iblk)` are defined as

$$\begin{aligned} i1bc(1,iblk) &= 1, \\ i1bc(2,iblk) &= [n1(iblk) + 3] [n2(iblk) + 1] + 1, \\ i1bc(3,iblk) &= 1, \\ i1bc(4,iblk) &= n1(iblk) + 2, \end{aligned} \quad (5.12)$$

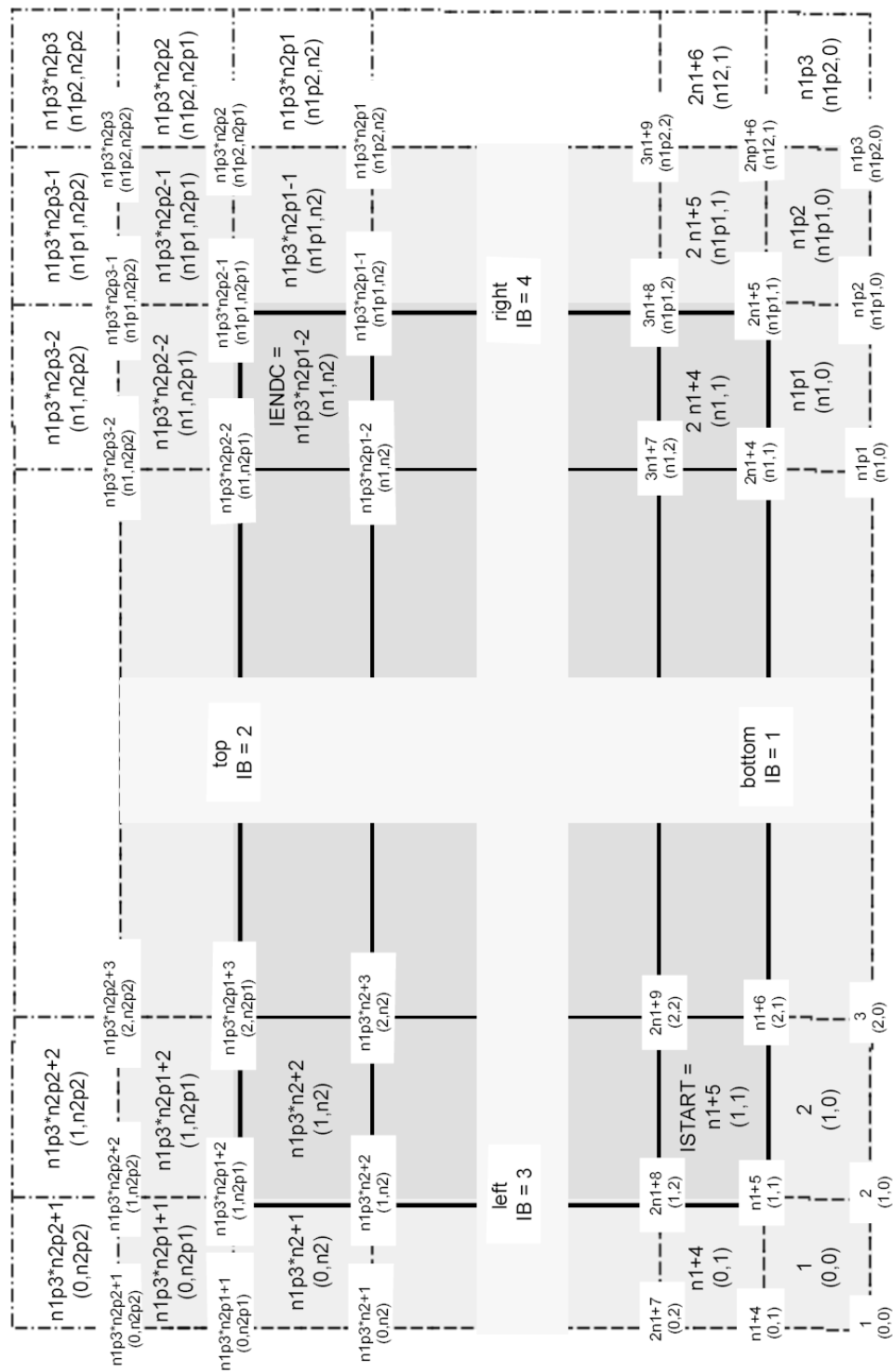


FIG. 5.1: Mesh indices and parameters inside a single block.

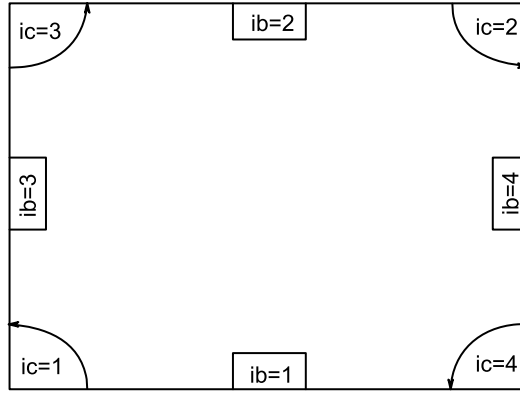


FIG. 5.2: Edge and corner numbering in a block.

$$\begin{aligned}
 i2bc(1, iblk) &= n1(iblk) + 3, \\
 i2bc(2, iblk) &= [n1(iblk) + 3] [n2(iblk) + 2], \\
 i2bc(3, iblk) &= [n1(iblk) + 3] [n2(iblk) + 2] + 1, \\
 i2bc(4, iblk) &= msz(iblk) - 1,
 \end{aligned} \tag{5.13}$$

$$\begin{aligned}
 i3bc(1, iblk) &= 1, \\
 i3bc(2, iblk) &= 1, \\
 i3bc(3, iblk) &= n1(iblk) + 3, \\
 i3bc(4, iblk) &= n1(iblk) + 3.
 \end{aligned} \tag{5.14}$$

Here $i1bc(ib, iblk)$ is the block-local single index of the *first* ghost cell along edge ib of block $iblk$; $i2bc(ib, iblk)$ is the block-local single index of the *last* ghost cell along edge ib of block $iblk$; $i3bc(ib, iblk)$ is the single-index *stride* along edge ib of block $iblk$; see Fig. 5.3. Then, a do-loop

```

do i=i1bc(ib, iblk), i2bc(ib, iblk), i3bc(ib, iblk)
...
enddo

```

will sweep over all ghost cells along edge ib of block $iblk$.

To perform do-loops over physical cells along block edges, an array

$$\begin{aligned}
 idelbc(1, iblk) &= n1(iblk) + 3, \\
 idelbc(2, iblk) &= -n1(iblk) - 3, \\
 idelbc(3, iblk) &= 1, \\
 idelbc(4, iblk) &= -1.
 \end{aligned} \tag{5.15}$$

of offsets towards physical domain is defined; see Fig. 5.3. Thus, a do-loop

```

i1=i1bc(ib, iblk)+i3bc(ib, iblk)+idelbc(ib, iblk)
i2=i2bc(ib, iblk)-2*i3bc(ib, iblk)+idelbc(ib, iblk)
do i=i1, i2, i3bc(ib, iblk)
...
enddo

```

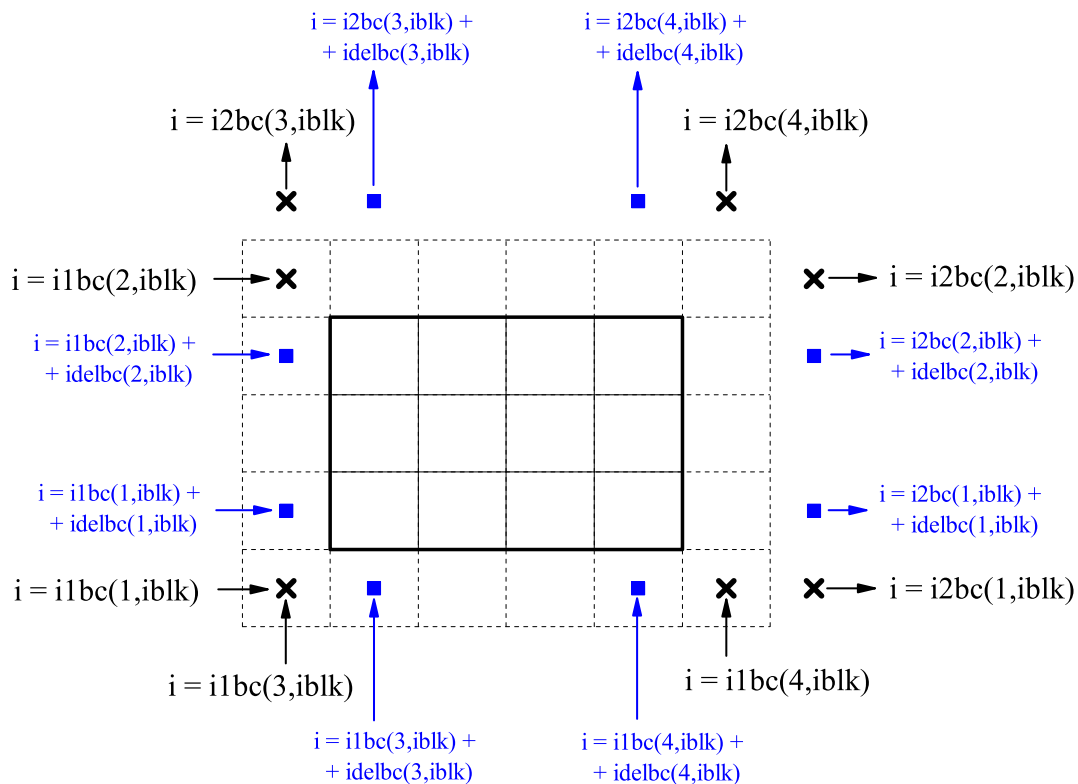


FIG. 5.3: Offsets for do-loops along block edges.

sweeps over all physical cells along edge ib of block $iblk$.

Do-loops over edge vertices and boundary cell faces can be arranged by using the same arrays. For example, a double do-loop over all boundary cell faces (or physical vertices) of block $iblk$ may be programmed as

```

do ib=1,4
    i1=i1bc(ib,iblk)+i3bc(ib,iblk)+mod(ib,2)*idelbc(ib,iblk)
    i2=i2bc(ib,iblk)-2*i3bc(ib,iblk)+mod(ib,2)*idelbc(ib,iblk)
    do i=i1,i2,i3bc(ib,iblk)
        ...
    enddo
enddo

```

6. INTERBLOCK COMMUNICATION

6.1. General quantities

To provide control over edge orientation, the following 4-value arrays are permanently defined as

$$\mathbf{ia}(\mathbf{ib}) = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \end{pmatrix}, \quad \mathbf{iend}(\mathbf{ib}) = \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \quad \mathbf{ileft}(\mathbf{ib}) = \begin{pmatrix} 3 \\ 4 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{irght}(\mathbf{ib}) = \begin{pmatrix} 4 \\ 3 \\ 1 \\ 2 \end{pmatrix}. \quad (6.1)$$

The value of $\mathbf{ia}(\mathbf{ib})$ gives mesh direction along edge \mathbf{ib} ; $\mathbf{ileft}(\mathbf{ib})$ gives the neighboring edge number on the left side of edge \mathbf{ib} (oriented such that the block body lies above edge \mathbf{ib}); $\mathbf{irght}(\mathbf{ib})$ gives the neighboring edge number on the right side of edge \mathbf{ib} ; $\mathbf{iend}(\mathbf{ib})$ gives the direction to the primary (left) corner of edge \mathbf{ib} . Arrays $\mathbf{ileft}(\mathbf{ib})$ and $\mathbf{irght}(\mathbf{ib})$ have also zeroth components $\mathbf{ileft}(0)=1$ and $\mathbf{irght}(0)=1$.

If a block \mathbf{iblk} locks onto block \mathbf{jblk} along its edge \mathbf{ib} , which physically coincides with the edge \mathbf{jb} of block \mathbf{jblk} , then we have

$$\mathbf{jblk} = \mathbf{nbc}(1, \mathbf{ib}, \mathbf{iblk}), \quad \mathbf{jb} = \mathbf{nbc}(2, \mathbf{ib}, \mathbf{iblk}) \quad (6.2)$$

$$\mathbf{nedg}(m, \mathbf{iblk}) = \begin{cases} \mathbf{n1}(\mathbf{iblk}) + 3 \equiv \mathbf{n1p3}, & m = 1, \\ \mathbf{n2}(\mathbf{iblk}) + 3 \equiv \mathbf{n2p3}, & m = 2, \end{cases} \quad (6.3)$$

$$\mathbf{nedgf}(m, \mathbf{iblk}) = \mathbf{nedg}(m, \mathbf{iblk}) - 2. \quad (6.4)$$

Block communication is accomplished by using predefined strides and offsets for the global index

$$\mathbf{I} = \mathbf{mob}(\mathbf{iblk}) + j \cdot [\mathbf{n1}(\mathbf{iblk}) + 3] + i + 1, \quad \begin{cases} i = 0, 1, \dots, \mathbf{n1p2}, \\ j = 0, 1, \dots, \mathbf{n2p2}. \end{cases} \quad (6.5)$$

The acceptor block \mathbf{IBLK} receives information for its ghost cells along edge \mathbf{IB} from the donor block \mathbf{JBLK} along its matching edge \mathbf{JB} . Stride along edge \mathbf{IB} of the acceptor block \mathbf{IBLK} is given by

$$\mathbf{ms1}(\mathbf{ib}, \mathbf{iblk}) = \mathbf{i3bc}(\mathbf{ib}, \mathbf{iblk}) = \begin{pmatrix} 1 \\ 1 \\ \mathbf{n1p3} \\ \mathbf{n1p3} \end{pmatrix}. \quad (6.6)$$

Stride along edge \mathbf{JB} of the donor block \mathbf{JBLK} is given by

$$\mathbf{ls1}(\mathbf{ib}, \mathbf{iblk}) = \mathbf{io} \cdot \mathbf{i3bc}(\mathbf{jb}, \mathbf{jblk}), \quad (6.7)$$

where $\mathbf{IO} = +1$ for mesh-parallel block contact (like edge $\mathbf{IB}=1$ to edge $\mathbf{JB}=2$), and $\mathbf{IO} = -1$ for mesh-anti-parallel block contact (like edge $\mathbf{IB}=1$ to edge $\mathbf{JB}=1$).

Global offsets for the acceptor block \mathbf{IBLK} are given by

$$\mathbf{moc}(\mathbf{ib}, \mathbf{iblk}) = \mathbf{mob}(\mathbf{iblk}) + \begin{pmatrix} 0 \\ \mathbf{n2p1} \cdot \mathbf{n1p3} \\ -\mathbf{n1p2} \\ -1 \end{pmatrix}, \quad (6.8)$$

Interblock communication: global indices for cell centers along touching boundaries

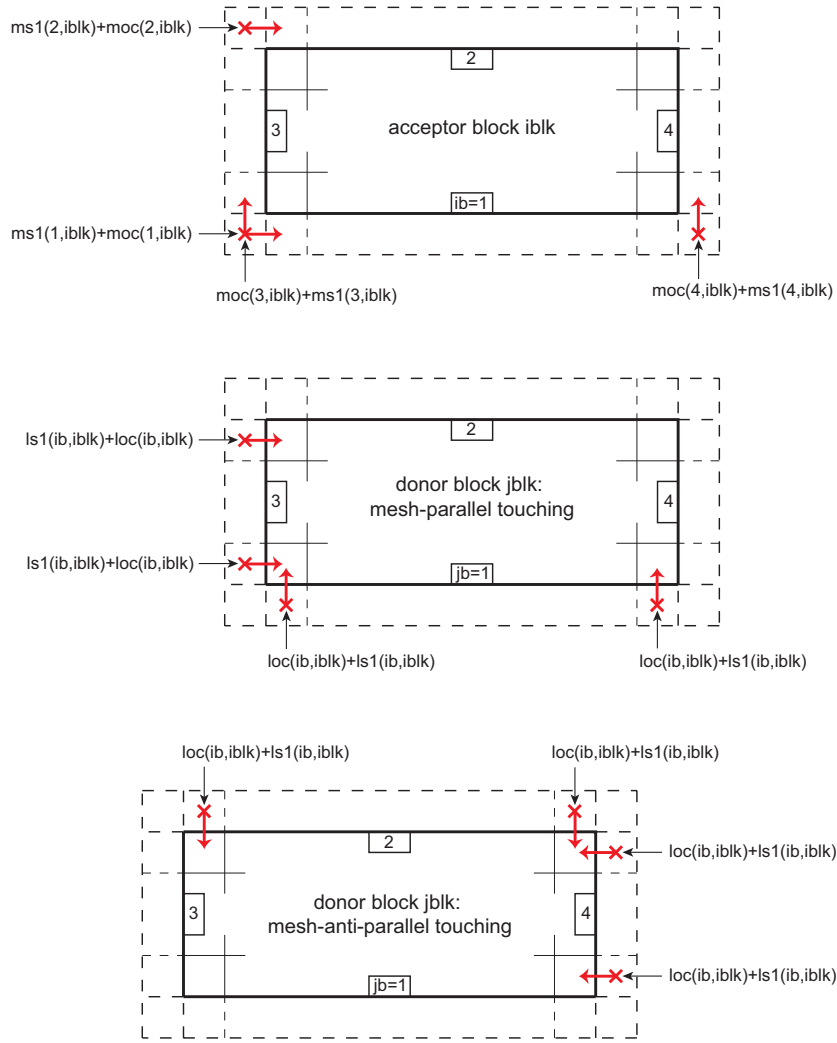


FIG. 6.1: Global indices for cell-centered quantities along communicating block boundaries.

$$mov(ib, iblk) = mob(iblk) + \begin{pmatrix} 0 \\ n2p2 \cdot n1p3 \\ -n1p2 \\ 0 \end{pmatrix}, \quad (6.9)$$

$$moe(ib, iblk) = mob(iblk) + \begin{pmatrix} n1p3 \\ n2p1 \cdot n1p3 \\ -n1p1 \\ -1 \end{pmatrix}, \quad (6.10)$$

$$mof(ib, iblk) = mob(iblk) + \begin{pmatrix} 1 + msz(iblk) \\ 1 + n2p1 \cdot n1p3 + msz(iblk) \\ 1 \\ n1p2 \end{pmatrix}. \quad (6.11)$$

MOC(IB,IBLK)+MS1(IB,IBLK) is the first cell center to accept information along edge IB (see Fig. 6.1). Analogously, MOV(IB,IBLK)+MS1(IB,IBLK) is the first vertex to accept information along edge IB (see Fig. 6.2), MOE(IB,IBLK)+MS1(IB,IBLK) is the first vertex to accept information along nearest sequence parallel to edge IB (see Fig. 6.3), MOF(IB,IBLK)+MS1(IB,IBLK)+ MOB(IBLK) is the first face, perpendicular to edge IB, which is to accept information along edge IB (see Fig. ??). If one is interested in physical vertices only, then MOE(IB,IBLK)+2*MS1(IB,IBLK) is the first physical vertex along edge IB of block IBLK; respectively, MOE(IB,IBLK)+[NEDG(IA(IB)),IBLK]-1]*MS1(IB,IBLK) is the last physical vertex along edge IB of block IBLK. All the above m -quantities are defined for all boundaries of all blocks, irrespective of whether they are, or are not, in contact with other blocks.

Strides and global offsets for donor blocks with mesh-parallel contact are

$$ls1(ib, iblk) = i3bc(jb, jblk) = \begin{cases} 1, & jb = 1, \\ 1, & jb = 2, \\ n1(jblk) + 3, & jb = 3, \\ n1(jblk) + 3, & jb = 4, \end{cases} \quad (6.12)$$

$$loc(ib, iblk) = mob(jblk) + \begin{cases} n1p3, & jb = 1, \\ n2 \cdot n1p3, & jb = 2, \\ -n1p1, & jb = 3, \\ -2, & jb = 4, \end{cases} \quad (6.13)$$

$$lov(ib, iblk) = mob(jblk) + \begin{cases} 2 \cdot n1p3, & jb = 1, \\ n2 \cdot n1p3, & jb = 2, \\ -n1, & jb = 3, \\ -2, & jb = 4, \end{cases} \quad (6.14)$$

$$loe(ib, iblk) = mob(jblk) + \begin{cases} n1p3, & jb = 1, \\ n2p1 \cdot n1p3, & jb = 2, \\ -n1p1, & jb = 3, \\ -1, & jb = 4, \end{cases} \quad (6.15)$$

For l -quantities the values $n1p3 = n1(jblk) + 3$, $n2p2 = n2(jblk) + 2$ and similar are calculated for the donor block JBLK; the columns of 4 values are ordered after donor edges JB =1,2,3,4.

Strides and global offsets for donor blocks with mesh-anti-parallel contact are given by

$$ls1(ib, iblk) = -i3bc(jb, jblk) = \begin{cases} -1, & jb = 1, \\ -1, & jb = 2, \\ -n1p3, & jb = 3, \\ -n1p3, & jb = 4, \end{cases} \quad (6.16)$$

$$loc(ib, iblk) = mob(jblk) + \begin{cases} 2 \cdot n1p3, & jb = 1, \\ n2p1 \cdot n1p3, & jb = 2, \\ n2p2 \cdot n1p3 + 2, & jb = 3, \\ n2p3 \cdot n1p3 - 2, & jb = 4, \end{cases} \quad (6.17)$$

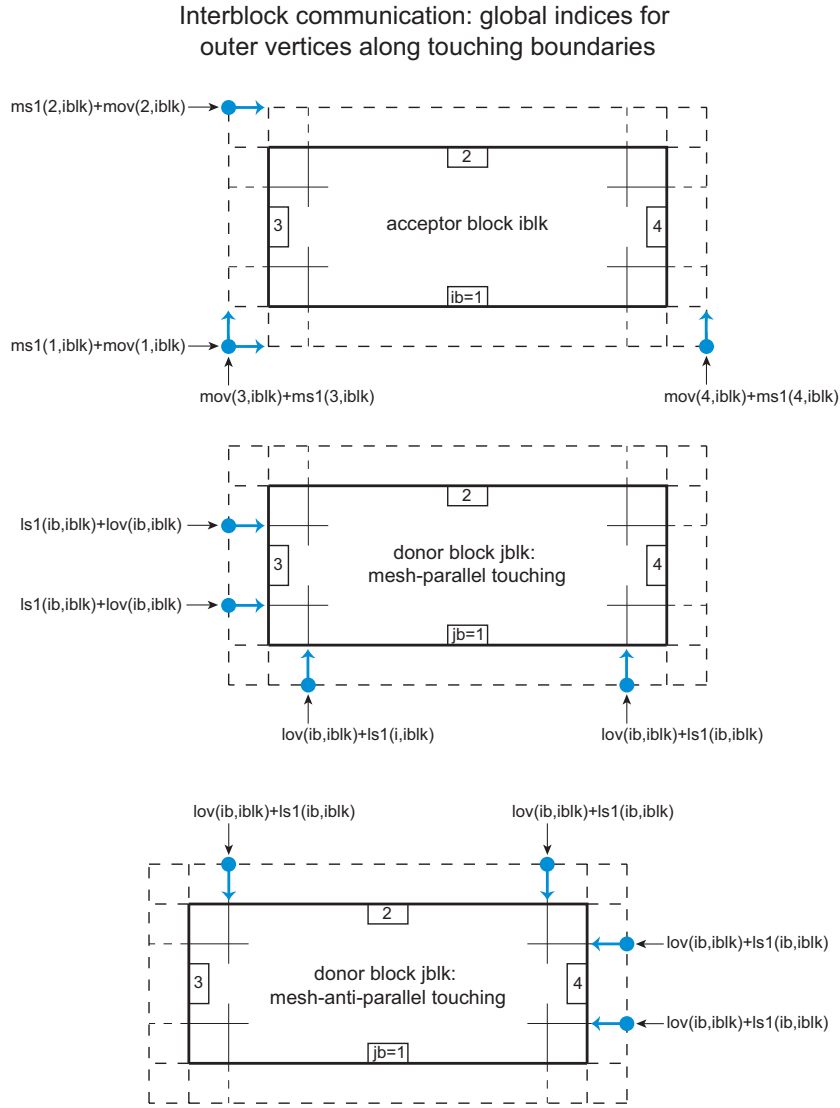


FIG. 6.2: Global indices for outer (along ghost edges) vertices along communicating block boundaries.

$$lov(ib, iblk) = mob(jblk) + \begin{cases} 3 \cdot n1p3 + 1, & jb = 1, \\ n2p1 \cdot n1p3 + 1, & jb = 2, \\ n2p3 \cdot n1p3 + 3, & jb = 3, \\ n2p4 \cdot n1p3 - 2, & jb = 4, \end{cases} \quad (6.18)$$

$$loe(ib, iblk) = mob(jblk) + \begin{cases} 2 \cdot n1p3 + 1, & jb = 1, \\ n2p2 \cdot n1p3 + 1, & jb = 2, \\ n2p3 \cdot n1p3 + 2, & jb = 3, \\ n2p4 \cdot n1p3 - 1, & jb = 4, \end{cases} \quad (6.19)$$

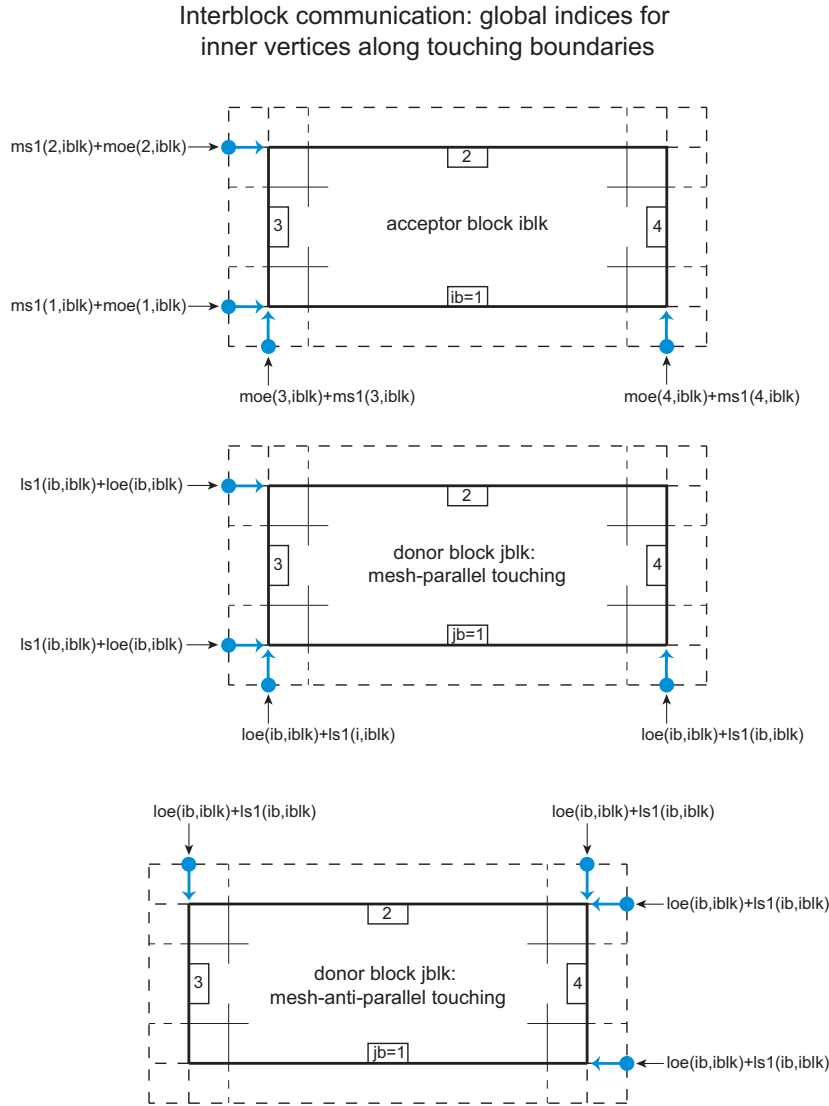


FIG. 6.3: Global indices for inner (along physical edges) vertices along communicating block boundaries.

6.2. 4-block meeting point

If a corner IB in a block IBLK is a 4-block meeting point (a 4-bk point, no void is allowed!), the flag $I4BK(IB, IBLK)$ is set equal to 1. In this case the vertex coordinates and physical parameters of all ghost cells around the 4-bk point are uniquely defined by the corresponding physical cells in corresponding donor blocks.

Regular boundary-to-boundary communication procedure: correct values are assigned to all acceptor ghost cell parameters except for (i) the ghost-corner vertex $MAV(IB, IBLK)$, and (ii) the ghost-corner cell center $MAC(IB, IBLK)$ (see Fig. 6.4) — which get some spurious values.

Special action: (i) the x, y coordinates of the ghost-corner $MAV(IB, IBLK)$ in the acceptor block IBLK are set equal to the x, y coordinates of the physical vertex $MDV(IB, IBLK)$ in the donor block $IDONR(IB, IBLK)$; (ii) the cell-centered values of ghost-corner cell $MAC(IB, IBLK)$

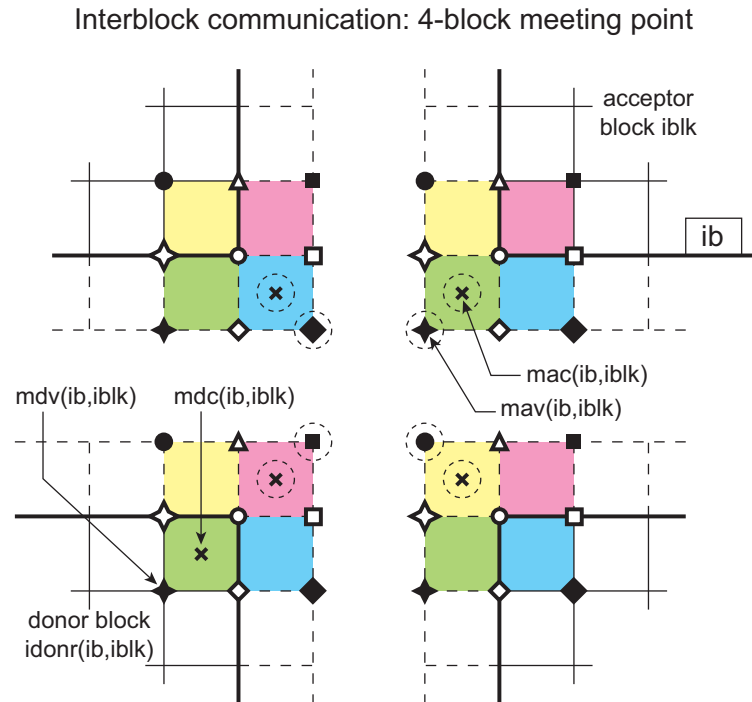


FIG. 6.4: Block communication at a 4-block meeting point. Physically identical vertices are marked with the same symbols. Vertices and cell centers, which require special action, are dash-encircled.

in the acceptor block IBLK are set equal to the cell-centered values in the physical cell $MDC(IB, IBLK)$ of the donor block $IDONR(IB, IBLK)$; see Fig. 6.4.

As a result, the physical corner vertex at a 4-bk point “sees” identical patterns around itself from any of the 4 contacting blocks.

6.3. 3-block meeting point

If a corner IB in a block IBLK is a 3-block meeting point with no void left (a 3-bk point), the flag $I3BK(IB, IBLK)$ is set equal to 1 (this is done for contacting corners in each of the 3 contacting blocks). In this case the vertex coordinates and physical parameters of all ghost cells around the 3-bk point — with the exception of ghost-corner cells — are also uniquely defined by the corresponding physical cells in corresponding donor blocks. The ghost-corner cell of each contacting block degenerates into a single cell edge (i.e. has zero volume), along which the physical-corner cells of two other blocks contact one another.

Regular boundary-to-boundary communication procedure: correct values are assigned to all acceptor ghost cell parameters except for (i) the ghost-corner vertex $MAV(IB, IBLK)$, and (ii) the ghost-corner cell center $MAC(IB, IBLK)$ (see Fig. 6.5) — which get some spurious values.

Special action: (i) the x, y coordinates of the ghost-corner $MAV(IB, IBLK)$ are set identical to those of one of the two neighboring ghost vertices (which coincide in the physical space!), and (ii) the cell-centered values in the ghost-corner cell $MAC(IB, IBLK)$ are determined by interpolation between the cell-centered values of the two neighboring cells.

As a result, in the 4-surrounding-cells picture, the physical 3-bk corner vertex sees different patterns around itself from the 3 different contacting blocks. It sees identical

Interblock communication: 3-block meeting point

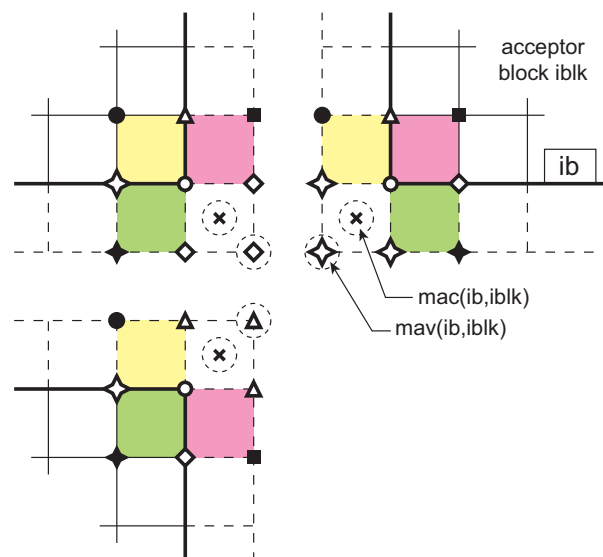


FIG. 6.5: Block communication at a 3-block meeting point. Physically identical vertices are marked with the same symbols. Vertices and cell centers, which require special action, are dash-encircled.

patterns only when the degenerate ghost-corner cell is ignored in each of the 3 contacting blocks.

Interblock communication: 3-block-void meeting point

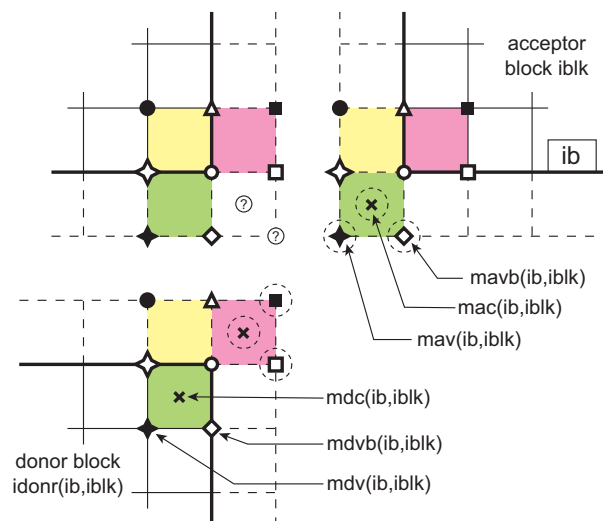


FIG. 6.6: Block communication at a 3-block-void meeting point. Physically identical vertices are marked with the same symbols. Vertices and cell centers, which require special action, are dash-encircled.

6.4. 3-block-void meeting point

When 3 blocks meet in an L-shape configuration and a void is left (a 3-vd meeting point), one block has a central position and a “donor-donor” contact; the other two meeting blocks, which have a “void-donor” type of contact, are diagonally opposite to one another (similar to the 4-bk meeting point); see Fig. 6.6. In such a case the flag $I3VD(IC, IBLK)$ is set equal to 1 in each of the two “void-donor” blocks (but not in the central “donor-donor” block!) for a corresponding corner IC at the 3-vd meeting point. To mark the corresponding 3-vd corner IC in the central “donor-donor” block $IBLK$, another special flag $I3DD(IC, IBLK)$ is introduced (in the CAVEAT-TR version) and set equal to 1 for that corner.

Regular boundary-to-boundary communication procedure: correct values are assigned to the acceptor ghost cell parameters except for (i) the ghost-corner vertex $MAV(IB, IBLK)$, and (ii) the ghost-corner cell center $MAC(IB, IBLK)$ (see Fig. 6.6) — which may (or may not, depends on the order of assignment sequence) get some spurious values.

Special action is taken only for the ghost-corner cells of the 2 “void-donor” blocks:

1. the x, y coordinates of the ghost vertex $mav(ib, iblk)$ in the acceptor block $iblk$ are set equal to the x, y coordinates of the physical vertex $mdv(ib, iblk)$ in the donor block $idonr(ib, iblk)$;
2. the cell-centered values of the ghost-corner cell $mac(ib, iblk)$ in the acceptor block $iblk$ are set equal to the cell-centered values in the physical cell $mdc(ib, iblk)$ of the donor block $idonr(ib, iblk)$;
3. in addition, the x, y coordinates of the ghost vertex $mavb(ib, iblk)$ in the acceptor block $iblk$ are set equal to the x, y coordinates of the physical vertex $mdvb(ib, iblk)$ in the donor block $idonr(ib, iblk)$ (see fig. 6.6); to ensure the correct sign of the ghost-face normal velocity u^* (at the face connecting the corner ib with the ghost vertex $mavb(ib, iblk)$), the value of $nsig3(ib, iblk)$ is set equal either to +1 (the unit normals $fn(i, m)$ of communicating faces have the same direction in space), or to -1 (the corresponding unit normals have opposite directions in space).

As a result, in the 4-surrounding-cells picture, the physical 3-vd corner vertex sees different patterns around itself from the 3 different contacting blocks. It sees identical patterns only when the ghost-corner cell is ignored in the central “donor-donor” block, and corresponding corner-neighboring ghost cells are ignored in each of the 2 “void-donor” blocks.

Important: a mesh with a 3-vd meeting point on a reflective boundary is not allowed! Such a mesh would lead to two contradictory prescriptions for calculating the coordinates of the ghost vertex $MAVB(IB, IBLK)$.

Important: since no special action is taken for the central “donor-donor” block, the coordinates of its $MAV(IB, IBLK)$ ghost vertex, and the cell-centered quantities in the $MAC(IB, IBLK)$ ghost cell retain their spurious values!

6.5. Changes in RALEF-2D

1. An additional flag $i3dd(ib, iblk)$ is introduced, which is set equal to 1 for the contacting corner ib of the “donor-donor” central block $iblk$ in a 3-block-void-meeting situation.
2. The values of $mav(ib, iblk)$ and $mac(ib, iblk)$ are calculated for all blocks in the problem — similar to $ms1(ib, iblk)$, $mov(ib, iblk)$, $moc(ib, iblk)$, ...

7. FLAG ARRAY IFLG(I)

7.1. Flags and masks

Certain key information on individual cells and vertices is contained in a compact form in a mesh-wide integer-4 array `iflg(I)`. For every vertex (cell) `I`, the integer value of `iflg(I)` is composed of a set of *flags*; each flag occupies only several bits of `iflg(I)`. The value of a flag `xxx` can be retrieved by applying the logical `iand` operator to `iflg(I)` with a corresponding mask `mskxxx`

$$xxx = iand(iflg(I), mskxxx). \quad (7.1)$$

Evidently, any single-bit flag can have a value of either 0 or 2^{k-1} , where k is the sequential number of the corresponding bit. Table I lists the flags and their masks used in the RALEF code.

TABLE I: Masks for reading the `iflg(I)` array.

mask	bits	value	description; allocation
<code>mskmat</code>	1-6	$2^6 - 1$	Yields material number; cell-centered.
<code>mskgst</code>	7	2^6	Is 'on' in ghost cells; cell-centered.
<code>msklvx</code>	8	2^7	Indicates a strictly Lagrangian vertex that cannot be rezoned; vertex-centered. Note that <code>msklvx</code> and <code>mskivx</code> are never both 'on'.
<code>msklf1</code>	9	2^8	Indicates a no-fluxing cell face along mesh direction 1; face-centered.
<code>msklf2</code>	10	2^9	Indicates a no-fluxing cell face along mesh direction 2; face-centered.
<code>mskivx</code>	11	2^{10}	Indicates a vertex on a Lagrangian (material) interface that can be tangentially rezoned; vertex-centered.
<code>msktr1</code>	12	2^{11}	Indicates that a vertex can be tangentially rezoned along mesh direction 1; vertex-centered.
<code>msktr2</code>	13	2^{12}	Indicates that a vertex can be tangentially rezoned along mesh direction 2; vertex-centered.
<code>mskfix</code>	14	2^{13}	Indicates that a vertex remains fixed in space; vertex-centered.
<code>mskrev</code>	15	2^{14}	Mask for Sesame EOS
<code>mskheb</code>	16	2^{15}	Mask for rezone inhibition on a burn front.
<code>mskmx</code>	17	2^{16}	Indicates that a cell has a mixed EOS; cell-centered.
<code>mskbnd</code>	18-20	7×2^{17}	Yields edge number of a 1st-row ghost cell; cell-centered.
<code>mskgcc</code>	21	2^{20}	Indicates that a cell is a corner ghost cell; cell-centered.
<code>mskbov</code>	22	2^{21}	Indicates that a vertex is a physical vertex along a block boundary; vertex-centered.
<code>mskpag</code>	23	2^{22}	Indicates that a cell is a Physically Associated Ghost Cell (PAGC); cell-centered.
<code>mskcnv</code>	24	2^{23}	Indicates that a vertex is a physical block-corner vertex; vertex-centered.

In more detail, the rules for setting the key flags in the `iflg(I)` array are as follows:

- flag `mat`, mask `mskmat`:
 - flag `mat` is cell-centered, significant in both physical and ghost cells;
 - is set `=matnum` in all physical cells;

- is set `=matnum` in all non-corner ghost cells along the boundaries with `ibc=3` (outflow) and `ibc=4` (inflow);
- is set `=matnum` of the corresponding donor cell in all ghost cells (corner + non-corner) of the 1st row along the interblock boundaries with `ibc=5`; for the ghost corner cell with `i3bk=1` the donor is the neighboring physical corner cell of the same block;
- flag `gst`, mask `mskgst`:
 - flag `gst` is cell-centered, serves to distinguish between physical and ghost cells; significant in both physical and ghost cells;
 - is set `=1` in all ghost cells (both 1st and 2nd rows);
- flag `bnd`, mask `mskbnd`:
 - flag `bnd` is cell-centered, significant in ghost cells only;
 - is set `=ib` in all ghost cells (1st + 2nd rows) along block edge `ib`; in the corner cells it takes the values of 3 or 4;
- flag `fix`, mask `mskfix`:
 - flag `fix` is vertex-centered, marks mesh vertices that must remain fixed in space (unless the entire mesh is translated in space); significant at physical vertices only;
 - is set `=1` for all physical block corners at intersection of fixed in space boundaries — i.e. boundaries with `ibc = 1, 3, 4` or 8;
 - is set `=1` at all physical vertices along fixed in space boundaries with `ibc = 1, 3, 4` or 8, for which tangential rezoning was forbidden by setting `ifnotr12bc(ib,iblk)=.true.;`
- flag `lvx`, mask `msklvx`:
 - flag `lvx` is vertex-centered, marks strictly Lagrangian mesh vertices that move with the fluid and are not subject to rezoning; significant at physical vertices only;
 - is set `=1` at all physical block corners that do not lie on any of the interblock boundaries with `ibc=5`, or are 3-*vd* or 3-*dd* block meeting points, and do not have flag `fix` set on;
 - is set `=1` at all physical vertices along a physical edge with `ibc=0` (a center-of-convergence type of boundary);
 - is set `=1` at all vertices (physical + ghost) where two rezoning directions are simultaneously indicated with the flags `tr1` and `tr2`.
 - is set `=1` at all physical vertices along moving boundaries with `ibc = 2, 6` or 9, for which tangential rezoning was forbidden by setting `ifnotr12bc(ib,iblk)=.true.;`
- flag `ivx`, mask `mskivx`:
 - flag `ivx` is vertex-centered, marks physical vertices on Lagrangian (material) interfaces (as well as on the inflow/outflow boundaries!) that can only be tangentially rezoned along these interfaces; significant at physical vertices only; for any single vertex only one of the three flags `fix`, `lvx` or `ivx` is allowed to be set on;

- is set =1 at all physical vertices that are not marked as strictly Lagrangian with $lvx=1$ but have either $tr1=1$ or $tr2=1$.
- flag $lf1$, mask $msklf1$:
 - flag $lf1$ is face-centered, marks cell faces along mesh direction 1 across which no fluxing of material is allowed; significant on physical faces only;
 - is set =1 on all PR-faces (i.e. physical cell faces + those that protrude into the belt of ghost cells) along mesh direction $m = 1$ that separate cells with different materials;
 - is set =1 on all physical faces along a center-of-convergence boundary (i.e. one with $ibc=0$) along mesh direction $m = 1$.
- flag $lf2$, mask $msklf2$:
 - same as flag $lf1$ but for mesh direction 2.
- flag $tr1$, mask $msktr1$:
 - flag $tr1$ is vertex-centered, marks vertices that can only be tangentially rezoned along mesh direction 1; significant at physical vertices only;
 - is set =1 at both ends of all PR-faces along mesh direction $m = 1$ that have $lf1=1$;
 - is set =1 at all physical vertices on an inflow/outflow [i.e. with $(ibc=3.or.ibc=4)$] boundary along mesh direction $m = 1$.
- flag $tr2$, mask $msktr2$:
 - same as flag $tr1$ but for mesh direction 2.

Note that it is important for all the ghost cells across “Lagrangian” boundaries (i.e. with $ibc = 1, 2, 6, 8$ or 9) to have the “unphysical” (“vacuum”) material number $matnum = 0$. As a consequence of the above setting rules, **every physical vertex along a block edge, which does not lie on an interblock boundary with $ibc=5$, is marked with either $fix=1$, or $lvx=1$, or $ivx=1$; 3-*vd* and 3-*dd* block meeting corners are marked with either $fix=1$ or $lvx=1$.** Figures 7.1 and 7.2 give two examples of flag setting on a single-block and a three-block meshes.

7.2. Layout of the subroutine FLAGSET

Step 1:

1. Initialize all flags to zero, $iflg=0$.
2. Set flag $mat=matnum$ in all physical cells, part by part.
3. Set flags $gst=1$ and $bnd=ib$ in all ghost cells (1st + 2nd rows).
4. Set flag $mat=matnum$ in all ghost cells (1st + 2nd rows) along the inflow/outflow boundaries with $(ibc=3.or.ibc=4)$ by copying from the neighboring cells of the same block.

Step 2:

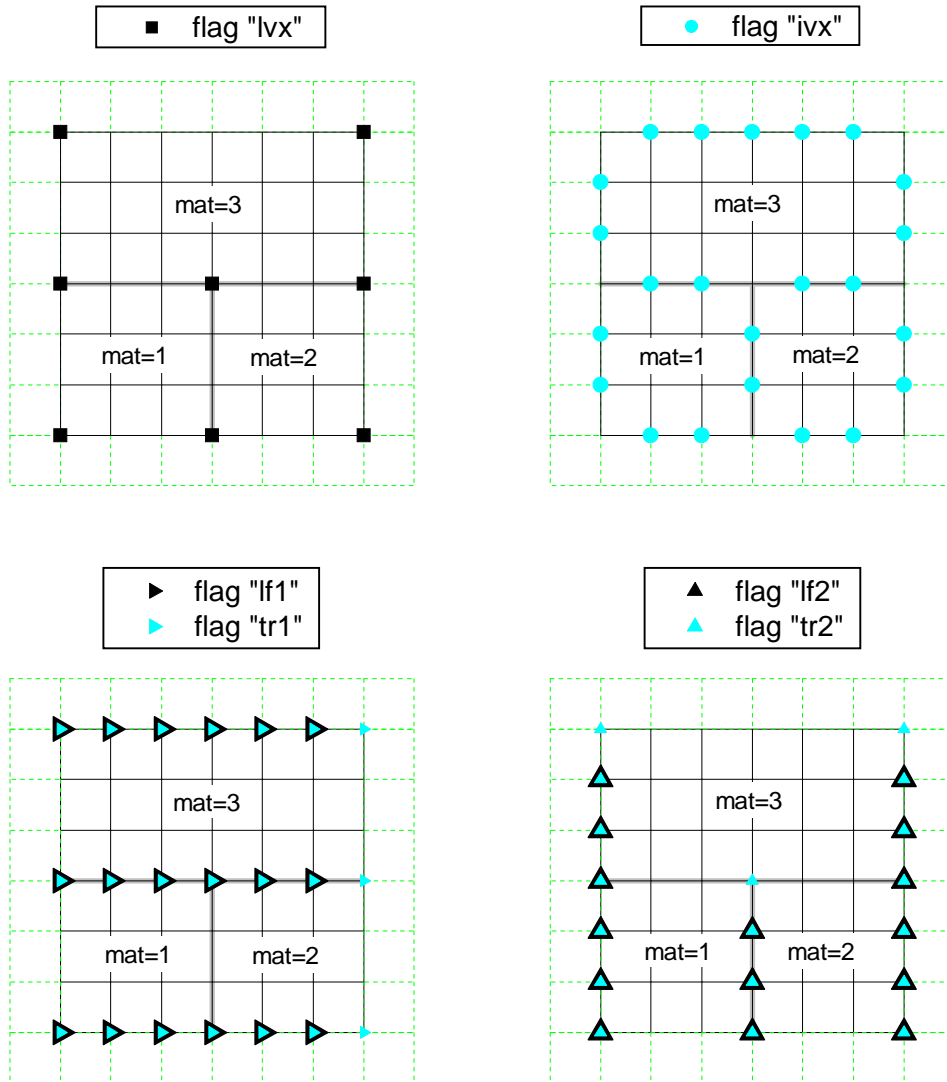


FIG. 7.1: Example of flag setting in a single-block mesh with 3 materials.

Set flag `mat=matnum` of the corresponding donor cell for all ghost cells (corner + non-corner) in the 1st row along all the interblock (i.e. with `ibc=5`) boundaries.

Step 3:

1. Set flag `cnv=1` for physical block corners.
2. Set flag `fix=1` for all physical block corners at intersections of fixed in space physical boundaries with `ibc = 1, 3, 4` or `8` (including the 3-*vd* and 3-*dd* corners); for corners not satisfying this latter condition and not lying on interblock (i.e. with `ibc=5`) boundaries (or being 3-*vd* or 3-*dd* block meeting corners) set flag `lvx=1`.

Step 4:

1. Set flag `lf1(lf2)=1` for all PR-faces along mesh direction $m = 1(2)$ (i.e. physical cell faces + those that protrude into the belt of ghost cells) which separate cells with different materials; set flag `tr1(tr2)=1` for both end vertices of such faces.

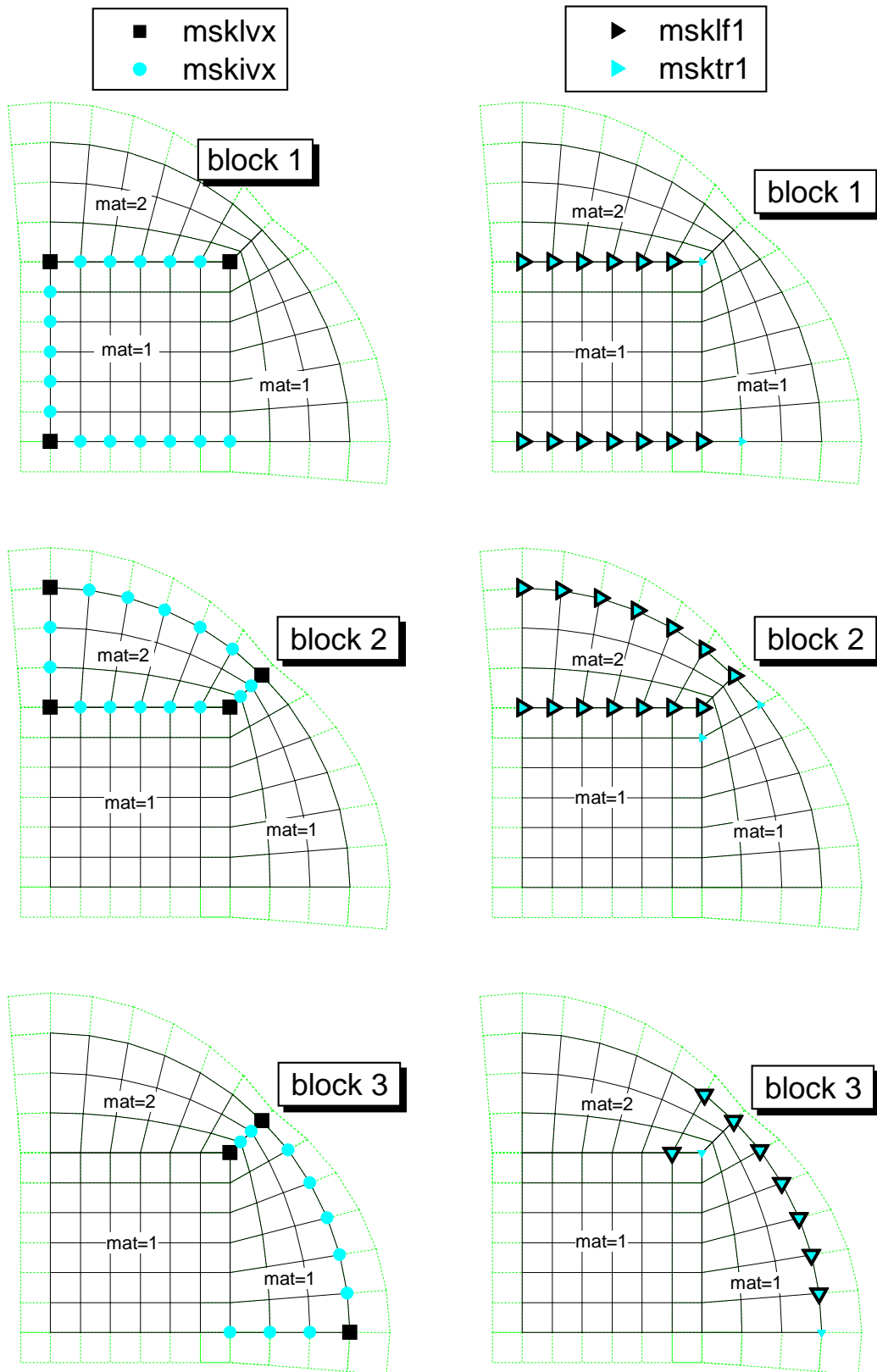


FIG. 7.2: Example of flag setting in a three-block mesh with 2 materials and a 3bk meeting point.

2. Set flag `tr1(tr2)=1` at physical vertices along the inflow/outflow boundaries [i.e. with `(ibc=3.or.ibc=4)` and mesh direction $m = 1(2)$] — which are not material interfaces!
3. Set flag `lvx=1` at physical vertices, and `lf1(lf2)=1` at physical faces along a center-of-convergence boundary with `ibc=0` along mesh direction $m = 1(2)$.
4. Set flag `lvx=1` at all vertices (physical + ghost) where the flags `tr1` and `tr2` are both 'on'.

Step 5:

1. Set flag `fix=1` for all physical vertices along fixed in space boundaries with `ibc = 1, 3, 4` or `8`, for which tangential rezoning was forbidden by setting `ifnotr12bc(ib,iblk)=.true..`
2. Set flag `lvx=1` for all physical vertices along moving physical boundaries with `ibc = 2, 6` or `9`, for which tangential rezoning was forbidden by setting `ifnotr12bc(ib,iblk)=.true..`

Step 6:

Set flag `lvx=1` for all vertices that lie in mesh parts excluded by the user from rezoning by assigning `iflagprt(ip,iblk)=.true.`, and which do not have `fix=1`.

Step 7:

Set flag `ivx=1` at all physical vertices that are not marked with `lvx=1` (strictly Lagrangian) or `fix=1` (fixed in space) but lie on a material interface (boundaries with vacuum included), i.e. have either `tr1=1` or `tr2=1`.

Step 8:

Load array `ivx_(i)` of block-local indices for all `ivx`-vertices, and array `nvx_(iblk)`.

Step 9:

Set flag `gcc=1` for all ghost corner cells.

Step 10:

Load array `MBLKIV(I)`, which provides the block number for any cell with a global index `I`.

Step 11:

Set flag `bov=1` for all physical vertices along all block edges.

Step 12:

Set flag `pag=1` for all physically associated ghost cells (PAGC).

8. GEOMETRIC QUANTITIES

The geometric quantities given below are calculated in the subroutine `GEOM`. The primary variables which define the mesh are the vertex coordinates $\vec{x}_i = (x_i, y_i)$. Here index i combines the two mesh indices (i, j) along directions $m = 1$ and $m = 2$ respectively; face im is the edge of mesh cell i in mesh direction m ; see Fig. 8.1.

For the neighbor mesh vertices we use the following index convention: i_{1+} is the vertex which lies next to vertex i along direction 1; i_{2+} is the vertex lying next to vertex i along direction 2; i_{12+} is the vertex lying next to vertex i along both direction 1 and 2 (across the diagonal of cell i); i_{1-} is the vertex preceding vertex i along direction 1; i_{2-} is the vertex preceding vertex i along direction 2. In the double-indexing convention, where $i = (i, j)$, we have $i_{1+} = (i+1, j)$, $i_{2+} = (i, j+1)$, $i_{12+} = (i+1, j+1)$, $i_{1-} = (i-1, j)$, $i_{2-} = (i, j-1)$. Then face $i1$ is the line segment between vertices i and i_{1+} ; face $i2$ is the line segment between vertices i and i_{2+} .

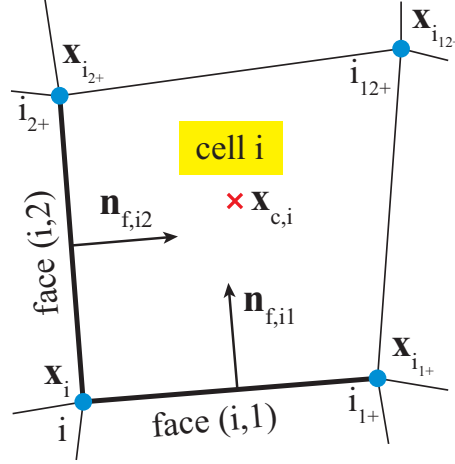


FIG. 8.1: Grid notation in a quadrilateral cell.

The length λ_{im} of face im is calculated as

$$\lambda_{im} = \sqrt{(x_{i_{m+}} - x_i)^2 + (y_{i_{m+}} - y_i)^2}. \quad (8.1)$$

The components of the unit normal vectors $\vec{n}_{f,im}$ are

$$\begin{aligned} \vec{n}_{f,i1} &= \left\{ \lambda_{im}^{-1}(y_i - y_{i_{1+}}), \lambda_{im}^{-1}(x_{i_{1+}} - x_i) \right\}, \\ \vec{n}_{f,i2} &= \left\{ \lambda_{im}^{-1}(y_{i_{2+}} - y_i), \lambda_{im}^{-1}(x_i - x_{i_{2+}}) \right\}. \end{aligned} \quad (8.2)$$

The area S_{im} of face im (not to be mixed with its length λ_{im}) is given by

$$S_{im} = \int_{\text{face } im} R d\lambda = \frac{1}{2} (R_i + R_{i_{m+}}) \lambda_{im}. \quad (8.3)$$

To evaluate any integral of the form $\int_{A_i} \Phi(x, y) dx dy$ over the area A_i of cell i , where the integrand $\Phi(x, y)$ is defined at mesh nodes (x_i, y_i) , we use a quadrature formula

$$\int_{A_i} \Phi(x, y) dx dy = \frac{1}{6} [\Phi_i (A_i + A_i^1) + \Phi_{i_{1+}} (A_i + A_i^2) + \Phi_{i_{12+}} (A_i + A_i^3) + \Phi_{i_{2+}} (A_i + A_i^4)], \quad (8.4)$$

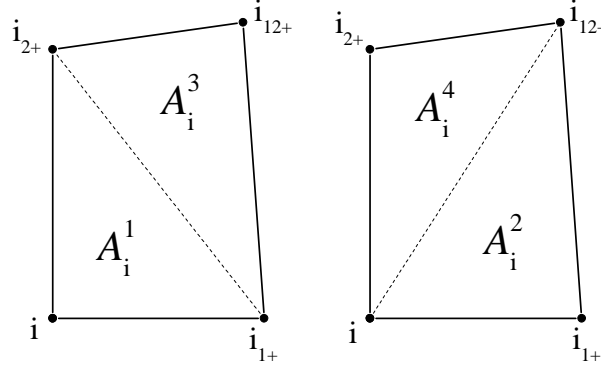


FIG. 8.2: Splitting of the mesh quadrilateral into two constituent triangles.

where A_i^α ($\alpha = 1, 2, 3, 4$) are the areas of the constituent triangles in two possible triangulations of the quadrilateral i ; see Fig. 8.2. Note that

$$A_i = A_i^1 + A_i^3 = A_i^2 + A_i^4. \quad (8.5)$$

The formula (8.4) is obtained by splitting the cell quadrilateral into two triangles (as shown in Fig. 8.2), applying to each triangle the quadrature formula

$$\int_{\Delta} \Phi(x, y) dx dy = \frac{1}{3} (\Phi_1 + \Phi_2 + \Phi_3) A_{\Delta}, \quad (8.6)$$

and subsequent averaging over the two possible triangulations. Because formula (8.6) is exact for any linear function $\Phi = a + b\vec{x}$, formula (8.4) is also exact for any such function. It is also important that this formula is symmetric with respect to the 4 vertices of each mesh quadrilateral. On an orthogonal mesh we have $A_i^\alpha = \frac{1}{2}A_i$, and Eq. (8.4) simplifies to an obvious expression

$$\int_{A_i} \Phi(x, y) dx dy = \frac{1}{4} (\Phi_i + \Phi_{i_{1+}} + \Phi_{i_{12+}} + \Phi_{i_{2+}}) A_i. \quad (8.7)$$

The areas of constituent triangles are given by

$$\begin{aligned} A_i^1 &= \frac{1}{2} (\vec{x}_{i_{1+}} - \vec{x}_i) \times (\vec{x}_{i_{2+}} - \vec{x}_i), \\ A_i^2 &= \frac{1}{2} (\vec{x}_{i_{12+}} - \vec{x}_{i_{1+}}) \times (\vec{x}_i - \vec{x}_{i_{1+}}), \\ A_i^3 &= \frac{1}{2} (\vec{x}_{i_{2+}} - \vec{x}_{i_{12+}}) \times (\vec{x}_{i_{1+}} - \vec{x}_{i_{12+}}), \\ A_i^4 &= \frac{1}{2} (\vec{x}_i - \vec{x}_{i_{2+}}) \times (\vec{x}_{i_{12+}} - \vec{x}_{i_{2+}}). \end{aligned} \quad (8.8)$$

These areas are all positive for any convex cell quadrilateral in a right-handed mesh.

The cell volume V_i is a particular case of integral (8.4), with $\Phi = R$ being either x or y . In this case we can simplify Eq. (8.4) to

$$V_i = \int_{A_i} R dx dy = \frac{1}{3} (R_{i_{1+}} + R_{i_{2+}} + R_i) A_i^1 + \frac{1}{3} (R_{i_{1+}} + R_{i_{2+}} + R_{i_{12+}}) A_i^3. \quad (8.9)$$

If a physical cell with $V_i < 0$ is found on a right-handed mesh, this is diagnosed as a mesh crash and computation is aborted. Note that all the algebraic expressions in Eqs. (8.1)–(8.3), (8.8), (8.9) are exact on an arbitrary quadrilateral mesh in both the Cartesian (x, y) and the cylindrical (r, z) geometries.

The mean cylindrical radius of cell i is defined as

$$R_{av,i} = \frac{V_i}{A_i}. \quad (8.10)$$

The geometrical center of cell i is calculated as

$$\vec{x}_{c,i} = \frac{1}{4} (\vec{x}_i + \vec{x}_{i_{1+}} + \vec{x}_{i_{2+}} + \vec{x}_{i_{12+}}). \quad (8.11)$$

Correspondence with the code variables:

λ_{im}	= <code>flngth(i,m)</code>	length of face im ;
$n_{f,i,m,x}$	= <code>fn(i,m,1)</code>	x -component of the unit normal to face im ;
$n_{f,i,m,y}$	= <code>fn(i,m,2)</code>	y -component of the unit normal to face im
S_{im}	= <code>fa(i,m)</code>	area of face im ;
A_i^α	= <code>a3angl(i,\alpha)</code>	area of a constituent triangle α ($= 1, 2, 3, 4$) of cell i ;
V_i	= <code>vol(i)</code>	volume of cell i ;
$x_{c,i}$	= <code>xc(i,1)</code>	x -component of the geometrical cell center;
$y_{c,i}$	= <code>xc(i,2)</code>	y -component of the geometrical cell center;
$R_{av,i}$	= <code>rav(i)</code>	average cylindrical radius of cell i ;

9. MESH LIBRARY

There are a number of preprogrammed options for mesh construction in the RALEF code. They are distinguished by the value of parameter `igeom`. Some of these options (usually with single-digit values of `igeom`) are more general than the others. Each type of mesh has a number of free parameters, that can usually be specified via the `namelist/input/`, and, possibly, several fixed parameters that are assigned automatically. Below all the principal parameters, which control the mesh properties, are divided into three groups:

- **fixed:** these are the parameters, for which there is no freedom of choice in this particular mesh option;
- **user-must:** these are the parameters that *must* be specified by the user in this particular mesh option; for these parameters there are no default values;
- **user-can:** the user may either set new values of these parameters or stay by their default values; the corresponding default values are indicated.

9.1. `igeom=1` or `2`: a multi-block rectangular x - y or r - z mesh

In this mesh option the number of blocks `nblks` is arbitrary, provided that `nblks` \leq `nb`. Each mesh block is a rectangle. “ x - y mesh” means that in each block mesh direction 1 is along the global x -axis, and mesh direction 2 is along the global y -axis, which is perpendicular to the x -axis. For `iradial=1` we have a rectangular r - z mesh, with r being identical with x . For `iradial=2` we have a rectangular r - z mesh, with r being identical with y .

For `igeom=1` the value of `iradial` is fixed at `iradial=0` (historically). For `igeom=2` the value of `iradial` is free to choose. An example of a single-block progressive (x, y) mesh with 2 parts along the x -axis is shown in Fig. 9.1.

Mesh parameters for `igeom=1`, uniform mesh:

- **fixed:** `iradial=0`
- **user-must:** `ncell(iprt,m,iblk)`
`dx(iprt,m,iblk)`
- **user-can:** `nblks` def = 1
`nprts(m,iblk)` def = 1
`x0(m,iblk)` def = (0.0,0.0)
`ibc(ib,iblk)` def = 2
`nbc(k,ib,iblk)` def = 0
`ghwidth` def = 10^{-4}

Mesh parameters for `igeom=1`, progressive mesh:

- **fixed:** `iradial=0`
- **user-must:** `ncell(iprt,m,iblk)`
`xxl(iprt,m,iblk)`
- **user-can:** `nblks` def = 1
`nprts(m,iblk)` def = 1
`fdx(iprt,m,iblk)` def = 1.0
`ibc(ib,iblk)` def = 2
`nbc(k,ib,iblk)` def = 0
`ghwidth` def = 10^{-4}

Mesh parameters for `igeom=2`, uniform mesh:

- **fixed:** none
- **user-must:** `ncell(iprt,m,iblk)`
`dx(iprt,m,iblk)`
- **user-can:** `iradial` def = 0
`nblks` def = 1
`nprts(m,iblk)` def = 1
`x0(m,iblk)` def = (0.0,0.0)
`ibc(ib,iblk)` def = 2
`nbc(k,ib,iblk)` def = 0
`ghwidth` def = 10^{-4}

Mesh parameters for `igeom=2`, progressive mesh:

- **fixed:** none

- **user-must:** ncell(iprt,m,iblk)
 xxl(iprt,m,iblk)
- **user-can:** iradial def = 0
 nblks def = 1
 nprts(m,iblk) def = 1
 fdx(iprt,m,iblk) def = 1.0
 ibc(ib,iblk) def = 2
 nbc(k,ib,iblk) def = 0
 ghwidth def = 10^{-4}

Explanations:

- nprts(m,iblk) is the number of parts along mesh direction m in part iprt of block iblk;
- ncell(iprt,m,iblk) is the number of physical cells along mesh direction m in part iprt of block iblk;
- x0(1,iblk) is the x coordinate of the lower left corner (corner 1) of block iblk;
- x0(2,iblk) is the y coordinate of the lower left corner (corner 1) of block iblk;
- dx(iprt,m,iblk) is the cell size along mesh direction m (i.e. either along the global x -axis or along the global y -axis) in part iprt of block iblk;
- xxl(iprt,1,iblk) and xxl(iprt+1,1,iblk) are the x coordinates of the left and right bounds of part iprt in block iblk;
- xxl(iprt,2,iblk) and xxl(iprt+1,2,iblk) are the y coordinates of the lower and upper bounds of part iprt in block iblk;
- fdx(iprt,m,iblk) is the ratio of successive cell sizes along mesh direction m in part iprt of block iblk; “good” values must be not too far from $\text{fdx}(\text{iprt},\text{m},\text{iblk}) = 1.0$, say, within the interval $0.9 \lesssim \text{fdx}(\text{iprt},\text{m},\text{iblk}) \lesssim 1.1$;
- ghwidth is the relative width of the ghost-cell bands;

A progressive-mesh option is chosen automatically when at least one value of $\text{xxl}(\text{iprt},\text{m},\text{iblk})$ is set different from its default value of $\text{undef} = -123456789.0$. In a progressive mesh, the values of $\text{x0}(\text{m},\text{iblk})$ and $\text{dx}(\text{iprt},\text{m},\text{iblk})$ are calculated from the values of $\text{xxl}(\text{iprt},\text{m},\text{iblk})$ and $\text{fdx}(\text{iprt},\text{m},\text{iblk})$; in particular, $\text{x0}(1,\text{iblk}) = \text{xxl}(1,1,\text{iblk})$, $\text{x0}(2,\text{iblk}) = \text{xxl}(1,2,\text{iblk})$.

9.2. igeom=3 or 4: a multi-block polar r - θ mesh

In this mesh option each block is a circular sector, with $x_2 = r$ being the radius, and $x_1 = \theta$ being the polar angle. The number of blocks nblks can be arbitrary, provided that $\text{nblks} \leq \text{nb}$. the polar angle θ is measured from the vertical y -axis in degrees of arc, so that $x = r \sin \theta$, $y = r \cos \theta$. Hence, the mesh parameters $\text{x0}(1,\text{iblk})$, $\text{dx}(\text{iprt},1,\text{iblk})$, and $\text{xxl}(\text{iprt},1,\text{iblk})$ must be given in degrees.

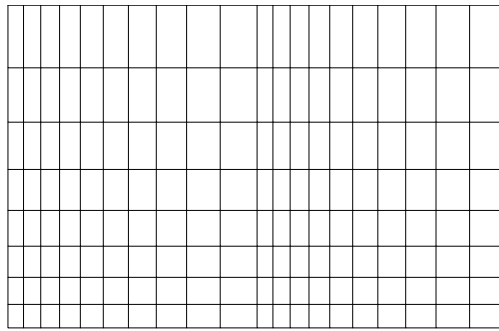


FIG. 9.1: A progressive (x, y) mesh with 2 parts along the x -axis in a single block.

For `igeom=3` the value of `iradial` is fixed at `iradial=0` (historically). For `igeom=4` the value of `iradial` is free to choose between 0, 1 and 2. Figure 9.2 shows an example of a single-block polar mesh, consisting of 3 parts along the $x_1 = \theta$ axis.

Mesh parameters for `igeom=3`, uniform mesh:

- **fixed:** `iradial=0`
- **user-must:** `ncell(iprt,m,iblk)`
`dx(iprt,m,iblk)`
- **user-can:** `nblks` def = 1
`nprts(m,iblk)` def = 1
`x0(m,iblk)` def = (0.0,0.0)
`ibc(ib,iblk)` def = 2
`nbc(k,ib,iblk)` def = 0
`ghwidth` def = 10^{-4}

Mesh parameters for `igeom=3`, progressive mesh:

- **fixed:** `iradial=0`
- **user-must:** `ncell(iprt,m,iblk)`
`xxl(iprt,m,iblk)`
- **user-can:** `nblks` def = 1
`nprts(m,iblk)` def = 1
`fdx(iprt,m,iblk)` def = 1.0
`ibc(ib,iblk)` def = 2
`nbc(k,ib,iblk)` def = 0
`ghwidth` def = 10^{-4}

Mesh parameters for `igeom=4`, uniform mesh:

- **fixed:** none

- **user-must:** ncell(iprt,m,iblk)
dx(iprt,m,iblk)
- **user-can:** iradial def = 0
nblks def = 1
nprts(m,iblk) def = 1
x0(m,iblk) def = (0.0,0.0)
ibc(ib,iblk) def = 2
nbc(k,ib,iblk) def = 0
ghwidth def = 10⁻⁴

Mesh parameters for igeom=4, progressive mesh:

- **fixed:** none
- **user-must:** ncell(iprt,m,iblk)
xxl(iprt,m,iblk)
- **user-can:** iradial def = 0
nblks def = 1
nprts(m,iblk) def = 1
fdx(iprt,m,iblk) def = 1.0
ibc(ib,iblk) def = 2
nbc(k,ib,iblk) def = 0
ghwidth def = 10⁻⁴

Explanations:

- nprts(m,iblk) is the number of parts along mesh direction m in part iprt of block iblk;
- ncell(iprt,m,iblk) is the number of physical cells along mesh direction m in part iprt of block iblk;
- x0(1,iblk) is the θ coordinate (in degrees) of the lower left corner (corner 1) of block iblk;
- x0(2,iblk) is the r coordinate of the lower left corner (corner 1) of block iblk;
- dx(iprt,m,iblk) is the cell size along mesh direction m [i.e. either along the θ -axis (m=1) or along the r -axis (m=2)] in part iprt of block iblk;
- xxl(iprt,1,iblk) and xxl(iprt+1,1,iblk) are respectively the θ coordinates (in degrees) of the left and right bounds of part iprt in block iblk;
- xxl(iprt,2,iblk) and xxl(iprt+1,2,iblk) are respectively the r coordinates of the lower and upper bounds of part iprt in block iblk;
- fdx(iprt,m,iblk) is the ratio of successive cell sizes along mesh direction m in part iprt of block iblk; “good” values must be not too far from fdx(iprt,m,iblk) = 1.0, say, within the interval $0.9 \lesssim \text{fdx(iprt,m,iblk)} \lesssim 1.1$;
- ghwidth is the relative width of the ghost-cell bands;

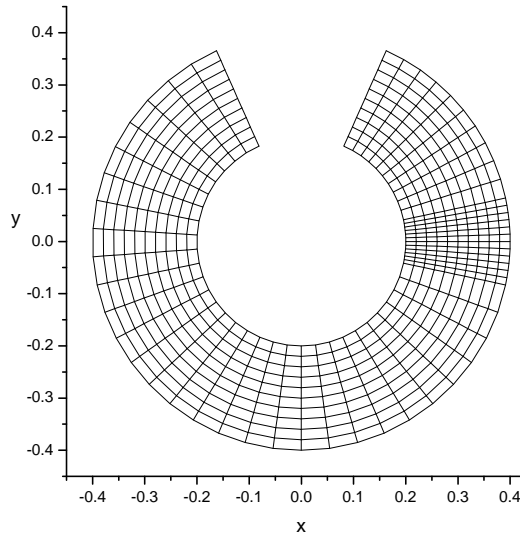


FIG. 9.2: A uniform (r, θ) mesh with 3 parts along the $x_1 = \theta$ axis in a single block.

The progressive-mesh option is chosen automatically when at least one value of `xxl(iprt,m,iblk)` is set different from its default value of `undef = -123456789.0`. In a progressive mesh, the values of `x0(m,iblk)` and `dx(iprt,m,iblk)` are calculated from the values of `xxl(iprt,m,iblk)` and `fdx(iprt,m,iblk)`; in particular, `x0(1,iblk) = xxl(1,1,iblk)`, `x0(2,iblk) = xxl(1,2,iblk)`.

9.3. `igeom=41`: a “snaky” band-like mesh

1. General description

This mesh is intended for computational domains in the form of narrow curved bands — to simulate, for example, thin curved foils. It may consist of an arbitrary number of blocks `nblks`. Every subsequent block `iblk+1` is attached to the previous block `iblk` along the edge `ib = 1` in the “upper” block `iblk+1`, and the edge `ib = 2` in the “lower” block `iblk`. The corresponding values of the block communication arrays `ibc` and `nbc` are assigned automatically in the subroutine `MSHP41`. A “snaky” mesh always meanders along mesh direction 2 in all blocks. An example of a “snaky” mesh, composed of two blocks, is shown in Fig. 9.3.

A “snaky” mesh consists of individual segments along mesh direction 2. Each segment is represented by a single part `jpvt` along mesh direction 2 in a current block `iblk`. Any given segment can be either straight or curved along a circular arc. More precisely, a “snaky” mesh can be thought of as built around a continuous reference curve — a “spinal chord”, which consists of straight and circular-arc segments. The “spinal chord” may lie anywhere between the left and the right boundaries of the mesh band. In Fig. 9.3 the “spinal chord” passes through the middle of the mesh band. The global rotation angle Θ of the mesh band is defined as the rotation angle (in the counter-clockwise direction) of the local normal to the “spinal chord” with respect to the global x -axis. If the starting value $\Theta_0 = 0$, then the first mesh segment starts in the vertical direction along the global y -axis (as in Fig. 9.3).

The full set of user-defined parameters, which completely specify the “snaky” mesh and are to be assigned via the namelist `input` in file ‘`in2d`’, is as follows

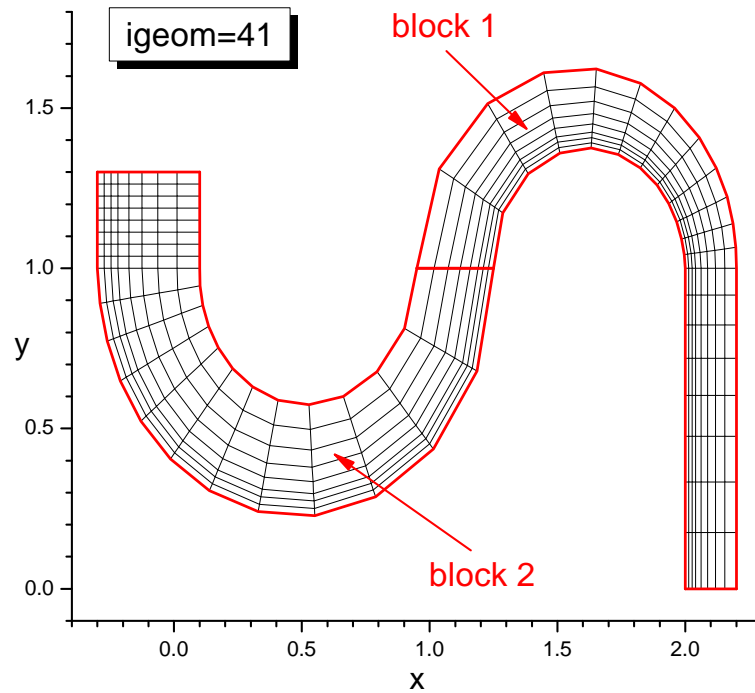


FIG. 9.3: A “snaky” mesh with 2 blocks; each block has 2 parts along each mesh direction.

- **user-must:** `ncell(1:nprts(1,1),1,1)`
`ncell(1:nprts(2,1),2,1:nblks)`
`dx(1:nprts(1,1),1,1)`
`dx(1:nprts(2,1),2,1:nblks)`
- **user-can:** `iradial, nblks,`
`nprts(1:2,1:nblks),`
`xxl(1:nprts(2,iblk)+1,1:2,1:nblks),`
`fdx(1:nprts(m,iblk),1:2,1:nblks),`
`x0(1:2,1).`

The parameters in the **user-must** group have no default values and must be specified by the user. The parameters in the **user-can** group have certain reasonable default values and are free to be left unspecified by the user.

2. The meaning of the mesh parameters

Number of cells

For mesh direction 1, the number of cells `ncell(ip,1,1)` must be prescribed for each part $ip = 1, 2, \dots, \text{nprts}(1,1)$ but only in the first block. In all other blocks the same values are used. The mesh in Fig. 9.3 has `ncell(1,1,1) = 3`, `ncell(2,1,1) = 5`.

For mesh direction 2, one has to load the values of `ncell(jp,2,iblk)` for all parts $jp = 1, 2, \dots, \text{nprts}(2,iblk)$ in all blocks $iblk = 1, 2, \dots, \text{nblks}$. The mesh in Fig. 9.3 has `ncell(1,2,1) = 8`, `ncell(2,2,1) = 12`, `ncell(1,2,2) = 12`, `ncell(2,2,2) = 8`.

Starting corner and edge

The coordinates (x_0, y_0) of the lower-left corner (corner # 1) in block 1 are given by the values of

$$\begin{aligned} x_0 &= \text{xxl}(1,1,1), \\ y_0 &= \text{xxl}(1,2,1). \end{aligned} \tag{9.1}$$

If no value is assigned to $\text{xxl}(1,1,1)$ [and/or $\text{xxl}(1,2,1)$] in the input file ‘in2d’, then the initial corner position is set to $x_0 = 0$ [and/or $y_0 = 0$]; in Fig. 9.3 we have $(x_0, y_0) = (2, 0)$.

The initial tilt angle Θ_0 of edge $\text{ib} = 1$ of block 1 is set by using the mesh parameter $\text{x0}(2,1)$:

$$\Theta_0 = \text{x0}(2,1). \tag{9.2}$$

The default value is $\Theta_0 = \text{x0}(2,1) = 0$.

Position of the “spinal chord”

The relative depth δ_{sc} of the “spinal chord” inside the mesh is defined by the parameter

$$0 \leq \delta_{sc} = \text{x0}(1,1) \leq 1. \tag{9.3}$$

In subroutine MSHP41 the value of $\text{x0}(1,1)$, assigned by the user, is automatically trimmed to satisfy condition (9.3). If $\delta_{sc} = 0$ (the default value), the “spinal chord” coincides with the left boundary of the mesh band; if $\delta_{sc} = 1$, it coincides with the right boundary of the mesh band. In Fig. 9.3 the value $\delta_{sc} = \text{x0}(1,1) = 0.5$ is used.

Dimensions

The principal dimensions of individual mesh segments are set by the dx array. The bottom width d_0 of the starting mesh segment, equal to the length of edge $\text{ib} = 1$ in block 1, is defined by the summed values of subarray $\text{dx}(1:\text{nprts}(1,1), 1, 1)$,

$$d_0 = \sum_{\text{ip}=1}^{\text{nprts}(1,1)} \text{dx}(\text{ip}, 1, 1), \tag{9.4}$$

which are to be set for the block 1 only. More precisely,

$$\text{dx}(\text{ip}, 1, 1)$$

is the length of part ip along mesh direction 1 at the lower edge 1 in block 1. In Fig. 9.3 we have $d_0 = 0.2$.

The mesh extension along the “spinal chord” (i.e. along mesh direction 2) is defined by the subarray

$$\text{dx}(1:\text{nprts}(2,1), 2, 1:\text{nblks}),$$

whose values must be assigned for all parts in all blocks along the mesh direction 2. The meaning of the elements of this subarray depends on whether the corresponding mesh segment is straight or circular:

- for a straight mesh segment, $\text{dx}(\text{jp}, 2, \text{iblk})$ is the length of the corresponding part jp along mesh direction 2 in block iblk ;
- for a circular mesh segment, $\text{dx}(\text{jp}, 2, \text{iblk})$ is the turning angle $\Delta\Theta_{jp}$ in degrees of arc of the local normal to the “spinal chord” over the corresponding part jp of block iblk .

For straight segments $dx(jp,2,iblk)$ must be positive, for circular segments $dx(jp,2,iblk)$ can be both positive and negative. In Fig. 9.3 we have $dx(1,2,1) = 1$, $dx(2,2,1) = 180$, $dx(1,2,2) = -180$, $dx(2,2,2) = 0.3$.

Curvature radii and width scale factors

The curvature and the width of different mesh segments are controlled by the array $xxl(1:ns+1,1:2,1:nb)$. In contrast to the dx array, the values of $xxl(1:ns+1,1:2,1:nb)$ refer to the interfaces between block parts along mesh direction 2, i.e. both the $xxl(jp+1,1,iblk)$ and the $xxl(jp+1,2,iblk)$ refer to the same interface between part jp and part $jp+1$ along mesh direction 2 in block $iblk$.

The meaning of $xxl(jp+1,1,iblk)$ is as follows:

- if $xxl(jp+1,1,iblk) > 0$, the mesh segment in the corresponding part jp along mesh direction 2 in block $iblk$ is curved along a circular arc, with

$$R_{jp} = xxl(jp+1,1,iblk) \tag{9.5}$$

being the radius of curvature of the “spinal chord” in this part;

- if $xxl(jp+1,1,iblk) \leq 0$ (the default case), the mesh segment in the corresponding part jp along mesh direction 2 in block $iblk$ is straight, and the value of $xxl(jp+1,1,iblk)$ is not used; the default value is $xxl(jp+1,1,iblk) = \text{undef} = -123456789$.

The mesh in Fig. 9.3 was constructed with $xxl(3,1,1) = 0.5$, $xxl(2,1,2) = 0.6$. Recall that the turning direction is defined by the sign of $dx(jp,2,iblk)$.

The value of $xxl(jp+1,2,iblk)$ defines the stretch factor for the mesh width in the cross-section $jp+1$ (i.e. between part jp and part $jp+1$ along mesh direction 2 in block $iblk$). More precisely, the width d_{jp} of the mesh band at the cross-section $jp+1$ is given by

$$d_{jp} = d_0 \cdot xxl(jp+1,2,iblk), \tag{9.6}$$

where d_0 is the initial width of the starting edge, defined in Eq. (9.4). Inside a given part jp , the mesh width varies linearly along the “spinal chord” from d_{jp} at the bottom to d_{jp+1} at the top. The mesh in Fig. 9.3 was constructed with $xxl(3,2,1) = 1.5$, $xxl(2,2,2) = xxl(3,2,2) = 2.0$.

Progression factors

The mesh progression factors $fdx(1:ns,1:2,1:nblks)$ have their usual meaning: $fdx(ip,m,iblk)$ is the ratio of successive cell sizes along mesh direction m in part ip of block $iblk$; the default value is $fdx(ip,m,iblk) = 1$. For the mesh direction 1, it is sufficient to assign the values $fdx(1:nprts(1,1),1,1)$ in the first block only; the same values are used in all other blocks.

The mesh in Fig. 9.3 was constructed with the values $fdx(1:2,1,1) = 1.0, 1.2$, $fdx(1:2,2,1) = 0.9, 1.15$, $fdx(1:2,2,2) = 0.9, 1.0$.

9.4. igeom=5 or 6: a cylindrical (or spherical) mesh in a 180° semi-circle with a free-float center

This mesh consists of $nblks = 4$ blocks and represents a half-circle with a rectangle in the central region. It is shown in Fig. 9.4. The central rectangle has edge ratio 1:2, so that

when the mesh is reflected with respect to the y -axis, it becomes a full circle with a square in the center. It has 4 main free parameters: two radii r_0 — the radius of the circumcircle around the central square, and r_{max} — the outer radius of the mesh.

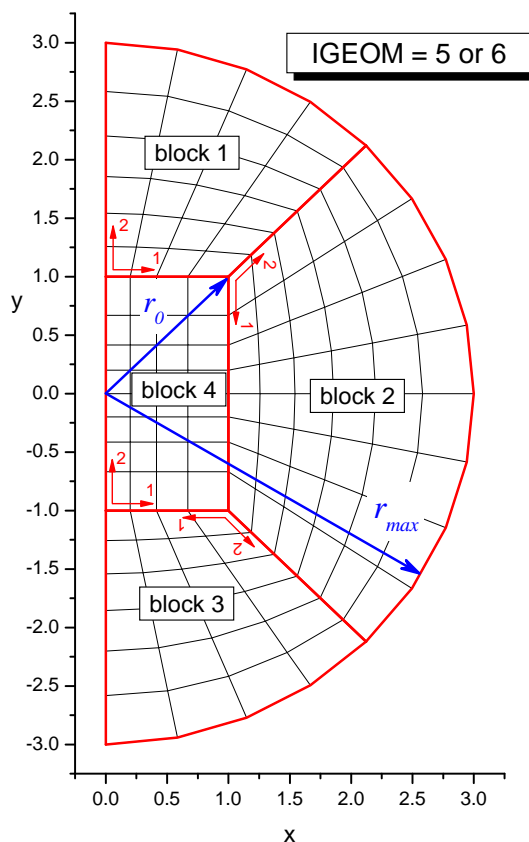


FIG. 9.4: A 180° 4-block cylindrical (or spherical) mesh with a free-float center.

Mesh parameters for $igeom=5$:

- **fixed:**
 - `iradial=0`
 - `nblks=4`
 - `nprts(m,iblk) =1`
 - `ncell(1,m,iblk) for $iblk \geq 2$`
 - `ibc(ib,iblk) (except for ibc(2,1) and ibc(3,1))`
 - `nbc(ib,iblk)`
- **user-must:**
 - `ncell(1,1,1)`
 - `ncell(1,2,1)`
 - `x0(1,1) = r_0`
 - `x0(2,1) = r_{max}`
- **user-can:**
 - `ibc(2,1)` `def = 2`
 - `ibc(3,1)` `def = 2`
 - `ghwidth` `def = 10^{-4}`

Mesh parameters for *igeom=6* (a full sphere):

- **fixed:** `iradial=1`
 `nblks=4`
 `nprts(m,iblk) =1`
 `ncell(1,m,iblk) for $iblk \geq 2$`
 `ibc(ib,iblk) (except for ibc(2,1) and`
 `ibc(3,1)`)
 `nbc(ib,iblk)`
- **user-must:** `ncell(1,1,1)`
 `ncell(1,2,1)`
 `x0(1,1) = r_0`
 `x0(2,1) = r_{max}`
- **user-can:** `ibc(2,1)` `def = 2`
 `ibc(3,1)` `def = 2`
 `ghwidth` `def = 10^{-4}`

Explanations:

- `ncell(1,1,1)` is the number of physical cells along the short edge of the central rectangle in block 4; its long edge has `2ncell(1,1,1)` mesh cells;
- `ncell(1,2,1)` is the number of physical cells along the radial direction in blocks 1, 2 and 3;
- `x0(1,1)` is the smaller radius r_0 ;
- `x0(2,1)` is the larger radius $r_{max} > r_0$;
- `ibc(2,1)` is the type of boundary condition along the circular part of the outer boundary; typically `ibc(2,1)=2`;
- `ibc(3,1)` is the type of boundary condition along the diameter part of the outer boundary; typically `ibc(3,1)=1`;

9.5. *igeom=22*: a multi-block arbitrary-quadrangle (AQ) *x-y* or *r-z* mesh

In this mesh option the number of blocks `nblks` is arbitrary, provided that `nblks` \leq `nb`. Every mesh block is an arbitrary quadrangle. All mesh lines are straight. An AQ mesh is always assumed to be progressive along each quadrangle edge with independent common ratios.

Mesh parameters for *igeom=22*:

- **fixed:** none
- **user-must:** `ncell(iprt,m,iblk)`
 `x0aq(m,ic,iblk)`

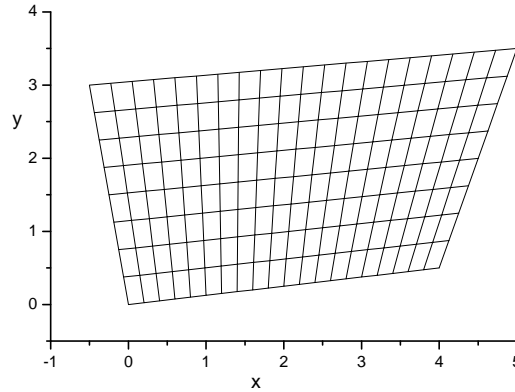


FIG. 9.5: An arbitrary-quadrangle mesh in a single block.

• user-can:	<code>iradial</code>	<code>def = 0</code>
	<code>nblks</code>	<code>def = 1</code>
	<code>nprts(m,iblk)</code>	<code>def = 1</code>
	<code>fprtaq(iprt,ib,iblk)</code>	<code>def = 1.0</code>
	<code>fdxaq(iprt,ib,iblk)</code>	<code>def = 1.0</code>
	<code>ibc(ib,iblk)</code>	<code>def = 2</code>
	<code>nbc(k,ib,iblk)</code>	<code>def = 0</code>
	<code>ghwidth</code>	<code>def = 10⁻⁴</code>

Explanations:

- `nblks` is the total number of mesh blocks;
- `nprts(m,iblk)` is the number of parts along mesh direction `m` in block `iblk`;
- `ncell(iprt,m,iblk)` is the number of physical cells along mesh direction `m` in part `iprt` of block `iblk`;
- `x0aq(1,ic,iblk)` is the x ($= x_1$) coordinate of corner `ic` of block `iblk`; corners are numbered according to the CAVEAT convention, i.e. corner 1 is the lower-left one, corner 2 is the upper-right one, corner 3 is the upper-left one, corner 4 is the lower-right one;
- `x0aq(2,ic,iblk)` is the y ($= x_2$) coordinate of corner `ic` of block `iblk`;
- `fprtaq(iprt,ib,iblk)` are the weights for part lengths along edge `ib` in block `iblk`, i.e. `fprtaq(i,ib,iblk) : fprtaq(i+1,ib,iblk)` is the ratio of lengths of parts `i` and `i+1` along edge `ib` in block `iblk`; normalization of `fprtaq(iprt,ib,iblk)` is arbitrary;
- `fdxaq(iprt,ib,iblk)` is the common ratio of successive cell sizes in part `iprt` of edge `ib` of block `iblk`; “good” values must be not too far from `fdxaq(iprt,ib,iblk) = 1.0`, say, within the interval $0.9 \lesssim \text{fdxaq}(iprt,ib,iblk) \lesssim 1.1$;
- `ghwidth` is the relative width of the ghost-cell bands;

9.6. `igeom=23`: a multi-block polygon-mosaic mesh

In this option the number of blocks `nblks` is a free parameter. Every mesh block `iblk` is composed of `nprts(1,iblk) × nprts(2,iblk)` parts, where each part is an arbitrary quadrangle. Within each part the mesh lines are straight. Within any given block the mesh lines are piece-wise straight. In general, the lengths of cell faces along every interface between different parts make a geometric progression with a common ratio, which is allowed to have individual values along different interfaces. In other words, each part of every block has an individual AQ mesh.

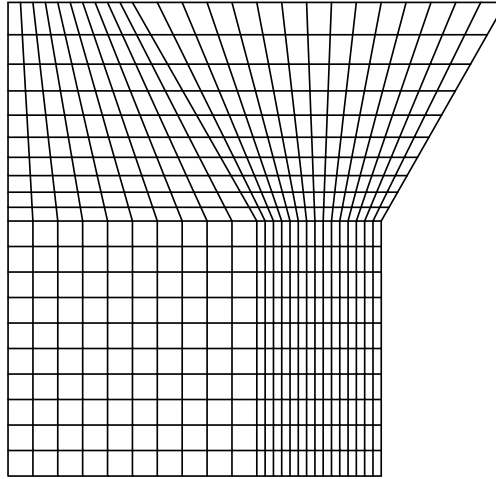


FIG. 9.6: An polygon mosaic mesh in a single block, which consists of two parts along each of the two mesh directions.

Mesh parameters for `igeom=23`:

- **fixed:** none
- **user-must:** `ncell(iprt,m,iblk)`
`xxp(1:nprts(1,iblk)+1,1:nprts(2,iblk)+1,1:2,iblk)`
- **user-can:** `iradial`
`nblks`
`nprts(m,iblk)`
`fdxp(1:nprts(1,iblk)+1,1:nprts(2,iblk)+1,1:2,iblk)`
`ibc(ib,iblk)`
`nbc(k,ib,iblk)`
`ghwidth`

Explanations:

- `nblks` is the total number of mesh blocks;
- `nprts(m,iblk)` is the number of parts along mesh direction `m` in block `iblk`;

- `ncell(iprt,m,iblk)` is the number of physical cells along mesh direction `m` in part `iprt` (along this direction) of block `iblk`;
- `xxp(iprt,jprt,1,iblk)` is the x ($= x_1$) coordinate of the lower-left corner of part `(iprt,jprt)` in block `iblk`; `iprt` is the part index along mesh direction 1, `jprt` is the part index along mesh direction 2; correspondingly, the lower-right, upper-left, and upper-right corners of part `(iprt,jprt)` in block `iblk` have the x -coordinates `xxp(iprt+1,jprt,1,iblk)`, `xxp(iprt,jprt+1,1,iblk)`, and `xxp(iprt+1,jprt+1,1,iblk)`; default value is `undef`;
- `xyp(iprt,jprt,2,iblk)` is the y ($= x_2$) coordinate of the lower-left corner of part `(iprt,jprt)` in block `iblk`; default value is `undef`;
- `fdxp(iprt,jprt,1,iblk)` is the common ratio of successive cell sizes along the part edge, starting from the lower-left corner of part `(iprt,jprt)` along mesh direction 1, in block `iblk`; correspondingly, `fdxp(iprt,jprt+1,1,iblk)` is the common ratio of successive cell sizes along the part edge, starting from the upper-left corner of part `(iprt,jprt)` along mesh direction 1; default value is 1.0;
- `fdyp(iprt,jprt,2,iblk)` is the common ratio of successive cell sizes along the part edge, starting from the lower-left corner of part `(iprt,jprt)` along mesh direction 2, in block `iblk`; correspondingly, `fdyp(iprt+1,jprt,1,iblk)` is the common ratio of successive cell sizes along the part edge, starting from the lower-right corner of part `(iprt,jprt)` along mesh direction 2; default value is 1.0;
- `ghwidth` is the relative width of the ghost-cell bands;

9.7. `igeom=50`: a 5-block cylindrical mesh in a full 360° circle with a free-float center

This is a mesh for a cylindrical shell, originally designed for the wobbler problem to study the ϕ asymmetry of cylindrical implosions. The 4 outer blocks represent an annular region with a void in the center; the optional 5-th block covers the central region (see Fig. 9.7).

Mesh parameters for `igeom=50`, radially uniform mesh:

- **fixed:**
 - `iradial=0`
 - `nprts(m,iblk) =1` along ϕ direction
 - `nprts(m,iblk)` along r direction for `iblk \geq 2`
 - `ncell(iprt,m,iblk)` for `iblk \geq 2`
 - `dx(iprt,1,iblk)`
 - `ibc(ib,iblk)` (except for `ibc(2,1)`)
 - `nbc(ib,iblk)`
- **user-must:**
 - `ncell(1,1,1)`
 - `ncell(iprt,2,1)`
 - `x0(2,1) = r_0`
 - `dx(iprt,2,1)`
- **user-can:**
 - `nblks (= 4 or 5)` def = 4
 - `nprts(2,1)` def = 1
 - `ibc(2,1)` def = 2
 - `ghwidth` def = 10^{-4}

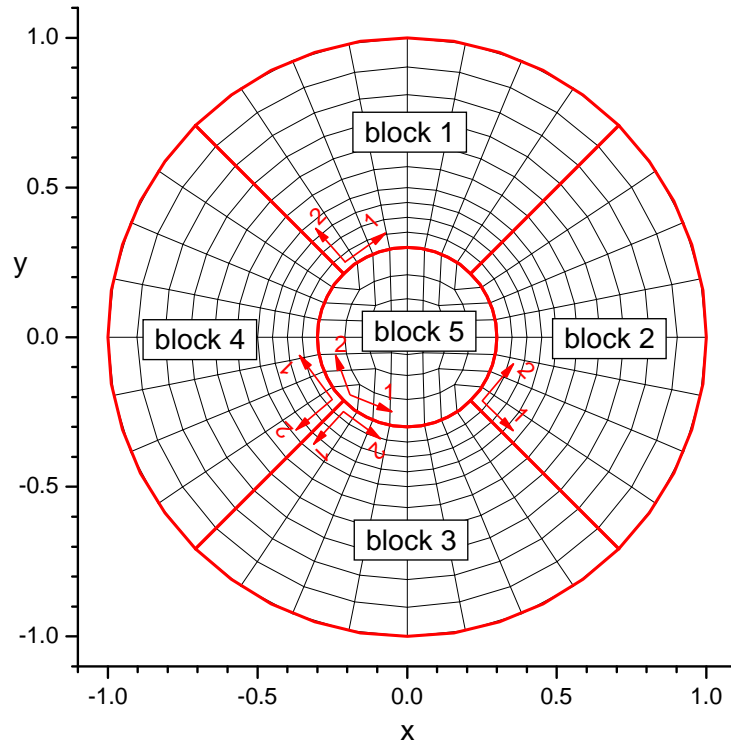


FIG. 9.7: A 360° full-circle 5-block cylindrical mesh with a free-float center.

Mesh parameters for *igeom=50*, radially progressive mesh:

- **fixed:**
 - `iradial=0`
 - `nprts(m,iblk) =1` along ϕ direction
 - `nprts(m,iblk)` along r direction for $iblk \geq 2$
 - `ncell(iprt,m,iblk)` for $iblk \geq 2$
 - `xxl(1,m,iblk)` along ϕ direction
 - `xxl(2,m,iblk)` along ϕ direction
 - `fdx(iprt,1,iblk)=1`
 - `ibc(ib,iblk)` (except for `ibc(2,1)`)
 - `nbc(ib,iblk)`
- **user-must:**
 - `ncell(1,1,1)`
 - `ncell(iprt,2,1)`
 - `xxl(1:nnprad+1,2,1)`
- **user-can:**
 - `nblks (= 4 or 5)` def = 4
 - `nprts(2,1) = nnprad` def = 1
 - `fdx(iprt,2,1)` def = 1.0
 - `ibc(2,1)` def = 2
 - `ghwidth` def = 10^{-4}

Explanations:

- `nprts(2,1)` is the number of parts along the radial direction in each of the 4 outer blocks;
- `ncell(1,1,1)` is the number of physical cells along the azimuth ϕ in each of the 4 outer blocks (i.e. over the ϕ interval of 90° ; it is also the number of cells along each edge of the central 5-th block);
- `ncell(iprt,2,1)` is the number of physical cells in part `iprt` along the radial direction in each of the 4 outer blocks;
- `x0(2,1)` is the radius $r_0 > 0$ of the central cavity (block 5);
- `dx(iprt,2,1)` is the cell size in part `iprt` along the radial direction in each of the 4 outer blocks;
- `xxl(iprt,2,1)` is the lower-end radius of part `iprt` along the radial direction in each of the 4 outer blocks;
- `fdx(iprt,2,1)` is the ratio of successive cell sizes in part `iprt` along the radial direction in each of the 4 outer blocks;
- `ibc(2,1)` is the type of boundary condition along the entire outer boundary; typically `ibc(2,1)=2`.

9.8. `igeom=51`: a multi-tier full-circle cylindrical mesh with a free-float center

1. General description

Here we construct a full-circle quasi-polar mesh with a free-float center, labeled as an H_4 -mesh (a “Hohlraum” mesh extending over the 4 quadrants of the polar angle θ). It is only suitable for Cartesian geometry with `iradial = 0`. The core of the H_4 -mesh consists of $n_t \geq 1$ full radial tiers. Each full tier extends over 360° and is comprised of 4 neighboring blocks. With one extra block in the center (which is chosen to be the block # 1), the total number of blocks in the mesh core is $4n_t + 1$.

Beside the $4n_t + 1$ core blocks, the H_4 -mesh can have 1, 2, or 3 extra protruding blocks (“cap” blocks) in the unfilled $n_t + 1$ -th tier, i.e. the total number of blocks in the H_4 -mesh is

$$5 \leq 4n_t + 1 \leq \text{nblks} < 4n_t + 5. \quad (9.7)$$

Generally, different “cap” blocks must not touch one another along their side edges.

An example of the H_4 -mesh with $n_t = 2$ full tiers and 2 extra “cap” blocks is shown in Fig. 9.8. The H_4 -mesh is constructed in the progressive-mesh mode only, i.e. all the mesh dimensions are supposed to be set by assigning the arrays `xxl(ns+1,2,nb)` and `fdx(ns,2,nb)`; the values of the mesh parameters `x0(2,nb)` and `dx(ns,2,nb)` are not used.

The full set of parameters that completely specify the physical part of the H_4 mesh consists of the following variables and arrays

$$\begin{aligned}
 & \text{nblks}, \\
 & \text{nprts}(1:2,1:\text{nb}), \\
 & \text{ncell}(1:\text{ns},1:2,1:\text{nb}), \\
 & \text{xxl}(1:\text{ns}+1,1:2,1:\text{nb}), \\
 & \text{fdx}(1:\text{ns},1:2,1:\text{nb}), \\
 & \text{nbc}(1:2,1:4,1:\text{nb}).
 \end{aligned} \quad (9.8)$$

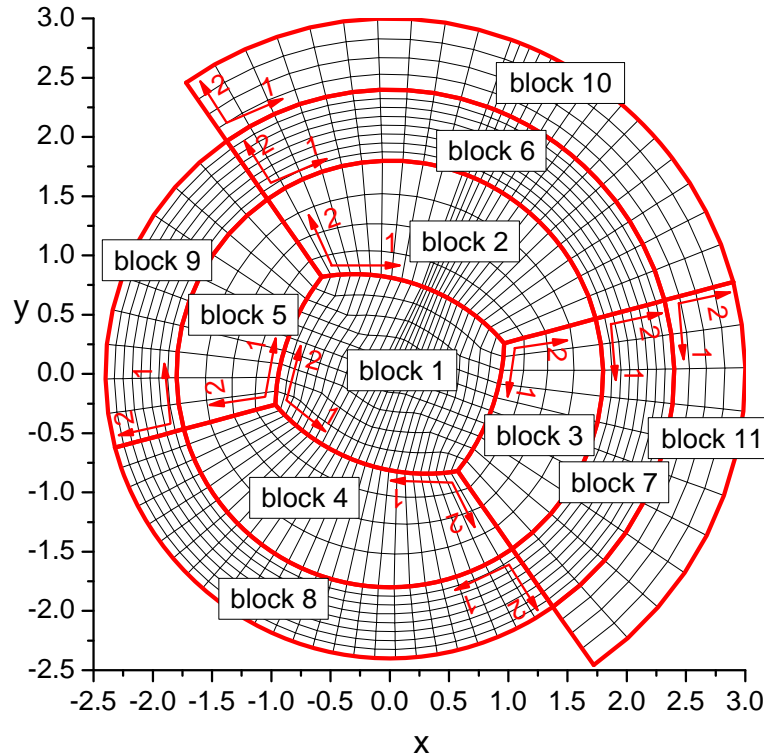


FIG. 9.8: A full-circle 11-block cylindrical mesh with a free-float center, 9 core blocks and 2 “cap” blocks (blocks 10 and 11).

The mesh directions $m = 1$ and $m = 2$ are rigidly fixed within every block: in all blocks other than the central block # 1 mesh direction 1 is always along the polar θ angle (as measured from the vertical axis in the clockwise direction), while mesh direction 2 is always along the cylindrical radius r .

The mesh in the central block # 1 can be continuously stretched between two extreme configurations: a rectangle and a full circle. The amount of stretch is independent along the two perpendicular directions and is controlled by the values of the two parameters $0 \leq \text{fdx}(1,1,1) \leq 1$ (the amount of stretch between blocks 4–1–2) and $0 \leq \text{fdx}(1,2,1) \leq 1$ (the amount of stretch between blocks 5–1–3). For the default values of $\text{fdx}(1,1,1) = \text{fdx}(1,2,1) = 1$ we have a rectangular shape of the central block, for $\text{fdx}(1,1,1) = \text{fdx}(1,2,1) = 0$ — a circular one. The four corners of block # 1 and its diagonals do not participate in the stretch. The mesh in Fig. 9.8 has been constructed with the values of $\text{fdx}(1,1,1) = \text{fdx}(1,2,1) = 0.5$.

In practice, to construct the H_4 mesh, one does not need the full set of parameters (9.8) but only its certain limited subset. We call the subset of the full list (9.8) that is actually used for mesh construction the *principal* mesh parameters. As usual, some of the principal mesh parameters have default values and must not (but can) be specified by the user, others do not have default values and must be specified in the `namelist/input/`.

A special feature of the H_4 mesh is that every individual block can be divided into parts in an arbitrary manner, independently of how other blocks are divided into parts. In other words, the values of `nprts(m,iblk)` can be chosen independently for each mesh direction $m = 1, 2$ in every block $\text{iblk} = 1, 2, \dots, \text{nblks}$. But then, because contacting blocks must have the same number of cells along common edges, the user-defined values of

`ncell(iprt,m,iblk)` must satisfy certain consistency conditions.

2. *Principal mesh parameters*

igeom=51: mesh parameters for the θ -direction

Parameters used for the physical θ -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(1,\text{iblk}), \\ \text{ncell}(1:\text{nprts}(1,\text{iblk}),1,\text{iblk}), \\ \text{xxl}(1:\text{nprts}(1,\text{iblk})+1,1,\text{iblk}), \\ \text{fdx}(1:\text{nprts}(1,\text{iblk}),1,\text{iblk}), \end{array} \right\} \text{iblk} = 2 \rightarrow 5. \quad (9.9)$$

Parameters that must be specified in the `namelist/input/`:

$$\begin{array}{l} \text{ncell}(1,1,2), \\ \text{ncell}(1,1,3), \\ \text{xxl}(1,1,2), \\ \text{xxl}(2,1,2). \end{array} \quad (9.10)$$

If more parameters are specified than in the “short” list (9.10), they must satisfy the following consistency conditions:

$$\begin{aligned} \sum_j \text{ncell}(j,1,4) &= \sum_i \text{ncell}(i,1,2), \\ \sum_j \text{ncell}(j,1,5) &= \sum_i \text{ncell}(i,1,3); \end{aligned} \quad (9.11)$$

$$\left. \begin{array}{l} \text{xxl}(1,1,\text{iblk}+1) = \text{xxl}(\text{nprts}(1,\text{iblk})+1,1,\text{iblk}), \\ \text{xxl}(\text{nprts}(1,\text{iblk}+1)+1,1,\text{iblk}+1) = \text{xxl}(1,1,\text{iblk}) + 180, \end{array} \right\} \text{iblk} = 2 \rightarrow 4. \quad (9.12)$$

$$\text{xxl}(i,1,\text{iblk}) < \text{xxl}(i+1,1,\text{iblk}), \quad \text{iblk} = 2 \rightarrow 5. \quad (9.13)$$

Note that the values of `xxl(i,1,iblk)` for `iblk = 2–5` specify the angular boundaries of the corresponding block parts in degrees of arc (as measured from the vertical axis in the clockwise direction). If only the values of `xxl(1:2,1,2)` are assigned in the `namelist/input/`, then the values of `xxl(1:2,1,iblk)` for `iblk = 3–5` are calculated from the consistency conditions (9.12).

igeom=51: mesh parameters for the r -direction in the core blocks

Parameters used for the physical r -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(2,\text{iblk}), \\ \text{ncell}(1:\text{nprts}(2,\text{iblk}),2,\text{iblk}), \end{array} \right\} \text{iblk} = 2 \rightarrow 4n_t + 1; \quad (9.14)$$

$$\left. \begin{array}{l} \text{xxl}(1:\text{nprts}(2,\text{iblk})+1,2,\text{iblk}), \\ \text{fdx}(1:\text{nprts}(2,\text{iblk}),2,\text{iblk}), \end{array} \right\} \text{iblk} = 2, 6, \dots, 4n_t - 2.$$

Parameters that must be specified in the `namelist/input/`:

$$\left. \begin{array}{l} \text{ncell}(1,2,\text{iblk}), \\ \text{xxl}(1,2,\text{iblk}), \\ \text{xxl}(2,2,4n_t - 2). \end{array} \right\} \text{iblk} = 2, 6, \dots, 4n_t - 2. \quad (9.15)$$

Consistency conditions:

$$\text{xxl}(1,2,\text{iblk}+4) = \text{xxl}(\text{nprts}(2,\text{iblk})+1,2,\text{iblk}), \quad \text{iblk} = 2, 3, \dots, 4n_t - 6. \quad (9.16)$$

$$\text{xxl}(i,2,\text{iblk}) < \text{xxl}(i+1,2,\text{iblk}), \quad \text{iblk} = 2, 6, \dots, 4n_t - 2. \quad (9.17)$$

igeom=51: mesh parameters for the “cap” blocks

Parameters used for the physical θ -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(1,\text{iblk}), \\ \text{ncell}(1:\text{nprts}(1,\text{iblk}),1,\text{iblk}), \\ \text{nbc}(1,1,\text{iblk}), \end{array} \right\} \text{iblk} = 4n_t + 2 \rightarrow \text{nblks}. \quad (9.18)$$

Parameters used for the physical r -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(2,\text{iblk}), \\ \text{ncell}(1:\text{nprts}(2,\text{iblk}),2,\text{iblk}), \end{array} \right\} \text{iblk} = 4n_t + 2 \rightarrow \text{nblks}; \quad (9.19)$$

$$\left. \begin{array}{l} \text{xxl}(1:\text{nprts}(2,\text{iblk})+1,2,\text{iblk}), \\ \text{fdx}(1:\text{nprts}(2,\text{iblk}),2,\text{iblk}), \end{array} \right\} \text{iblk} = 4n_t + 2.$$

Parameters that must be specified in the `namelist/input/`:

$$\left. \begin{array}{l} \text{ncell}(1,2,\text{iblk}), \\ \text{xxl}(2,2,\text{iblk}), \end{array} \right\} \text{iblk} = 4n_t + 2; \quad (9.20)$$

$$\text{nbc}(1,1,\text{iblk}), \quad \text{iblk} = 4n_t + 2, \dots, \text{nblks}.$$

Consistency conditions:

$$\sum_j \text{ncell}(j,1,\text{iblk}) = \sum_i \text{ncell}(i,1,\text{jblk}), \quad (9.21)$$

$$\text{jblk} = \text{nbc}(1,1,\text{iblk}), \quad \text{iblk} = 4n_t + 2 \rightarrow \text{nblks};$$

$$\text{xxl}(1,2,\text{iblk}+4) = \text{xxl}(\text{nprts}(2,\text{iblk})+1,2,\text{iblk}), \quad \text{iblk} = 4n_t - 2. \quad (9.22)$$

$$\text{xxl}(i,2,\text{iblk}) < \text{xxl}(i+1,2,\text{iblk}), \quad \text{iblk} = 4n_t + 2 \rightarrow \text{nblks}. \quad (9.23)$$

igeom=51: stretch parameters for the central block # 1

Parameters used to stretch the physical mesh in block # 1:

$$\begin{aligned} 0 &\leq \text{fdx}(1,1,1) \leq 1, \\ 0 &\leq \text{fdx}(1,2,1) \leq 1. \end{aligned} \quad (9.24)$$

3. Concluding remarks

Having specified the values of the above listed principal mesh parameters, one fully determines the physical part of the H_4 mesh. As usual, the relative width of the ghost-cell layer along the outer boundary is controlled by the user-defined parameter `ghwidth` (default value 10^{-4}). The type of the outer boundary condition is specified by the user-defined values of `ibc(ib,iblk)` along the outer block edges that border vacuum.

However, there remains freedom for dividing into parts those mesh blocks, whose values of `nprts(m,iblk)` and `ncell(ip,m,iblk)` are not required for mesh construction (like blocks 7–9 in Fig. 9.8. Hence, the user can, in addition, specify the values of these parameters in all the remaining blocks as well — provided that they are consistent with the number of cells along the common edges with other blocks. Such an additional division into parts can be used for complex distribution of different materials among different parts of any block in the mesh. The latter is facilitated by the fact that the mesh construction routine `MSHB51` does not change the values of `nprts(m,iblk)` and `ncell(ip,m,iblk)`. The mutual consistency of the values of these arrays is checked in the subroutine `MSHP51`, and the job is aborted if an inconsistency is found.

Finally, the assignment rules for the H_4 -mesh parameters can be summarized as follows.

- **fixed or restricted:** `iradial=0`,
`nblks \geq 5`,
`ibc(ib,iblk)=5` at common block faces.
- **user-must:** `nblks`
`ncell(1,1,2)`
`ncell(1,1,3)`
`xxl(1:2,1,2)`
`ncell(1,2,2), ncell(1,2,6), ...`
`xxl(1,2,2), xxl(1,2,6), ...`
`xxl(2,2,4nt - 2)`
`ncell(1,2,4nt + 2)`
`xxl(2,2,4nt + 2)`
`nbc(1,1,4nt + 2), nbc(1,1,4nt + 3), ...`
- **user-can:** the remaining principal mesh parameters

Explanations:

- `ncell(1,1,2)` is the number of physical cells along the polar angle θ in the angular sector 1 (blocks 4–1–2); it is also the number of cells along the “horizontal” edge of the central block 1;
- `ncell(1,1,3)` is the number of physical cells along the polar angle θ in the angular sector 2 (blocks 5–1–3); it is also the number of cells along the “vertical” edge of the central block 1;
- `xxl(1,1,2)` is the θ coordinate (in degrees of arc) of the lower-left corner of block 2;
- `xxl(nprts(1,2)+1,1,2)` is the θ coordinate (in degrees of arc) of the lower-right corner of block 2;
- `xxl(1,2,2)` is the r coordinate of the lower-left corner of block 2;

- $1 - \text{fdx}(1,1,1)$ is the stretch factor along angular sector 1 of the mesh in the central block 1 between rectangular and circular shapes;
- $1 - \text{fdx}(1,2,1)$ is the stretch factor along angular sector 2 of the mesh in the central block 1 between rectangular and circular shapes.

9.9. igeom=52: a multi-tier half-circle mesh with a free-float center

1. General description

Here we construct a half-circle quasi-polar mesh with a free-float center, labeled as an H_2 mesh (a “Hohlraum” mesh extending over 2 quadrants of the polar angle θ). It is suitable for both the Cartesian xy and the cylindrical rz geometries with $\text{iradial} = 0, 1, 2$. The *core* of the H_2 -mesh consists of $n_t \geq 1$ full radial tiers. Each full tier extends over 180° and comprises three neighboring blocks. With one extra block in the center (which is chosen to be the block # 1), the total number of blocks in the mesh core is $3n_t + 1$.

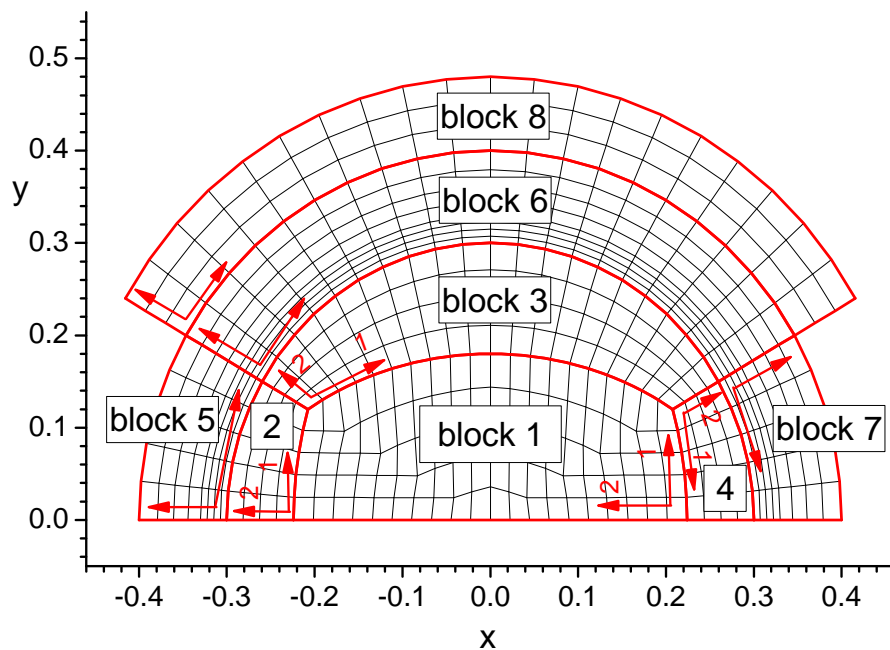


FIG. 9.9: An 8-block half-circle mesh with a free-float center, 7 core blocks, and one “cap” block (block 8).

Beside the $3n_t + 1$ *core blocks*, the H_2 -mesh can have 1 or 2 extra protruding blocks (*cap blocks*) in the unfilled $n_t + 1$ -th tier, i.e. the total number of blocks in the H_2 -mesh is

$$4 \leq 3n_t + 1 \leq \text{nblks} \leq 3n_t + 3. \tag{9.25}$$

Generally, different cap blocks must not touch one another along their side edges.

An example of the H_2 -mesh with $n_t = 2$ full tiers and one extra cap block is shown in Fig. 9.9. The H_2 -mesh is constructed in the progressive-mesh mode only, i.e. all the mesh dimensions are supposed to be set by assigning the arrays $\text{xx1}(ns+1, 2, nb)$ and $\text{fdx}(ns, 2, nb)$; the values of the mesh parameters $\text{x0}(2, nb)$ and $\text{dx}(ns, 2, nb)$ are not used.

The full set of parameters that completely specify the physical part of the H_2 mesh consists of the following variables and arrays

$$\begin{aligned}
 & \text{nblks}, \\
 & \text{nprts}(1:2,1:\text{nb}), \\
 & \text{ncell}(1:\text{ns},1:2,1:\text{nb}), \\
 & \text{xxl}(1:\text{ns}+1,1:2,1:\text{nb}), \\
 & \text{fdx}(1:\text{ns},1:2,1:\text{nb}), \\
 & \text{nbc}(1:2,1:4,1:\text{nb}).
 \end{aligned} \tag{9.26}$$

The mesh directions $m = 1$ and $m = 2$ are rigidly fixed within every block: in all blocks other than the central block 1 mesh direction 1 is always along the polar θ angle (as measured from the vertical axis in the clockwise direction), while mesh direction 2 is always along the polar radius r .

The mesh in the central block # 1 can be continuously stretched between two extreme configurations: a rectangle and a half-circle. The amount of stretch is independent along the two perpendicular directions and is controlled by the values of the two parameters $0 \leq \text{fdx}(1,1,1) \leq 1$ (the amount of stretch between blocks 4–1–2) and $0 \leq \text{fdx}(1,2,1) \leq 1$ (the amount of stretch between blocks 1–3). For the default values of $\text{fdx}(1,1,1) = \text{fdx}(1,2,1) = 1$ one obtains a rectangular shape of the central block. If the user sets $\text{fdx}(1,1,1) = \text{fdx}(1,2,1) = 0$, the result will be a circular central block. The two upper (along the y -axis) corners of block # 1 and the corresponding half-diagonals do not participate in the stretch. The mesh in Fig. 9.9 has been constructed with the values of $\text{fdx}(1,1,1) = \text{fdx}(1,2,1) = 0.5$.

In practice, to construct the H_2 mesh, one does not need the full set of parameters (9.26) but only its certain subset. We call the subset of the full list (9.26) that is actually used for mesh construction the *principal* mesh parameters. As usual, some of the principal mesh parameters have default values and must not (but can) be specified by the user, others do not have default values and must be specified in the `namelist/input/`.

A special feature of the H_2 mesh is that every individual block can be divided into parts in an arbitrary manner, independently of how other blocks are divided into parts. In other words, the values of `nprts(m,iblk)` can be chosen independently for each mesh direction $m = 1, 2$ in every block $\text{iblk} = 1, 2, \dots, \text{nblks}$. But then, because contacting blocks must have the same number of cells along common edges, the user-defined values of `ncell(iprt,m,iblk)` must satisfy certain consistency conditions.

2. Principal mesh parameters

igeom=52: mesh parameters for the θ -direction

Parameters used for the physical θ -mesh construction:

$$\left. \begin{aligned}
 & \text{nprts}(1, \text{iblk}), \\
 & \text{ncell}(1:\text{nprts}(1, \text{iblk}), 1, \text{iblk}), \\
 & \text{xxl}(1:\text{nprts}(1, \text{iblk})+1, 1, \text{iblk}), \\
 & \text{fdx}(1:\text{nprts}(1, \text{iblk}), 1, \text{iblk}),
 \end{aligned} \right\} \text{iblk} = 2, 3, 4. \tag{9.27}$$

Parameters that must be specified in the `namelist/input/`:

$$\begin{aligned} & \text{ncell}(1,1,2), \\ & \text{ncell}(1,1,3), \\ & \text{xxl}(1,1,2), \\ & \text{xxl}(2,1,2). \end{aligned} \tag{9.28}$$

Consistency conditions:

$$\sum_j \text{ncell}(j,1,4) = \sum_i \text{ncell}(i,1,2); \tag{9.29}$$

$$\begin{aligned} \text{xxl}(1,1,\text{iblk}+1) &= \text{xxl}(\text{nprts}(1,\text{iblk})+1,1,\text{iblk}), \quad \text{iblk} = 2, 3, \\ \text{xxl}(\text{nprts}(1,4)+1,1,4) &= \text{xxl}(1,1,2) + 180, \\ \text{xxl}(1,1,3) + \text{xxl}(\text{nprts}(1,3)+1,1,3) &= 2 \cdot \text{xxl}(1,1,\text{iblk}) + 180. \end{aligned} \tag{9.30}$$

Note that the values of $\text{xxl}(i,1,\text{iblk})$ for $\text{iblk} = 2-4$ specify the angular boundaries of the corresponding block parts in degrees of arc (as measured from the vertical axis in the clockwise direction). If only the values of $\text{xxl}(1:2,1,2)$ are assigned in the `namelist/input/`, then the values of $\text{xxl}(1:2,1,\text{iblk})$ for $\text{iblk} = 3$ and 4 are calculated from the consistency conditions (9.30).

igeom=52: mesh parameters for the r-direction in the core blocks

Parameters used for the physical r -mesh construction:

$$\begin{aligned} & \left. \begin{aligned} & \text{nprts}(2,\text{iblk}), \\ & \text{ncell}(1:\text{nprts}(2,\text{iblk}),2,\text{iblk}), \end{aligned} \right\} \text{iblk} = 2 \rightarrow 3n_t + 1. \\ & \left. \begin{aligned} & \text{xxl}(1:\text{nprts}(2,\text{iblk})+1,2,\text{iblk}), \\ & \text{fdx}(1:\text{nprts}(2,\text{iblk}),2,\text{iblk}), \end{aligned} \right\} \text{iblk} = 2, 5, \dots, 3n_t - 1. \end{aligned} \tag{9.31}$$

Parameters that must be specified in the `namelist/input/`:

$$\begin{aligned} & \left. \begin{aligned} & \text{ncell}(1,2,\text{iblk}), \\ & \text{xxl}(1,2,\text{iblk}), \end{aligned} \right\} \text{iblk} = 2, 5, \dots, 3n_t - 1. \\ & \text{xxl}(2,2,3n_t - 1). \end{aligned} \tag{9.32}$$

Consistency conditions:

$$\text{xxl}(1,2,\text{iblk}+3) = \text{xxl}(\text{nprts}(2,\text{iblk})+1,2,\text{iblk}), \quad \text{iblk} = 2, 3, \dots, 3n_t - 2. \tag{9.33}$$

$$\text{xxl}(i,2,\text{iblk}) < \text{xxl}(i+1,2,\text{iblk}), \quad \text{iblk} = 2, 3, \dots, 3n_t + 1. \tag{9.34}$$

igeom=52: mesh parameters for the "cap" blocks

Parameters used for the physical θ -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(1, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(1, \text{iblk}), 1, \text{iblk}), \\ \text{nbc}(1, 1, \text{iblk}), \end{array} \right\} \text{iblk} = 3n_t + 2 \rightarrow \text{nblks}. \quad (9.35)$$

Parameters used for the physical r -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(2, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \end{array} \right\} \text{iblk} = 3n_t + 2 \rightarrow \text{nblks}; \quad (9.36)$$

$$\left. \begin{array}{l} \text{xxl}(1:\text{nprts}(2, \text{iblk})+1, 2, \text{iblk}), \\ \text{fdx}(1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \end{array} \right\} \text{iblk} = 3n_t + 2;$$

Parameters that must be specified in the namelist/input/:

$$\left. \begin{array}{l} \text{ncell}(1, 2, \text{iblk}), \\ \text{xxl}(2, 2, \text{iblk}), \end{array} \right\} \text{iblk} = 3n_t + 2; \quad (9.37)$$

$$\text{nbc}(1, 1, \text{iblk}), \quad \text{iblk} = 3n_t + 2, \dots, \text{nblks}.$$

Consistency conditions:

$$\sum_j \text{ncell}(j, 1, \text{iblk}) = \sum_i \text{ncell}(i, 1, \text{jblk}), \quad (9.38)$$

$$\text{jblk} = \text{nbc}(1, 1, \text{iblk}), \quad \text{iblk} = 3n_t + 2 \rightarrow \text{nblks};$$

$$\text{xxl}(1, 2, \text{iblk}+3) = \text{xxl}(\text{nprts}(2, \text{iblk})+1, 2, \text{iblk}), \quad \text{iblk} = 3n_t - 1 \rightarrow \text{nblks} - 3. \quad (9.39)$$

$$\text{xxl}(i, 2, \text{iblk}) < \text{xxl}(i+1, 2, \text{iblk}), \quad \text{iblk} = 3n_t + 2 \rightarrow \text{nblks}. \quad (9.40)$$

igeom=52: stretch parameters for the central block # 1

Parameters used to stretch the physical mesh in block # 1:

$$\begin{aligned} 0 &\leq \text{fdx}(1, 1, 1) \leq 1, \\ 0 &\leq \text{fdx}(1, 2, 1) \leq 1. \end{aligned} \quad (9.41)$$

3. *Concluding remarks*

Having specified the values of the above listed principal mesh parameters, one fully determines the physical part of the H_2 mesh. As usual, the relative width of the ghost-cell layer along the outer boundary is controlled by the user-defined parameter `ghwidth` (default value 10^{-4}). The type of the outer boundary condition is specified by the user-defined values of `ibc(ib, iblk)` along the outer block edges that border vacuum.

However, there remains freedom for dividing into parts those mesh blocks, whose values of `nprts(m, iblk)` and `ncell(ip, m, iblk)` are not required for mesh construction. Hence,

the user can, in addition, specify the values of these parameters in all the remaining blocks as well — provided that they are consistent with the number of cells along the common edges with other blocks. Such an additional division into parts can be used for complex distribution of different materials among different parts of any block in the mesh. The latter is facilitated by the fact that the mesh construction routine `MSHB52` does not change the values of `nprts(m,iblk)` and `ncell(ip,m,iblk)`. The mutual consistency of the values of these arrays is checked in the subroutine `MSHP52`, and the job is aborted if an inconsistency is found.

Finally, the assignment rules for the H_2 -mesh parameters can be summarized as follows.

- **fixed or restricted:** `nblks` ≥ 4 ,
`ibc(ib,iblk)=5` at common block faces.
- **user-must:** `nblks`
`ncell(1,1,2)`
`ncell(1,1,3)`
`xxl(1:2,1,2)`
`ncell(1,2,2), ncell(1,2,5), ...`
`xxl(1,2,2), xxl(1,2,5), ...`
`xxl(2,2,3nt - 1)`
`ncell(1,2,3nt + 2)`
`xxl(2,2,3nt + 2)`
`nbc(1,1,3nt + 2), nbc(1,1,3nt + 3), ...`
- **user-can:** the remaining principal mesh parameters

Explanations:

- `ncell(1,1,2)` is the number of physical cells along the polar angle θ in the angular sector 1 (blocks 4-1-2); it is also the number of cells along the “horizontal” edge of the central block 1;
- `ncell(1,1,3)` is the number of physical cells along the polar angle θ in the angular sector 2 (blocks 1-3); it is also the number of cells along the “vertical” edge of the central block 1;
- `xxl(1,1,2)` is the θ coordinate (in degrees of arc) of the lower-left corner of block 2;
- `xxl(nprts(1,2)+1,1,2)` is the θ coordinate (in degrees of arc) of the lower-right corner of block 2;
- `xxl(1,2,2)` is the r coordinate of the lower-left corner of block 2;
- `1 - fdx(1,1,1)` is the stretch factor along angular sector 1 of the mesh in the central block 1 between rectangular and circular shapes;
- `1 - fdx(1,2,1)` is the stretch factor along angular sector 2 of the mesh in the central block 1 between rectangular and circular shapes.

9.10. igeom=211: a multi-block randomized x - y or r - z mesh

This is a distorted version of mesh constructed with $\text{IGEOM} = 1\text{--}4$. The distortion is set by shifting each inner vertex of each block to a random position on a circle of radius $0.2h$ around the original vertex location, where h is a minimum of the four distances to the four principal neighbors of the vertex in question; see Fig. 9.10. The vertices along the block edges are left at their original positions. Such a “random” mesh was proposed in Ref. [?].

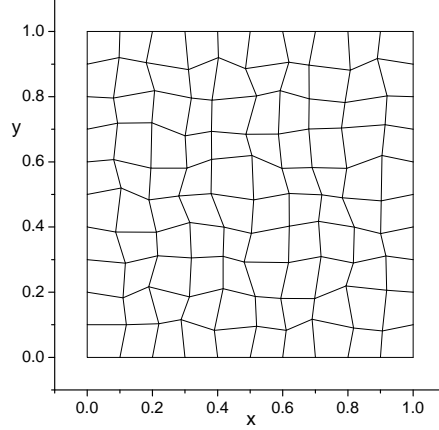


FIG. 9.10: A “random” 20%-distorted mesh in a single block $(x, y) \in [0, 1] \times [0, 1]$.

Mesh parameters for $\text{IGEOM} = 211$ are the same as for $\text{IGEOM} = 1, 2, 3$ or 4 .

9.11. igeom=212: a multi-block smoothly distorted x - y or r - z mesh

This is a distorted version of mesh constructed with $\text{IGEOM} = 1, 2$. In each block the distortion is set by a sine-modulated diagonal shift for each inner vertex of the originally generated mesh $\{x_{ij}, y_{ij}\}$

$$\begin{aligned} x'_{ij} &= x_{ij} + a_0 \sin(2\pi\xi_{ij}) \sin(2\pi\eta_{ij}), \\ y'_{ij} &= y_{ij} + a_0 \sin(2\pi\xi_{ij}) \sin(2\pi\eta_{ij}), \end{aligned} \tag{9.42}$$

where

$$\xi_{ij} = \frac{[(x_{ij} - x_{1j})^2 + (y_{ij} - y_{1j})^2]^{1/2}}{[(x_{N_x+1,j} - x_{1j})^2 + (y_{N_x+1,j} - y_{1j})^2]^{1/2}}, \tag{9.43}$$

$$\eta_{ij} = \frac{[(x_{ij} - x_{i1})^2 + (y_{ij} - y_{i1})^2]^{1/2}}{[(x_{i,N_y+1} - x_{i1})^2 + (y_{i,N_y+1} - y_{i1})^2]^{1/2}}; \tag{9.44}$$

see Fig. 9.11. The vertices along the block edges are left at their original positions. The parent mesh $\{x_{ij}, y_{ij}\}$ must have straight coordinate lines in the (x, y) plane. Evidently, transformation (9.42)–(9.44) can also be applied to the mesh option $\text{IGEOM} = 22$.

Mesh parameters for $\text{IGEOM} = 212$ are the same as for $\text{IGEOM} = 1$ or 2 .

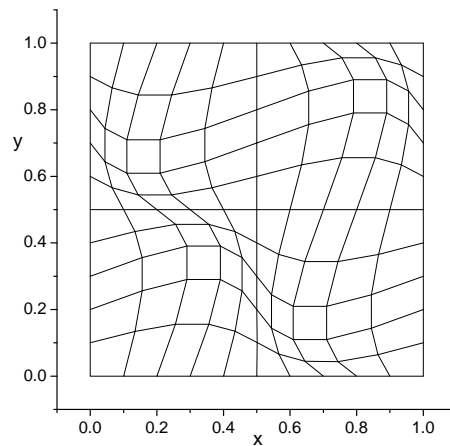


FIG. 9.11: A “wavy” distorted mesh in a single block $(x, y) \in [0, 1] \times [0, 1]$ with the perturbation amplitude $a_0 = 0.1$.

9.12. igeom=311: a multi-block randomized polar r - ϕ mesh in cylindrical (x, y) geometry

This is a distorted version of mesh constructed with $\text{IGEOM} = 3$. The distortion is set by shifting each inner vertex of each block to a random position on a circle of radius $0.2h$ around the original vertex location, where h is a minimum of the four distances to the four principal neighbors of the vertex in question; see Fig. 9.12. The vertices along the block edges are left at their original positions.

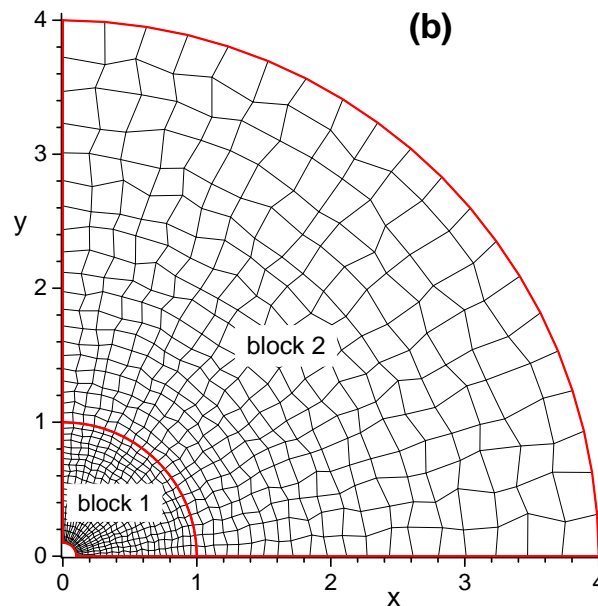


FIG. 9.12: A 2-block “random” 20%-distorted mesh in a quarter-circle.

Mesh parameters for $\text{IGEOM} = 311$ are the same as for $\text{IGEOM} = 3$.

9.13. igeom=513: a multi-tier full-circle mesh of polygon-mosaic type in the r, θ -coordinates

1. General description

Topologically, this mesh is equivalent to the above described H_4 mesh for $igeom = 51$. The only difference is that now in blocks 2, 3, ..., $nblks$ the mesh geometry in the r, θ -coordinates is piece-wise linear with a polygon-mosaic part-by-part structure — exactly as for $igeom = 23$ in the x, y -coordinates. We use the name H_4 -mosaic for this type of mesh. This mesh can be constructed for $iradial = 0$ only.

So far, construction of the *cap* blocks has not been implemented, and the mesh consists of $n_t \geq 1$ full radial tiers representing the *core* of the mesh. Each full tier consists of 4 neighboring blocks — but now only the *primary circle* of the mesh is assumed to span exactly 180° . The primary mesh circle is defined as a full circle composed of the three lower ($ib = 1$) edges of blocks 2, 3, 4, and 5 before the stretch transformation. With one extra block in the center (which is chosen to be the block # 1), the total number of blocks in the mesh core is $4n_t + 1$.

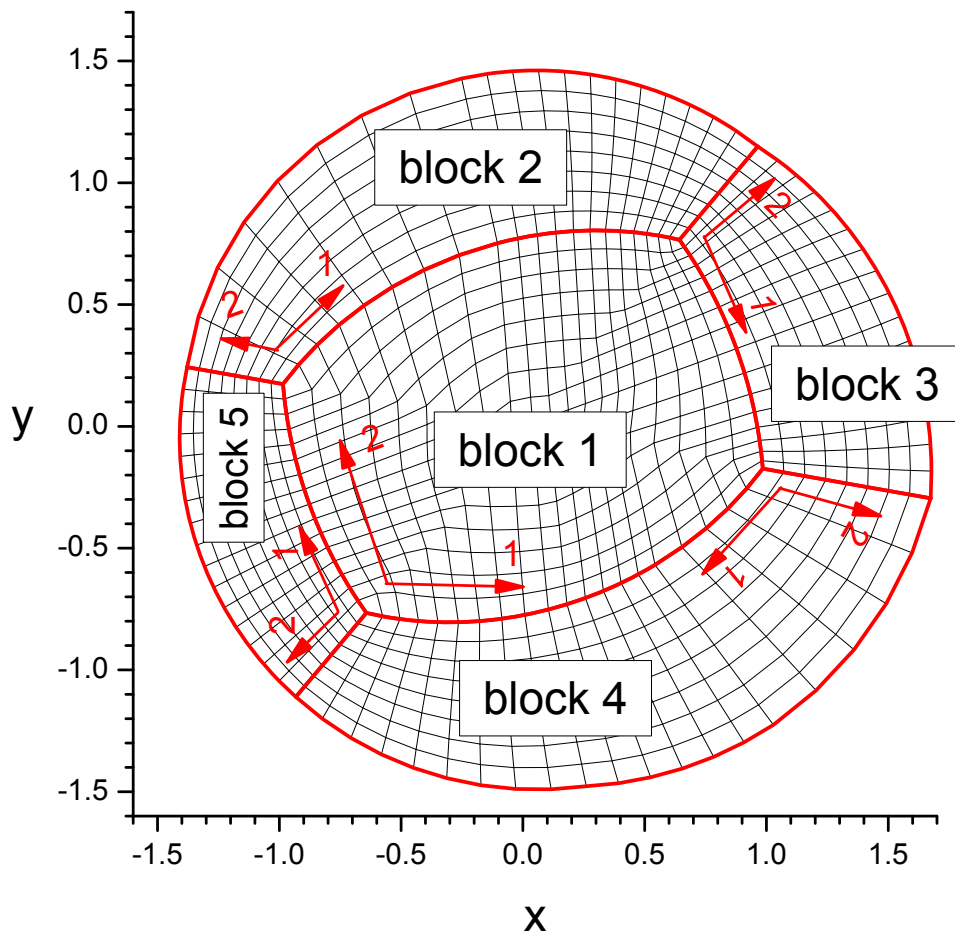


FIG. 9.13: A 5-block full-circle H_4 -mosaic mesh with a polygon-mosaic (r, θ) structure.

Beside the $4n_t + 1$ *core blocks*, this mesh can have up to 3 extra protruding blocks (*cap*

blocks) in the unfilled $n_t + 1$ -th tier, i.e. the total number of blocks is

$$4 \leq 4n_t + 1 \leq \text{nblks} \leq 4n_t + 4. \quad (9.45)$$

Generally, different cap blocks must not touch one another along their side edges. An example of the `igeom = 513` mesh with a single full tier is shown in Fig. 9.13.

The full set of parameters that completely specify the physical part of the H_4 -mosaic mesh includes the following variables and arrays

$$\begin{aligned} &\text{nblks}, \\ &\text{nprts}(1:2, 1:\text{nb}), \\ &\text{ncell}(1:\text{ns}, 1:2, 1:\text{nb}), \\ &\text{xxp}(1:\text{ns}+1, 1:\text{ns}+1, 1:2, 2:\text{nb}), \\ &\text{fdxp}(1:\text{ns}+1, 1:\text{ns}+1, 1:2, 1:\text{nb}), \\ &\text{nbc}(1:2, 1:4, 1:\text{nb}). \end{aligned} \quad (9.46)$$

The mesh directions $m = 1$ and $m = 2$ are rigidly fixed within every block: in all blocks, other than the central block 1, mesh direction 1 is always along the polar θ angle (as measured from the vertical axis in the clockwise direction), while mesh direction 2 is always along the polar radius r . Similarly to the case of `igeom = 23`, the mesh parameters `xxp` and `fdxp` specify the corner coordinates and the cell-size progression ratios for individual parts within every block, starting from block #2.

The mesh in the central block #1 can be continuously stretched between two extreme configurations: a rectangle and a full circle. The amount of stretch is independent along the two perpendicular directions and is controlled by the values of the two parameters $0 \leq \text{fdxp}(1,1,1,1) \leq 1$ (the amount of stretch between blocks 5–1–3) and $0 \leq \text{fdxp}(1,1,2,1) \leq 1$ (the amount of stretch between blocks 4–1–2). For the default values of $\text{fdxp}(1,1,1,1) = \text{fdxp}(1,1,2,1) = 1$ one obtains a rectangular shape of the central block. If the user sets $\text{fdxp}(1,1,1,1) = \text{fdxp}(1,1,2,1) = 0$, the result will be a circular central block. The four corners of block #1 and its diagonals do not participate in the stretch. The mesh in Fig. 9.13 was constructed with the values of $\text{fdxp}(1,1,1,1) = \text{fdxp}(1,1,2,1) = 0.5$.

In practice, to construct the H_4 -mosaic mesh, one does not need the full set of parameters (9.46) but only its certain subset. We call the subset of the full list (9.46), which is actually used for mesh construction, the *principal* mesh parameters. As usual, some of the principal mesh parameters have default values and must not (but can) be specified by the user, others do not have default values and must be specified in the `namelist/input/`.

A special feature of the H_4 -mosaic mesh is that every individual block can be divided into parts in an arbitrary manner, independently of the part structure of other blocks. In other words, the values of `nprts(m,iblk)` can be chosen independently for each mesh direction $m = 1, 2$ in every block $\text{iblk} = 1, 2, \dots, \text{nblks}$. But then, because contacting blocks must have the same number of cells along common edges, the user-defined values of `ncell(iprt,m,iblk)` must satisfy certain consistency conditions.

2. Principal mesh parameters

igeom=513: mesh parameters for the θ -direction

Parameters used for the physical θ -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(1, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(1, \text{iblk}), 1, \text{iblk}), \\ \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk}), 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \end{array} \right\} \text{iblk} = 2, 3, \dots, \text{nblks}. \quad (9.47)$$

Parameters that must be specified in the namelist/input/:

$$\begin{array}{l} \text{ncell}(1, 1, 2), \\ \text{ncell}(1, 1, 3), \\ \text{xxp}(1:2, 1:2, 1, 2:\text{nblks}), \end{array} \quad (9.48)$$

except for $\text{xxp}(1:2, 1, 1, 3)$, $\text{xxp}(1:2, 1, 1, 4)$, and $\text{xxp}(1:2, 1, 1, 5)$.

Consistency conditions:

$$\begin{aligned} \sum_j \text{ncell}(j, 1, 4) &= \sum_i \text{ncell}(i, 1, 2), \\ \sum_j \text{ncell}(j, 1, 5) &= \sum_i \text{ncell}(i, 1, 3), \end{aligned} \quad (9.49)$$

$$\begin{aligned} \text{xxp}(1, 1, 1, \text{iblk}+1) &= \text{xxp}(\text{nprts}(1, \text{iblk})+1, 1, 1, \text{iblk}), \quad \text{iblk} = 2, 3, 4, \\ \text{xxp}(1, 1, 1, 4) &= \text{xxp}(1, 1, 1, 2) + 180, \\ \text{xxp}(1, 1, 1, 5) &= \text{xxp}(1, 1, 1, 3) + 180. \end{aligned} \quad (9.50)$$

Also, one should of course pay attention that the coordinates of all meeting corners of different blocks have the same values.

Note that the values of $\text{xxp}(\text{ip}, \text{jp}, 1, \text{iblk})$ for $\text{iblk} = 2\text{--nblks}$ specify the angular boundaries of the corresponding block parts in degrees of arc (as measured from the vertical y -axis in the clockwise direction). If just the values of $\text{xxp}(1:2, 1, 1, 2)$ are assigned in the namelist/input/ [for $\text{nprts}(1, 2) = 1$], then the values of $\text{xxp}(1:2, 1, 1, \text{iblk})$ for $\text{iblk} = 3\text{--}5$ are calculated from the consistency conditions (9.50).

igeom=513: mesh parameters for the r -direction in the core blocks

Parameters used for the physical r -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(2, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \end{array} \right\} \text{iblk} = 2, 3, \dots, \text{nblks}.$$

$$\left. \begin{array}{l} \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 2, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \end{array} \right\} \text{iblk} = 2, 3, \dots, \text{nblks}. \quad (9.51)$$

Parameters that must be specified in the namelist/input/:

$$\begin{array}{l} \text{ncell}(1, 2, \text{iblk}), \quad \text{iblk} = 2, 6, \dots, 4n_t - 2. \\ \text{xxp}(1:2, 1:2, 2, \text{iblk}), \quad \text{iblk} = 2, 3, \dots, \text{nblks}, \end{array} \quad (9.52)$$

except for $\text{xxp}(2,1,2,2)$, $\text{xxp}(1:2,1,2,3)$, $\text{xxp}(1:2,1,2,4)$, and $\text{xxp}(1:2,1,2,5)$ because

$$\text{xxp}(2,1,2,2) = \text{xxp}(1:2,1,2,3) = \text{xxp}(1:2,1,2,4) = \text{xxp}(1:2,1,2,5) = \text{xxp}(1,1,2,2)$$
(9.53)

is the radius of the primary circle fixed by the specified value of $\text{xxp}(1,1,2,2)$.

igeom=513: mesh parameters for the “cap” blocks

Parameters used for the physical θ -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(1, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(1, \text{iblk}), 1, \text{iblk}), \\ \text{nbc}(1, 1, \text{iblk}), \\ \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk}), 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \end{array} \right\} \text{iblk} = 4n_t + 2 \rightarrow \text{nblks}.$$
(9.54)

Parameters used for the physical r -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(2, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \\ \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 2, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \end{array} \right\} \text{iblk} = 4n_t + 2 \rightarrow \text{nblks}.$$
(9.55)

igeom=513: stretch parameters for the central block # 1

Parameters used to stretch the physical mesh in block #1:

$$\begin{aligned} 0 &\leq \text{fdx}(1, 1, 1, 1) \leq 1, \\ 0 &\leq \text{fdx}(1, 1, 2, 1) \leq 1. \end{aligned}$$
(9.56)

Example:

The mesh shown in Fig. 9.13 has been constructed with the following set of parameters:

- geometry:
 - `igeom=513`
 - `iradial=0`
 - `nblks=5`
- theta-mesh:
 - `nprts(1,2)=2`
 - `nprts(1,3)=2`
 - `nprts(1,4)=3`
 - `ncell(1,1,2)=8,12`
 - `ncell(1,1,3)=8,8`
 - `ncell(1,1,4)=5,6,9`
 - `ncell(1,1,5)=16`

```

xyp(1,1,1,2)=-80.d0, -10.d0, 40.d0
xyp(1,2,1,2)=-80.d0, -10.d0, 40.d0
fdxp(1,1,1,2)=1.05d0, .95d0
xyp(1,1,1,3)=40.d0, 70.d0, 100.d0
xyp(1,2,1,3)=40.d0, 70.d0, 100.d0
fdxp(1,1,1,3)=1.15d0, .85d0
xyp(1,1,1,4)=100.d0, 140.d0, 170.d0, 220.d0
xyp(1,2,1,4)=100.d0, 140.d0, 170.d0, 220.d0
fdxp(1,1,1,4)=1.03d0, 1.00d0, .97d0
xyp(1,1,1,5)=220.d0, 280.d0
xyp(1,2,1,5)=220.d0, 280.d0

- block 1 is stretched 50%:
fdxp(1,1,1,1)=0.5d0
fdxp(1,1,2,1)=0.5d0

- r-mesh:
ncell(1,2,2)=8
ncell(1,2,3)=8
ncell(1,2,4)=8
ncell(1,2,5)=8
xyp(1,1,2,2)=1.d0, 1.0d0, 1.d0
xyp(1,2,2,2)=1.4d0, 1.45d0, 1.5d0
xyp(1,1,2,3)=1.d0, 1.d0, 1.d0
xyp(1,2,2,3)=1.5d0, 1.6d0, 1.7d0
xyp(1,1,2,4)=1.d0, 1.d0, 1.d0, 1.d0
xyp(1,2,2,4)=1.7d0, 1.6d0, 1.5d0, 1.45d0
xyp(1,1,2,5)=1.d0, 1.d0
xyp(1,2,2,5)=1.45d0, 1.4d0

```

3. Concluding remarks

Having specified the values of the above listed principal mesh parameters, one fully determines the physical part of the H_4 -mosaic mesh. As usual, the relative width of the ghost-cell layer along the outer boundary is controlled by the user-defined parameter `ghwidth` (default value 10^{-4}). The type of the outer boundary condition is specified by the user-defined values of `ibc(ib,iblk)` along the outer block edges that border vacuum. Mutual consistency of the mesh parameters assigned by the user is partially checked in the subroutine `MSHP513`, and the job is aborted if any inconsistency is found.

Explanations:

- `nprts(m,iblk)` is the number of parts along mesh direction `m` in block `iblk`;
- `ncell(iprt,m,iblk)` is the number of physical cells along mesh direction `m` in part `iprt` (along this direction) of block `iblk`;
- `xyp(iprt,jprt,1,iblk)` is the θ ($= x_1$) coordinate (in degrees of arc as measured from the vertical x_2 -axis) of the lower-left corner of part `(iprt,jprt)` in block `iblk`; `iprt` is the part index along mesh direction 1, `jprt` is the part index along mesh direction 2; correspondingly, the lower-right, upper-left, and upper-right corners of part `(iprt,jprt)` in block `iblk` have the x -coordinates `xyp(iprt+1,jprt,1,iblk)`,

- $\text{xxp}(\text{iprt}, \text{jprt}+1, 1, \text{iblk})$, and $\text{xxp}(\text{iprt}+1, \text{jprt}+1, 1, \text{iblk})$; here $\text{iblk} = 2, 3, \dots, \text{nblks}$; default value is `undef`;
- $\text{xxp}(\text{iprt}, \text{jprt}, 2, \text{iblk})$ is the r ($= x_2$) coordinate of the lower-left corner of part $(\text{iprt}, \text{jprt})$ in block iblk ; here $\text{iblk} = 2, 3, \dots, \text{nblks}$; default value is `undef`;
- $\text{fdxp}(\text{iprt}, \text{jprt}, 1, \text{iblk})$ is the common ratio of successive cell sizes along the part edge, starting from the lower-left corner of part $(\text{iprt}, \text{jprt})$ along mesh direction 1, in block iblk ; correspondingly, $\text{fdxp}(\text{iprt}, \text{jprt}+1, 1, \text{iblk})$ is the common ratio of successive cell sizes along the part edge, starting from the upper-left corner of part $(\text{iprt}, \text{jprt})$ along mesh direction 1; here $\text{iblk} = 2, 3, \dots, \text{nblks}$; default value is 1.0;
- $\text{fdxp}(\text{iprt}, \text{jprt}, 2, \text{iblk})$ is the common ratio of successive cell sizes along the part edge, starting from the lower-left corner of part $(\text{iprt}, \text{jprt})$ along mesh direction 2, in block iblk ; correspondingly, $\text{fdxp}(\text{iprt}+1, \text{jprt}, 1, \text{iblk})$ is the common ratio of successive cell sizes along the part edge, starting from the lower-right corner of part $(\text{iprt}, \text{jprt})$ along mesh direction 2; here $\text{iblk} = 2, 3, \dots, \text{nblks}$; default value is 1.0;
- $\text{xxp}(1, 1, 2, 2)$ is the radius of the primary circle of the H_4 -mosaic mesh;
- $1 - \text{fdxp}(1, 1, 1, 1)$ is the stretch factor along angular sector 1 of the mesh in the central block 1 between rectangular and circular shapes;
- $1 - \text{fdxp}(1, 1, 2, 1)$ is the stretch factor along angular sector 2 of the mesh in the central block 1 between rectangular and circular shapes.

9.14. igeom=523: a multi-tier half-circle mesh of polygon-mosaic type in the r, θ -coordinates

1. General description

Topologically, this mesh is equivalent to the above described H_2 mesh for `igeom = 52`. The only difference is that now in blocks $2, 3, \dots, 3n_t + 1$ the mesh geometry in the r, θ -coordinates is piece-wise linear with a polygon-mosaic part-by-part structure — exactly as for `igeom = 23` in the x, y -coordinates. We will use the name H_2 -mosaic for this type of mesh.

So far, construction of the *cap* blocks has not been implemented, and the mesh consists of $n_t \geq 1$ full radial tiers representing the *core* of the mesh. Each full tier consists of 3 neighboring blocks — but now only the *primary circle* of the mesh is assumed to span exactly 180° . The primary mesh circle is defined as a circular arc composed of the three lower ($\text{ib} = 1$) edges of blocks 2, 3, and 4 before the stretch transformation. With one extra block in the center (which is chosen to be the block # 1), the total number of blocks in the mesh core is $3n_t + 1$.

Beside the $3n_t + 1$ *core blocks*, this mesh can have 1 or 2 extra protruding blocks (*cap blocks*) in the unfilled $n_t + 1$ -th tier, i.e. the total number of blocks is

$$4 \leq 3n_t + 1 \leq \text{nblks} \leq 3n_t + 3. \tag{9.57}$$

Generally, different cap blocks must not touch one another along their side edges. An example of the `igeom = 523` mesh with a single full tier is shown in Fig. 9.14.

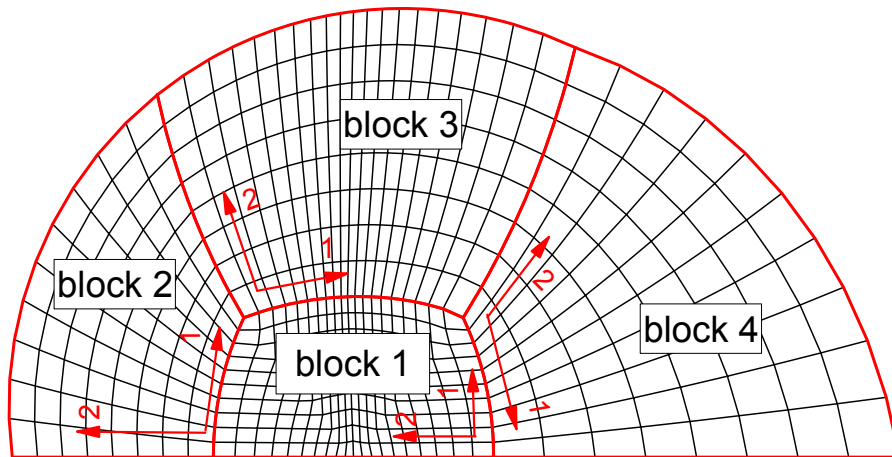


FIG. 9.14: A 4-block half-circle H_2 -mosaic mesh with a polygon-mosaic (r, θ) structure.

The full set of parameters that completely specify the physical part of the H_2 -mosaic mesh includes the following variables and arrays

$$\begin{aligned}
 & \text{nblks}, \\
 & \text{nprts}(1:2, 1:\text{nb}), \\
 & \text{ncell}(1:\text{ns}, 1:2, 1:\text{nb}), \\
 & \text{xxp}(1:\text{ns}+1, 1:\text{ns}+1, 1:2, 2:\text{nb}), \\
 & \text{fdxp}(1:\text{ns}+1, 1:\text{ns}+1, 1:2, 1:\text{nb}), \\
 & \text{NBC}(1:2, 1:4, 1:\text{nb}).
 \end{aligned} \tag{9.58}$$

The mesh directions $m = 1$ and $m = 2$ are rigidly fixed within every block: in all blocks other than the central block #1 mesh direction 1 is always along the polar θ angle (as measured from the vertical axis in the clockwise direction), while mesh direction 2 is always along the polar radius r . Similarly to the case of `igeom = 23`, the mesh parameters `xxp` and `fdxp` specify the corner coordinates and the cell-size progression ratios for individual parts within every block, starting from block #2.

The mesh in the central block #1 can be continuously stretched between two extreme configurations: a rectangle and a half-circle. The amount of stretch is independent along the two perpendicular directions and is controlled by the values of the two parameters $0 \leq \text{fdxp}(1,1,1,1) \leq 1$ (the amount of stretch between blocks 4–1–2) and $0 \leq \text{fdxp}(1,1,2,1) \leq 1$ (the amount of stretch between blocks 1–3). For the default values of $\text{fdxp}(1,1,1,1) = \text{fdxp}(1,1,2,1) = 1$ one obtains a rectangular shape of the central block. If the user sets $\text{fdxp}(1,1,1,1) = \text{fdxp}(1,1,2,1) = 0$, the result will be a circular central block. The two upper (along the y -axis) corners of block #1 and the corresponding half-diagonals do not participate in the stretch. The mesh in Fig. 9.14 was constructed with the values of $\text{fdxp}(1,1,1,1) = \text{fdxp}(1,1,2,1) = 0.5$.

In practice, to construct the H_2 -mosaic mesh, one does not need the full set of parameters (9.58) but only its certain subset. We call the subset of the full list (9.58) that is actually used for mesh construction the *principal* mesh parameters. As usual, some of the principal mesh parameters have default values and must not (but can) be specified by the user, others do not have default values and must be specified in the `namelist/input/`.

A special feature of the H_2 -mosaic mesh is that every individual block can be divided into parts in an arbitrary manner, independently of the part structure of other blocks.

In other words, the values of `nprts(m,iblk)` can be chosen independently for each mesh direction $m = 1, 2$ in every block $iblk = 1, 2, \dots, nblks$. But then, because contacting blocks must have the same number of cells along common edges, the user-defined values of `ncell(iprt,m,iblk)` must satisfy certain consistency conditions.

2. *Principal mesh parameters*

igeom=523: mesh parameters for the θ -direction

Parameters used for the physical θ -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(1, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(1, \text{iblk}), 1, \text{iblk}), \\ \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk}), 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \end{array} \right\} \text{iblk} = 2, 3, \dots, \text{nblks}. \quad (9.59)$$

Parameters that must be specified in the `namelist/input/`:

$$\begin{array}{l} \text{ncell}(1, 1, 2), \\ \text{ncell}(1, 1, 3), \\ \text{xxp}(1:2, 1:2, 1, 2:\text{nblks}), \end{array} \quad (9.60)$$

except for `xxp(1:2,1,1,3)` and `xxp(1:2,1,1,4)`.

Consistency conditions:

$$\sum_j \text{ncell}(j, 1, 4) = \sum_i \text{ncell}(i, 1, 2), \quad (9.61)$$

$$\begin{array}{l} \text{xxp}(1, 1, 1, \text{iblk}+1) = \text{xxp}(\text{nprts}(1, \text{iblk})+1, 1, 1, \text{iblk}), \quad \text{iblk} = 2, 3, \\ \text{xxp}(\text{nprts}(1, 4)+1, 1, 1, 4) = \text{xxp}(1, 1, 1, 2) + 180, \\ \text{xxp}(1, 1, 1, 3) + \text{xxp}(\text{nprts}(1, 3)+1, 1, 1, 3) = 2 \cdot \text{xxp}(1, 1, 1, \text{iblk}) + 180. \end{array} \quad (9.62)$$

Also, one should of course pay attention that the coordinates of all meeting corners of different blocks have the same values.

Note that the values of `xxp(ip,jp,1,iblk)` for $iblk = 2$ – $nblks$ specify the angular boundaries of the corresponding block parts in degrees of arc (as measured from the vertical y -axis in the clockwise direction). If only the values of `xxp(1:2,1,1,2)` are assigned in the `namelist/input/` [for `nprts(1,2) = 1`], then the values of `xxp(1:2,1,1,iblk)` for $iblk = 3$ and 4 are calculated from the consistency conditions (9.62).

igeom=523: mesh parameters for the r -direction in the core blocks

Parameters used for the physical r -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(2, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \end{array} \right\} \text{iblk} = 2, 3, \dots, \text{nblks}.$$

$$\left. \begin{array}{l} \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 2, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \end{array} \right\} \text{iblk} = 2, 3, \dots, \text{nblks}.$$
(9.63)

Parameters that must be specified in the namelist/input/:

$$\begin{array}{l} \text{ncell}(1, 2, \text{iblk}), \quad \text{iblk} = 2, 5, \dots, 3n_t - 1. \\ \text{xxp}(1:2, 1:2, 2, \text{iblk}), \quad \text{iblk} = 2, 3, \dots, \text{nblks}, \end{array}$$
(9.64)

except for $\text{xxp}(2, 1, 2, 2)$, $\text{xxp}(1:2, 1, 2, 3)$ and $\text{xxp}(1:2, 1, 2, 4)$ because

$$\text{xxp}(2, 1, 2, 2) = \text{xxp}(1:2, 1, 2, 3) = \text{xxp}(1:2, 1, 2, 4) = \text{xxp}(1, 1, 2, 2)$$
(9.65)

is the radius of the primary circle fixed by the specified value of $\text{xxp}(1, 1, 2, 2)$.

igeom=523: mesh parameters for the “cap” blocks

Parameters used for the physical θ -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(1, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(1, \text{iblk}), 1, \text{iblk}), \\ \text{nbc}(1, 1, \text{iblk}), \\ \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk}), 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \end{array} \right\} \text{iblk} = 3n_t + 2 \rightarrow \text{nblks}.$$
(9.66)

Parameters used for the physical r -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(2, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \\ \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 2, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \end{array} \right\} \text{iblk} = 3n_t + 2 \rightarrow \text{nblks}.$$
(9.67)

Parameters that must be specified in the namelist/input/:

$$\begin{array}{l} \text{ncell}(1, 2, \text{iblk}), \quad \text{iblk} = 3n_t + 2, \\ \text{nbc}(1, 1, \text{iblk}), \quad \text{iblk} = 3n_t + 2, \dots, \text{nblks}, \\ \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 1:2, \text{iblk}), \quad \text{iblk} = 3n_t + 2, \dots, \text{nblks}. \end{array}$$
(9.68)

igeom=523: stretch parameters for the central block # 1

Parameters used to stretch the physical mesh in block #1:

$$\begin{array}{l} 0 \leq \text{fdx}(1, 1, 1, 1) \leq 1, \\ 0 \leq \text{fdx}(1, 1, 2, 1) \leq 1. \end{array}$$
(9.69)

Example:

The mesh shown in Fig. 9.14 has been constructed with the following set of parameters:

```

- geometry:
  igeom=523
  iradial=0
  nblks=4

- theta-mesh:
  nprts(1,3)=2
  ncell(1,1,2)=10
  ncell(1,1,3)=10,10
  ncell(1,1,4)=10
  xxp(1,1,1,2)=-90.d0, -40.d0
  xxp(1,2,1,2)=-90.d0, -30.d0
  xxp(1,1,1,3)=-40.d0, 0.d0, 40.d0
  xxp(1,2,1,3)=-30.d0, 0.d0, 30.d0
  xxp(1,1,1,4)=40.d0, 90.d0
  xxp(1,2,1,4)=30.d0, 90.d0
  fdxp(1,1,1,3)=.9d0, 1.1d0
  fdxp(1,2,1,3)=.9d0, 1.1d0

- block 1 is stretched 50%:
  fdxp(1,1,1,1)=0.5d0
  fdxp(1,1,2,1)=0.5d0

- r-mesh:
  ncell(1,2,2)=8
  xxp(1,1,2,2)=1.d0, 1.d0
  xxp(1,2,2,2)=2.d0, 2.3d0
  xxp(1,1,2,3)=1.d0, 1.d0, 1.d0
  xxp(1,2,2,3)=2.3d0, 2.45d0, 2.6d0
  xxp(1,1,2,4)=1.d0, 1.d0
  xxp(1,2,2,4)=2.6d0, 3.2d0

```

3. Concluding remarks

Having specified the values of the above listed principal mesh parameters, one fully determines the physical part of the H_2 -mosaic mesh. As usual, the relative width of the ghost-cell layer along the outer boundary is controlled by the user-defined parameter `ghwidth` (default value 10^{-4}). The type of the outer boundary condition is specified by the user-defined values of `ibc(ib,iblk)` along the outer block edges that border vacuum. Mutual consistency of the mesh parameters assigned by the user is partially checked in the subroutine `MSHP523`, and the job is aborted if an inconsistency is found.

Explanations:

- `nprts(m,iblk)` is the number of parts along mesh direction `m` in block `iblk`;
- `ncell(iprt,m,iblk)` is the number of physical cells along mesh direction `m` in part `iprt` (along this direction) of block `iblk`;

- $\text{xxp}(\text{iprt}, \text{jpvt}, 1, \text{iblk})$ is the θ ($= x_1$) coordinate (in degrees of arc as measured from the vertical x_2 -axis) of the lower-left corner of part $(\text{iprt}, \text{jpvt})$ in block iblk ; iprt is the part index along mesh direction 1, jpvt is the part index along mesh direction 2; correspondingly, the lower-right, upper-left, and upper-right corners of part $(\text{iprt}, \text{jpvt})$ in block iblk have the x -coordinates $\text{xxp}(\text{iprt}+1, \text{jpvt}, 1, \text{iblk})$, $\text{xxp}(\text{iprt}, \text{jpvt}+1, 1, \text{iblk})$, and $\text{xxp}(\text{iprt}+1, \text{jpvt}+1, 1, \text{iblk})$; here $\text{iblk} = 2, 3, \dots, \text{nblks}$; default value is `undef`;
- $\text{xxp}(\text{iprt}, \text{jpvt}, 2, \text{iblk})$ is the r ($= x_2$) coordinate of the lower-left corner of part $(\text{iprt}, \text{jpvt})$ in block iblk ; here $\text{iblk} = 2, 3, \dots, \text{nblks}$; default value is `undef`;
- $\text{fdxp}(\text{iprt}, \text{jpvt}, 1, \text{iblk})$ is the common ratio of successive cell sizes along the part edge, starting from the lower-left corner of part $(\text{iprt}, \text{jpvt})$ along mesh direction 1, in block iblk ; correspondingly, $\text{fdxp}(\text{iprt}, \text{jpvt}+1, 1, \text{iblk})$ is the common ratio of successive cell sizes along the part edge, starting from the upper-left corner of part $(\text{iprt}, \text{jpvt})$ along mesh direction 1; here $\text{iblk} = 2, 3, \dots, \text{nblks}$; default value is 1.0;
- $\text{fdxp}(\text{iprt}, \text{jpvt}, 2, \text{iblk})$ is the common ratio of successive cell sizes along the part edge, starting from the lower-left corner of part $(\text{iprt}, \text{jpvt})$ along mesh direction 2, in block iblk ; correspondingly, $\text{fdxp}(\text{iprt}+1, \text{jpvt}, 1, \text{iblk})$ is the common ratio of successive cell sizes along the part edge, starting from the lower-right corner of part $(\text{iprt}, \text{jpvt})$ along mesh direction 2; here $\text{iblk} = 2, 3, \dots, \text{nblks}$; default value is 1.0;
- $\text{xxp}(1, 1, 2, 2)$ is the radius of the primary circle of the H_2 -mosaic mesh;
- $1 - \text{fdxp}(1, 1, 1, 1)$ is the stretch factor along angular sector 1 of the mesh in the central block 1 between rectangular and circular shapes;
- $1 - \text{fdxp}(1, 1, 2, 1)$ is the stretch factor along angular sector 2 of the mesh in the central block 1 between rectangular and circular shapes.

9.15. `igeom=533` or `534`: a multi-tier quarter-circle mesh of polygon-mosaic type in the r, θ -coordinates

1. General description

Topologically, this mesh is a $\pi/4$ version of the above described H_2 mesh for `igeom = 523`. For `igeom = 533` it consists of an arbitrary number of tiers n_t , each tier having 2 blocks covering the polar angle of 90° : tier 1 consists of blocks 2 and 3, tier 2 consists of blocks 4 and 5, etc. Block 1 is the central block. We use the name H_1 -mosaic for this type of mesh. For `igeom = 534` the `igeom = 533` mesh, called here the *core* of the `igeom = 534` mesh, is augmented by $n_t + 1$ side blocks along the right edge of the $\pi/4$ angle. Thus, the total number of blocks in the mesh is $2n_t + 1$ for `igeom = 533`, and $3n_t + 2$ for `igeom = 534`. The primary mesh circle is defined as a circular arc composed of the two lower ($\text{ib} = 1$) edges of blocks 2 and 3 before the eventual stretch transformation. The center of the primary circle (i.e. the coordinate origin in block 1) is assumed to be at $(x, y) \equiv (x_1, x_2) = (0, 0)$.

The full set of parameters that completely specify the physical part of the H_1 -mosaic

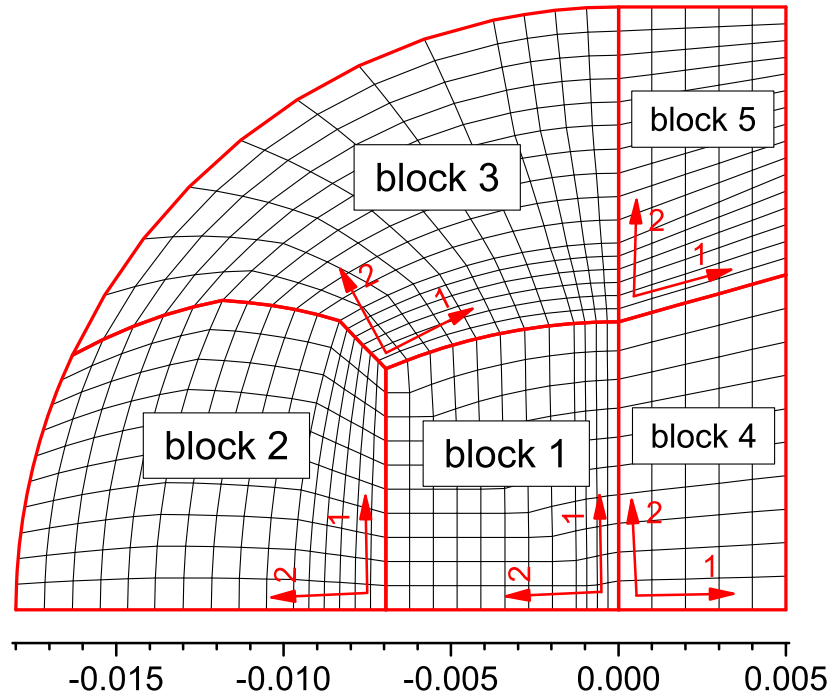


FIG. 9.15: A 5-block single-tier quarter-circle H_1 -mosaic mesh with a polygon-mosaic (r, θ) structure.

mesh includes the following variables and arrays

$$\begin{aligned}
 & \text{nblks,} \\
 & \text{nprts}(1:2, 1:\text{nb}), \\
 & \text{ncell}(1:\text{ns}, 1:2, 1:\text{nb}), \\
 & \text{xxp}(1:\text{ns}+1, 1:\text{ns}+1, 1:2, 1:\text{nb}), \\
 & \text{fdxp}(1:\text{ns}+1, 1:\text{ns}+1, 1:2, 1:\text{nb}), \\
 & \text{NBC}(1:2, 1:4, 1:\text{nb}),
 \end{aligned} \tag{9.70}$$

of which not all are mutually independent. The mesh directions $m = 1$ and $m = 2$ are rigidly fixed within every block: in all blocks other than the central block #1 mesh direction 1 is always along the polar θ angle (as measured from the vertical axis in the clockwise direction), while mesh direction 2 is always along the polar radius r . Similarly to the case of `igeom = 23`, the mesh parameters `xxp` and `fdxp` specify the corner coordinates and the cell-size progression ratios for individual parts within every block, starting from block #2.

If bloc #1 has more than one part in any of the two mesh directions, the `xxp` and `fdxp` values must be specified for this block as well.

Important: in such a case the `xpx` values for both mesh directions in block #1 must be given in degrees of arc.

The mesh in the central block #1 can be continuously stretched between two extreme configurations: a rectangle and a half-circle. The amount of stretch is independent along the two perpendicular directions and is controlled by the values of the two parameters $0 \leq \text{fdxp}(1, \text{nprts}(2,1)+1,1,1) \leq 1$ (the amount of stretch between blocks 1-2) and $0 \leq \text{fdxp}(2, \text{nprts}(2,1)+1,1,1) \leq 1$ (the amount of stretch between blocks 1-3) [**note the difference with the case of `igeom = 523` !**]. For the default values of $\text{fdxp}(1, \text{nprts}(2,1)+1,1,1) = \text{fdxp}(2, \text{nprts}(2,1)+1,1,1) = 1$ one obtains a rectangular shape of the central block. If the user sets $\text{fdxp}(1, \text{nprts}(2,1)+1,1,1) = \text{fdxp}(2, \text{nprts}(2,1)+1,1,1) = 0$, the result will be a circular central block. The two upper (along the y -axis) corners of block #1 and the corresponding half-diagonals do not participate in the stretch. The mesh in Fig. 9.15 was constructed with the values of $\text{fdxp}(1, \text{nprts}(2,1)+1,1,1) = 1, \text{fdxp}(2, \text{nprts}(2,1)+1,1,1) = 0.5$.

In practice, to construct the H_1 -mosaic mesh, one does not need the full set of parameters (9.70) but only its certain subset. We call the subset of the full list (9.70) that is actually used for mesh construction the *principal* mesh parameters. As usual, some of the principal mesh parameters have default values and must not (but can) be specified by the user, others do not have default values and must be specified in the `namelist/input/`.

A special feature of the H_1 -mosaic mesh is that every individual block can be divided into parts in an arbitrary manner, independently of the part structure of other blocks. In other words, the values of `nprts(m, iblk)` can be chosen independently for each mesh direction $m = 1, 2$ in every block $\text{iblk} = 1, 2, \dots, \text{nblks}$. But then, because contacting blocks must have the same number of cells along common edges, the user-defined values of `ncell(iprt, m, iblk)` must satisfy the necessary consistency conditions.

2. Principal mesh parameters

igeom=533 or 534: mesh parameters for the θ -direction

Parameters used for the physical θ -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(1, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(1, \text{iblk}), 1, \text{iblk}), \\ \text{xpx}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk}), 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \end{array} \right\} \text{iblk} = 2, 3, \dots, \text{nblks}. \quad (9.71)$$

Parameters that must be specified in the `namelist/input/`:

$$\begin{array}{l} \text{ncell}(1, 1, 2), \\ \text{ncell}(1, 1, 3), \\ \text{xpx}(1:2, 1:2, 1, 2:\text{nblks}), \end{array} \quad (9.72)$$

except for `xpx(1:2, 1, 1, 3)`. If block 1 has more than one part in any direction m , the corresponding values of `ncell(1:nprts(m, 1), m, 1)` and `xpx(2:nprts(1, 1), 2:nprts(2, 1), 1:2, 1)` for the inner part corners must be also specified.

Consistency conditions:

$$\begin{array}{l} \text{xpx}(1, 1, 1, 3) = \text{xpx}(\text{nprts}(1, \text{iblk})+1, 1, 1, 2), \\ \text{xpx}(\text{nprts}(1, 3)+1, 1, 1, 3) = \text{xpx}(1, 1, 1, 2) + 90. \end{array} \quad (9.73)$$

Also, one should of course pay attention that the coordinates of all meeting corners of different blocks have the same values.

Note that the values of $\text{xxp}(\text{ip}, \text{jp}, 1, \text{iblk})$ for $\text{iblk} = 2, 3, \dots, 2n_t + 1$ specify the angular boundaries of the corresponding block parts in degrees of arc (as measured from the vertical y -axis in the clockwise direction). At the same time, all the values of $\text{xxp}(\text{ip}, \text{jp}, 1:2, \text{iblk})$ for the side blocks (when $\text{igeom} = 534$) $\text{iblk} = 2n_t + 2, \dots, 3n_t + 2$ are in usual length units.

igeom=533 or 534: mesh parameters for the r -direction in the core blocks

Parameters used for the physical r -mesh construction:

$$\left. \begin{array}{l} \text{nprts}(2, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \end{array} \right\} \text{iblk} = 2, 3, \dots, \text{nblks}.$$

$$\left. \begin{array}{l} \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 2, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \end{array} \right\} \text{iblk} = 2, 3, \dots, \text{nblks}.$$
(9.74)

Parameters that must be specified in the `namelist/input/`:

$$\begin{array}{l} \text{ncell}(1, 2, \text{iblk}), \quad \text{iblk} = 2, 4, \dots, 2n_t. \\ \text{xxp}(1:2, 1:2, 2, \text{iblk}), \quad \text{iblk} = 2, 3, \dots, \text{nblks}, \end{array}$$
(9.75)

except for $\text{xxp}(2, 1, 2, 2)$ and $\text{xxp}(1:2, 1, 2, 3)$ because

$$\text{xxp}(2, 1, 2, 2) = \text{xxp}(1:2, 1, 2, 3) = \text{xxp}(1, 1, 2, 2)$$
(9.76)

is the radius of the primary circle fixed by the specified value of $\text{xxp}(1, 1, 2, 2)$.

igeom=534: mesh parameters for the side blocks

Parameters used for the physical θ -mesh (or x -mesh) construction:

$$\left. \begin{array}{l} \text{nprts}(1, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(1, \text{iblk}), 1, \text{iblk}), \\ \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk}), 1:\text{nprts}(2, \text{iblk})+1, 1, \text{iblk}), \end{array} \right\} \text{iblk} = 2n_t + 2 \rightarrow \text{nblks}.$$
(9.77)

Parameters used for the physical r -mesh (or y -mesh) construction:

$$\left. \begin{array}{l} \text{nprts}(2, \text{iblk}), \\ \text{ncell}(1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \\ \text{xxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk})+1, 2, \text{iblk}), \\ \text{fdxp}(1:\text{nprts}(1, \text{iblk})+1, 1:\text{nprts}(2, \text{iblk}), 2, \text{iblk}), \end{array} \right\} \text{iblk} = 2n_t + 2 \rightarrow \text{nblks}.$$
(9.78)

Parameters that must be specified in the `namelist/input/`:

$$\begin{aligned} \text{ncell}(1,2,\text{iblk}), \quad \text{iblk} = 2n_t + 2, \\ \text{xxp}(2:\text{nprts}(1,\text{iblk})+1,1:\text{nprts}(2,\text{iblk})+1,1:2,\text{iblk}), \quad \text{iblk} = 2n_t + 2, \dots, \text{nblks}. \end{aligned} \quad (9.79)$$

igeom=533 or 534: stretch parameters for the central block # 1

Parameters used to stretch the physical mesh in block #1:

$$\begin{aligned} 0 \leq \text{fdx}(1,\text{nprts}(2,1)+1,1,1) \leq 1, \\ 0 \leq \text{fdx}(2,\text{nprts}(2,1)+1,1,1) \leq 1. \end{aligned} \quad (9.80)$$

Example:

The mesh shown in Fig. 9.14 has been constructed with the following set of parameters:

```
- geometry:
  igeom=534
  iradial=2
  nblks=5

- block 1:
  nprts(2,1)=2
  ncell(1,2,1)=4, 8
  xxp(1,2,1,1)=0.d0, 46.d0
  xxp(1,1,2,1)=0.d0, 0.d0
  xxp(1,2,2,1)=8.d0, 8.d0
  xxp(1,3,2,1)=44.d0, 44.d0

- theta-mesh:
  nprts(1,3)=2
  ncell(1,1,2)=10
  ncell(1,1,3)=8, 4
  xxp(1,1,1,2)=-90.d0, -44.d0
  xxp(1,2,1,2)=-90.d0, -44.d0
  xxp(1,3,1,2)=-90.d0, -52.d0
  xxp(1,4,1,2)=-90.d0, -65.d0
  fdxp(1,3,1,2)=1.003d0
  fdxp(1,4,1,2)=1.007d0
  xxp(1,1,1,3)=-44.d0, -8.d0, 0.d0
  xxp(1,2,1,3)=-44.d0, -10.d0, 0.d0
  xxp(1,3,1,3)=-52.d0, -12.d0, 0.d0
  xxp(1,4,1,3)=-65.d0, -12.d0, 0.d0
  fdxp(1,3,1,3)=1.003d0, 1.003d0
  fdxp(1,4,1,3)=1.007d0, 1.007d0

- block 1 is stretched 50% in vertical direction:
  fdxp(1,1,1,1)=0.5d0
  fdxp(1,3,1,1)=1.0d0, .50d0
```



```

- r-mesh:
  nprts(2,2)=3
  nprts(2,3)=3
  ncell(1,2,2)=6,5,5
  ncell(1,2,3)=6,5,5
  ncell(1,2,4)=6,5,5
  xxp(1,1,2,2)=.010d0, .010d0
  xxp(1,2,2,2)=.012d0, .012d0
  xxp(1,3,2,2)=.015d0, .015d0
  xxp(1,4,2,2)=.018d0, .018d0
  fdxp(1,2,2,2)=1.013d0, 1.013d0
  fdxp(1,3,2,2)=1.014d0, 1.013d0
  xxp(1,1,2,3)=.010d0, .010d0, .010d0
  xxp(1,2,2,3)=.012d0, .012d0, .012d0
  xxp(1,3,2,3)=.015d0, .015d0, .015d0
  xxp(1,4,2,3)=.018d0, .018d0, .018d0
  fdxp(1,2,2,3)=1.013d0, 1.012d0, 1.012d0
  fdxp(1,3,2,3)=1.013d0, 1.012d0, 1.012d0

- side blocks:
  ncell(1,1,4)=5
  ncell(1,2,4)=10
  xxp(1,1,1,4)=0.d0, .005d0
  xxp(1,2,1,4)=0.d0, .005d0
  xxp(1,1,2,4)=0.d0, .0d0
  xxp(1,2,2,4)=0.01d0, .01d0
  ncell(1,1,5)=5
  ncell(1,2,5)=16
  xxp(1,1,1,5)=0.d0, .005d0
  xxp(1,2,1,5)=0.d0, .005d0
  xxp(1,1,2,5)=0.01d0, .01d0
  xxp(1,2,2,5)=0.018d0, .018d0

```

3. Concluding remarks

As usual, the relative width of the ghost-cell layer along the outer boundary is controlled by the user-defined parameter `ghwidth` (default value 10^{-4}). The type of the outer boundary condition is specified by the user-defined values of `ibc(ib,iblk)` along the outer block edges that border vacuum. Mutual consistency of the mesh parameters assigned by the user is partially checked in the subroutine `MSHP534`, and the job is aborted if an inconsistency is found.

Explanations:

- `nprts(m,iblk)` is the number of parts along mesh direction `m` in block `iblk`;
- `ncell(iprt,m,iblk)` is the number of physical cells along mesh direction `m` in part `iprt` (along this direction) of block `iblk`;
- `xxp(iprt,jprt,1,iblk)` is the θ ($= x_1$) coordinate [in degrees of arc (blocks `iblk = 1, 2, 3, \dots, 2n_t + 1`) as measured from the vertical x_2 -axis] of the lower-left corner of part `(iprt,jprt)` in block `iblk`; `iprt` is the part index along mesh

direction 1, $jprt$ is the part index along mesh direction 2; correspondingly, the lower-right, upper-left, and upper-right corners of part $(iprt, jprt)$ in block $iblk$ have the x -coordinates $xxp(iprt+1, jprt, 1, iblk)$, $xxp(iprt, jprt+1, 1, iblk)$, and $xxp(iprt+1, jprt+1, 1, iblk)$; here $iblk = 2, 3, \dots, nblks$; default value is `undef`;

- $xxp(iprt, jprt, 2, iblk)$ is the $r (= x_2)$ coordinate of the lower-left corner of part $(iprt, jprt)$ in block $iblk$; here $iblk = 2, 3, \dots, nblks$; default value is `undef`;
- $fdxp(iprt, jprt, 1, iblk)$ is the common ratio of successive cell sizes along the part edge, starting from the lower-left corner of part $(iprt, jprt)$ along mesh direction 1, in block $iblk$; correspondingly, $fdxp(iprt, jprt+1, 1, iblk)$ is the common ratio of successive cell sizes along the part edge, starting from the upper-left corner of part $(iprt, jprt)$ along mesh direction 1; here $iblk = 1, 2, 3, \dots, nblks$; default value is 1.0;
- $fdxp(iprt, jprt, 2, iblk)$ is the common ratio of successive cell sizes along the part edge, starting from the lower-left corner of part $(iprt, jprt)$ along mesh direction 2, in block $iblk$; correspondingly, $fdxp(iprt+1, jprt, 1, iblk)$ is the common ratio of successive cell sizes along the part edge, starting from the lower-right corner of part $(iprt, jprt)$ along mesh direction 2; here $iblk = 1, 2, 3, \dots, nblks$; default value is 1.0;
- $xxp(1, 1, 2, 2)$ is the radius of the primary circle of the H_1 -mosaic mesh;
- $1 - fdxp(1, nprts(2, 1)+1, 1, 1)$ is the stretch factor along angular sector 1 of the mesh in the central block #1 between rectangular and circular shapes;
- $1 - fdxp(2, nprts(2, 1)+1, 1, 1)$ is the stretch factor along angular sector 2 of the mesh in the central block #1 between rectangular and circular shapes.

9.16. `igeom=2001`: a 4-block (or 5-block) “daisy-like” mesh in a rectangle

This is a special case of a 4-block skewed mesh in a rectangular region, designed primarily for test problems. The optional 5-th block has a height of $y_1 - y_0$ and is attached from below (see Fig. 9.16).

Mesh parameters for `igeom=2001`:

- **fixed:** $nprts(m, iblk) = 1$
 $n_{cell}(1, m, iblk)$ for $iblk \geq 2$
 $ibc(ib, iblk)$ [except for $ibc(2, 1)$,
 $ibc(3, 1)$, $ibc(2, 2)$, and $ibc(1, 5)$ (or
 $ibc(1, 3)$)]
 $nbc(ib, iblk)$
- **user-must:** $n_{cell}(1, 1, 1)$
 $n_{cell}(1, 2, 1)$
 $x_0(1, 1) = y_0$
 $x_0(2, 1) = y_1$

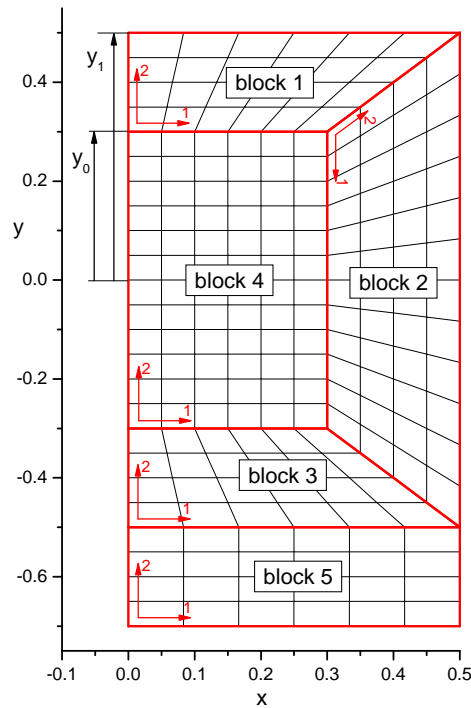


FIG. 9.16: A 5-block “daisy-like” mesh in a rectangular region for IGEOM = 2001.

- **user-can:**

<code>iradial</code>	<code>def = 0</code>
<code>nblks (= 4 or = 5)</code>	<code>def = 4</code>
<code>ibc(2,1)</code>	<code>def = 2</code>
<code>ibc(3,1)</code>	<code>def = 2</code>
<code>ibc(2,2)</code>	<code>def = 2</code>
<code>ibc(1,3) (or ibc(1,5) for 5 blocks)</code>	<code>def = 2</code>
<code>ghwidth</code>	<code>def = 10⁻⁴</code>

Explanations:

- `ncell(1,1,1)` is the number of physical cells along the short edge of the central rectangle in block 4; its long edge has `2*ncell(1,1,1)` mesh cells;
- `ncell(1,2,1)` is the number of physical cells along the radial direction in blocks 1, 2 and 3;
- `x0(1,1)` is the length of the short edge of block 4 equal to y_0 ;
- `x0(2,1)` is one half of the total extension of the 4-block mesh along y -axis equal to $y_1 > y_0$;

9.17. igeom=2002: a 5-block skewed Kershaw mesh in a rectangle

This is a special case of a skewed mesh in a rectangular region proposed by Kershaw [?]. In our case this mesh is constructed by using five blocks stacked vertically (along the y -axis)

on top of one another, each having two parts along the horizontal x -axis; see Fig. 9.17. The mesh is constructed by applying the MSHB22 subroutine (i.e. the AQ-mesh algorithm) to each block. No material differences between different blocks and parts are foreseen in this version. Concerning the material properties and the initial state, it is sufficient to set the necessary parameters (namely, `rho0(1,1)`, `pr0(1,1)`, `matnum(1,1)`) for the first part of the first block only: the same values are then automatically assigned in all other parts and blocks.

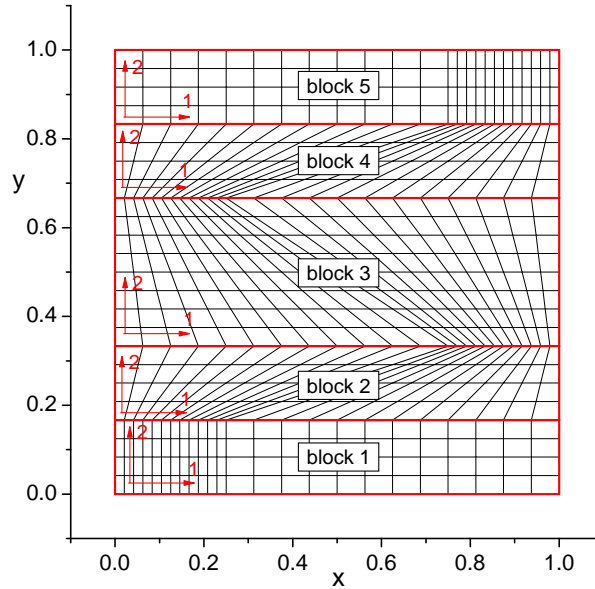


FIG. 9.17: A 5-block skewed Kershaw mesh in a rectangular region for IGEOM = 2002.

Identical boundary conditions are assigned along edges 3 and 4 of all five blocks, i.e. `ibc(3:4,iblk) = ibc(3:4,1)`, `ITCONBC(3:4,iblk) = ITCONBC(3:4,1)`, `TEMPBC(3:4,iblk) = TEMPBC(3:4,1)`, ... for all `iblk = 2, 3, 4, 5`.

Mesh parameters for `igeom=2002`:

- **fixed:**
 - `nblks = 5`
 - `nprts(1,iblk) = 2`
 - `nprts(2,iblk) = 1`
- **user-must:**
 - `ncell(1,1,1)`
 - `ncell(1,2,1)`
 - `xxl(1:3,1,1)`
 - `xxl(1:2,2,1)`
- **user-can:**
 - `iradial` def = 0
 - `fdxaq(iprt,ib,iblk)` def = 1.0
 - `ibc(1,1)` def = 2
 - `ibc(3,1)` def = 2
 - `ibc(4,1)` def = 2
 - `ibc(2,5)` def = 2
 - `ghwidth` def = 10^{-4}

Explanations:

- `ncell(1,1,1)` is the number of physical cells in part 1 of all five blocks along x -axis; part 2 contains equal number of cells;
- `ncell(1,2,1)` is the number of physical cells in blocks 1, 2, 4, and 5 along y -axis; block 3 has twice that number of cells along y ;
- `xxl(iprt,1,1)` is the x -coordinate of the left end of part `iprt` along x -axis;
- `xxl(1,2,1)` is y -coordinate of the lower-left corner of block 1;
- `xxl(2,2,1)` is y -coordinate of the upper-left corner of block 5 (!)

Note that `xxl(2,2,1) - xxl(1,2,1)` is the y -extension of the entire mesh, and not just of its first block.

9.18. `igeom=2003`: a 5-block skewed Kershaw mesh in a rectangle with attached 5 blocks of orthogonal mesh

This is an extension of the previous case of a 5-block Kershaw mesh with an attached 5-block rectangular region along x -axis.

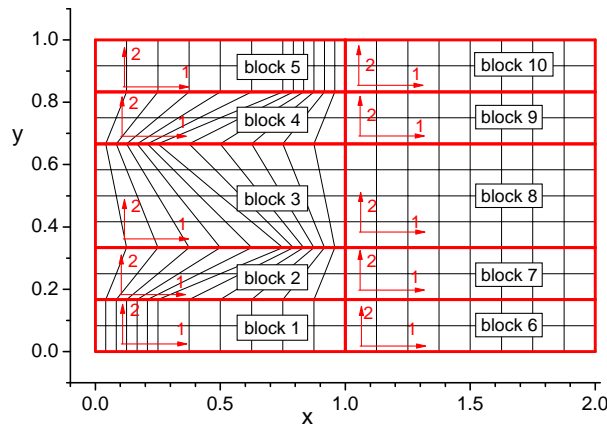


FIG. 9.18: A 5-block Kershaw mesh with an attached rectangle for `IGEOM = 2003`.

Identical boundary conditions are assigned along edges 3 of blocks 1–5, along edges 4 of blocks 6–10, along edges 1 of blocks 1 and 6, along edges 2 of blocks 5 and 10.

Mesh parameters for `igeom=2003`:

- **fixed:**
 - `nblks = 10`
 - `nprts(1,iblk) = 2, iblk = 1–5,`
 - `nprts(1,iblk) = 1, iblk = 6–10,`
 - `nprts(2,iblk) = 1`

- **user-must:** ncell(1,1,1)
ncell(1,2,1)
xxl(1:3,1,1)
xxl(1:2,2,1)
ncell(1,1,6)
xxl(2,1,6)
- **user-can:** iradial def = 0
fdx(1,1,6) def = 1.0
ibc(1,1) def = 2
ibc(3,1) def = 2
ibc(4,6) def = 2
ibc(2,5) def = 2
ghwidth def = 10^{-4}

Explanations:

- ncell(1,1,1) is the number of physical cells in part 1 of the first five blocks along x -axis; part 2 contains equal number of cells;
- ncell(1,2,1) is the number of physical cells in blocks 1, 2, 4, 5, 6, 7, 9, 10 along y -axis; blocks 3 and 8 have twice that number of cells along y ;
- ncell(1,1,6) is the number of physical cells along x -axis in blocks 6–10;
- xxl(iprt,1,1) is the x -coordinate of the left end of part iprt along x -axis;
- xxl(1,2,1) is y -coordinate of the lower-left corner of block 1;
- xxl(2,2,1) is y -coordinate of the upper-left corner of block 5 (!)
- xxl(2,1,6) is the x -coordinate of the right corners in blocks 6–10.

Note that $xxl(2,2,1) - xxl(1,2,1)$ is the y -extension of the entire mesh, and not just of its first block.

9.19. igeom=2005: a single-block skewed Saltzman mesh in a rectangle

The number of blocks $nblks = 1$ is fixed; fixed also is the number of parts $nprts(m,1) = 1$. Parameter *iradial* is free to choose. The values of $fdx(iprt,m,iblk)$ are irrelevant.

Mesh parameters for igeom=2005:

- **fixed:** nblks=1
nprts(m,1)=1
- **user-must:** ncell(1,m,1)
xxl(1,m,1)
xxl(2,m,1)
- **user-can:** ibc(ib,1) def = 2
ghwidth def = 10^{-4}

Explanations:

- `nprts(m,iblk)` is the number of parts along mesh direction `m` in part `iprt` of block `iblk`;
- `ncell(iprt,m,iblk)` is the number of physical cells along mesh direction `m` in part `iprt` of block `iblk`;
- `xxl(iprt,1,iblk)` and `xxl(iprt+1,1,iblk)` are the x coordinates of the left and right bounds of part `iprt` in block `iblk`;
- `xxl(iprt,2,iblk)` and `xxl(iprt+1,2,iblk)` are the y coordinates of the lower and upper bounds of part `iprt` in block `iblk`;
- `ghwidth` is the relative width of the ghost-cell bands;

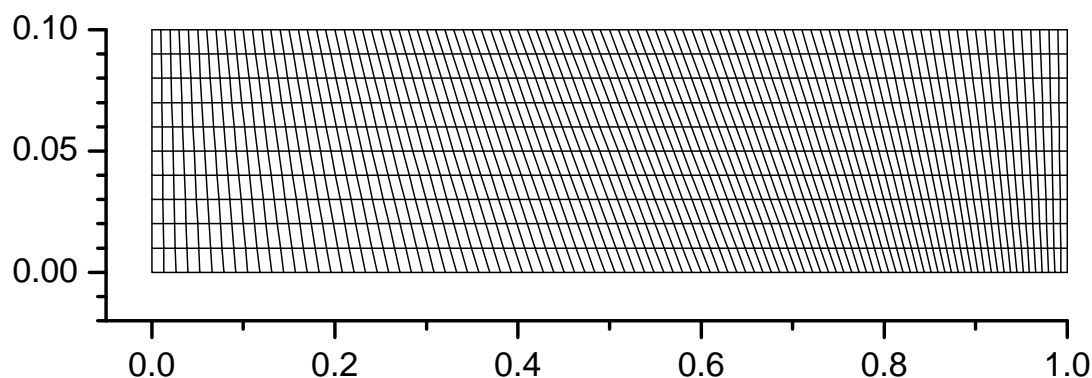


FIG. 9.19: A skewed Saltzman mesh in a rectangle.

9.20. `igeom=2201`: a 5-block “criss-cross” mesh in an arbitrary quadrangle

This is a special case of a 5-block skewed mesh in an arbitrary quadrangle; see Fig. 9.20.

Mesh parameters for `igeom=2201`:

- **fixed:** `nblks` (= 5)
- **user-must:** `ncell(iprt,m,1)`
`ncell(iprt,m,2)`
`x0aq(m,ic,1)` for `ic = 1,2,3,4`
`x0aq(m,ic,2)` for `ic = 2,3,4`
`x0aq(m,ic,3)` for `ic = 2,3`
`x0aq(m,3,4)`

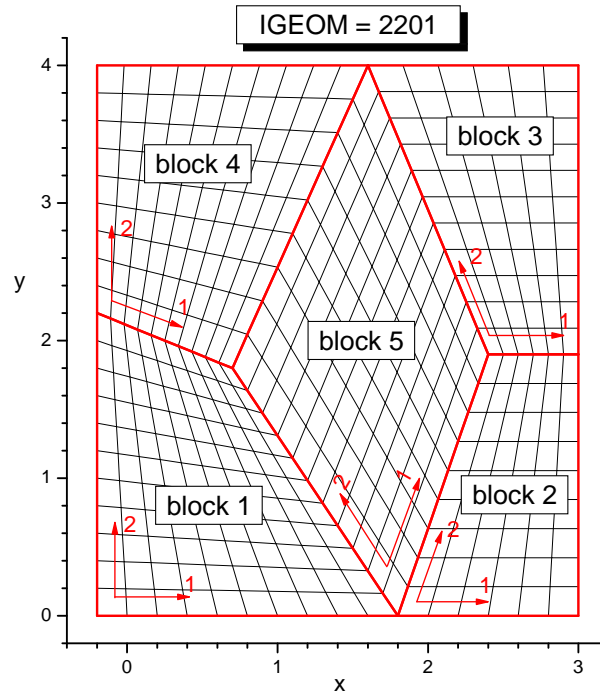


FIG. 9.20: A 5-block “criss-cross” mesh in an arbitrary quadrangle for IGEOM = 2201.

- **user-can:**

<code>iradial</code>	<code>def = 0</code>
<code>nprts(m,iblk) for iblk = 1,2</code>	<code>def = 1</code>
<code>fprtaq(iprt,ib,iblk) for iblk = 1,2</code>	<code>def = 1.0</code>
<code>fprtaq(iprt,ib,3) for ib = 2,3,4</code>	<code>def = 1.0</code>
<code>fprtaq(iprt,ib,4) for ib = 2,3,4</code>	<code>def = 1.0</code>
<code>fdxaq(iprt,ib,iblk) for iblk = 1,2</code>	<code>def = 1.0</code>
<code>fdxaq(iprt,ib,3) for ib = 2,3,4</code>	<code>def = 1.0</code>
<code>fdxaq(iprt,ib,4) for ib = 2,3,4</code>	<code>def = 1.0</code>
<code>ibc(ib,1) for ib = 1,3</code>	<code>def = 1.0</code>
<code>ibc(ib,2) for ib = 1,4</code>	<code>def = 2</code>
<code>ibc(ib,3) for ib = 2,4</code>	<code>def = 2</code>
<code>ibc(ib,4) for ib = 2,3</code>	<code>def = 2</code>
<code>ghwidth</code>	<code>def = 10⁻⁴</code>

The meaning of all the above mesh parameters is the same as for `igeom=22`.

9.21. `igeom=3001`: a multi-block distorted polar r - θ mesh

This version of a skewed polar mesh has been proposed in [?]; see Fig. 9.21. Each block is a circular sector with x_2 running along the radius r in the positive direction, and x_1 running along θ in the negative direction. The mesh parameters `x0(1,iblk)`, `dx(iprt,1,iblk)`, and `xx1(iprt,1,iblk)` should be assigned in angular degrees.

The meaning of all mesh parameters is the same as for `IGEOM = 3` and `4`. The mesh can be either uniform or progressive. As for `IGEOM = 4`, there are no fixed parameters. The

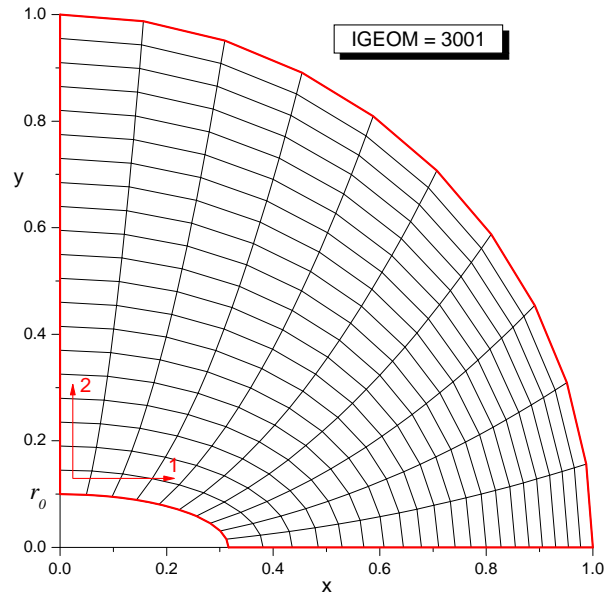


FIG. 9.21: A distorted polar mesh for IGEOM = 3001.

distortion is imposed by transformation

$$\begin{aligned} x_{ij} &= \sqrt{r_j} \cos \theta_i, \\ y_{ij} &= r_j \sin \theta_i, \end{aligned} \tag{9.81}$$

where

$$\begin{aligned} r_j &= r_0 + (j - 1) \Delta r, \\ \theta_i &= \theta_0 + (i - 1) \Delta \theta. \end{aligned} \tag{9.82}$$

9.22. igeom=6001: a quarter-circle cylindrical (or spherical) mesh with 3, 5, 7, ... blocks

This is a $(3 + 2k_t)$ -block quarter-circle mesh for cylindrical and spherical configurations (depending on the value `iradial`), which has been originally designed for tests. The central core inside the full circular domain is a square, represented by block 1; see Fig. 9.22. Above blocks 2 and 3 (each occupying 45°) there are k_t circular tiers, each comprised of 2 blocks. By setting reflective boundary conditions along the x - and y -axes, one can simulate half-circle or full-circle domains. This mesh is always progressive. The innermost corner (corner 1 in block 1) is always placed at $x_0(1,1) = x_0(2,1) = 0.0$.

Mesh parameters for `igeom=6001`, progressive mesh:

- **fixed:**

```

nprts(1,iblk) = 1
nprts(2,iblk) = 1, iblk = 1-3
ncell(1,m,iblk) for iblk ≠ 2
ibc(ib,iblk) (except for ibc(1,1),
ibc(3,1), and ibc(2,nblks-1))
nbc(ib,iblk)
        
```

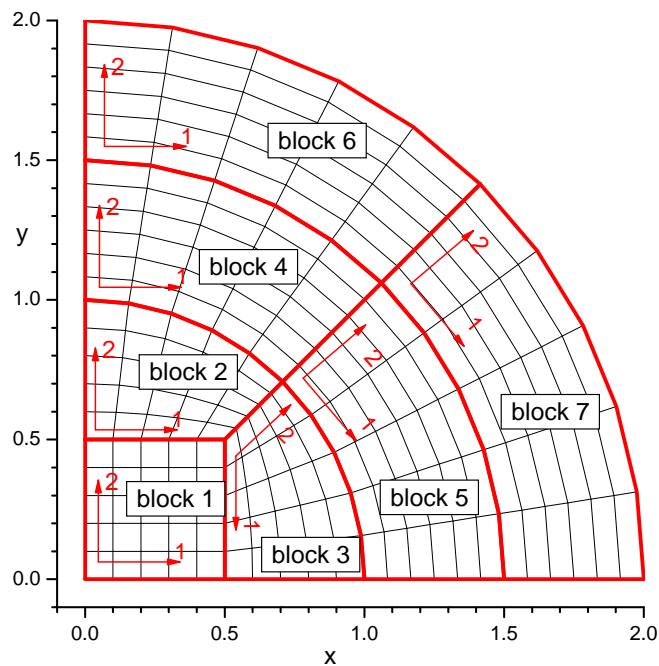


FIG. 9.22: A 7-block quarter-circle mesh.

- **user-must:**

```

ncell(1,1,1)
ncell(1,2,2)
ncell(1,2,2)
ncell(1:nprts(2,4),2,4)
ncell(1:nprts(2,6),2,6)
...
xxl(1,2,2)
xxl(2,2,2)
xxl(2:nprts(2,4)+1,2,4)
xxl(2:nprts(2,6)+1,2,6)
...

```
- **user-can:**

iradial	def = 0
nblks	def = 3
nprts(2,4)	def = 1
nprts(2,6)	def = 1
...	
fdx(1,1,2)	def = 1.0
fdx(1,2,2)	def = 1.0
fdx(1:nprts(2,4),2,4)	def = 1.0
fdx(1:nprts(2,6),2,6)	def = 1.0
...	
ibc(1,1)	def = 2
ibc(3,1)	def = 2
ibc(2,nblks-1)	def = 2
ghwidth	def = 10 ⁻⁴

Explanations:

- `nprts(2,4)` is the number of parts along radial direction in blocks 4 and 5;
- `nprts(2,6)` is the number of parts along radial direction in blocks 6 and 7;
- ...
- `ncell(1,1,1)` is the number of physical cells along each direction in the central block 1;
- `ncell(1,2,2)` is the number of physical cells along radial direction in blocks 2 and 3;
- `ncell(kp,2,4)` is the number of physical cells in part `kp` along radial direction in blocks 4 and 5;
- ...
- `xxl(1,2,2)` is the edge length of the central square (block 1);
- `xxl(2,2,2)` is the outer radius of blocks 2 and 3;
- `xxl(kp+1,2,4)` is the outer radius of part `kp` in blocks 4 and 5;
- ...
- `fdx(1,1,2)` is the ratio of successive cell sizes along each mesh direction in the central square (block 1);
- `fdx(1,2,2)` is the ratio of successive cell sizes along the radial mesh direction in the outer blocks 2 and 3;
- `fdx(kp,2,4)` is the ratio of successive cell sizes along the radial mesh direction in part `kp` of blocks 4 and 5;
- ...
- `ibc(1,1)` is the type of boundary condition along the x -axis in blocks 1 and 3; typically `ibc(1,1) = 1` or `2`.
- `ibc(3,1)` is the type of boundary condition along the y -axis in blocks 1 and 2; typically `ibc(3,1) = 1` or `2`.
- `ibc(2,nblks-1)` is the type of boundary condition along the outer circular boundary; default value `ibc(2,nblks-1)=2`.

10. PRACTICAL RECOMMENDATIONS FOR REZONING CONTROL

10.1. General remarks

There are many parameters that provide control over different aspects of the mesh rezoning and remapping algorithms. Some of them have been inherited from the CAVEAT code, others have been added in the RALEF package. The values of the most of these parameters can be adjusted via the `namelist/input/`.

The standard way to influence how the rezoning algorithm tends to construct a new mesh is by choosing an appropriate weight function `wt(i)`, calculated in the subroutine `WEIGHT`, file `'f05_remap.f'`. Usually, a new problem-specific formula for the rezoning weight function can be easily programmed by editing Step 2 in this subroutine.

10.2. A nearly Eulerian simulation

Sometimes it may be desirable to suppress the rezoning procedure so that the new mesh \vec{x}_i^{n+1} could be forced to remain arbitrarily close to the old one \vec{x}_i^n . In particular, this might be an efficient practical option when all the physical boundaries are defined as fixed in space (i.e. as reflective, inflow/outflow or free-slip with `ibc = 1, 3, 4, or 8`) and there are no Lagrangian material interfaces: it often helps to significantly suppress the numerical diffusion near sharp density contrasts in regions with negligible physical motion.

The amplitude $\max(|\vec{x}_i^{m+1} - \vec{x}_i^m|)$ of the mesh displacement during the rezoning procedure can be reduced to an arbitrarily small value

- by setting the value of $\alpha_{ALE} = \text{alecoef} \ll 1$ (say, $\alpha_{ALE} = 10^{-3}$) to ensure practically 100% retraction to \vec{x}_i^m when calculating the initial state $\vec{x}_i^{0,n+1}$ for the rezoning procedure combined with
- a sufficiently small (equal, say, to 0.01) value of the amplitude `aBrackbill` of the Brackbill rezoning algorithm.

10.3. Suppression of tangential rezoning along the mesh boundaries

In some cases it may be desirable to suppress tangential rezoning along certain mesh boundaries. This, in fact, may even be necessary to prevent the mesh being strongly pulled (and finally collapsed there) towards a block corner — which sometimes happens, for example, at an intersection of two outflow boundaries with a strong mass outflow through both of them. Tangential rezoning can be suppressed by setting the flag `ifnotr12bc(ib,iblk)=.true.` for any selected block edge `ib` — preferably the one, along which the mesh is most uniform and the least subject to adaptive changes due to variations of physical quantities (like concentration towards regions with higher matter density).

The effect of setting `ifnotr12bc(ib,iblk)=.true.` will be

- to mark all the physical vertices as fixed in space (i.e. with the flag `fix` set on) along a fixed in space boundary identified with `ibc = 1, 3, 4, or 8`, or
- to mark all the physical vertices as strictly Lagrangian (i.e. with the flag `lvx` set on) along a moving boundary identified with `ibc = 2, 6, or 9`.

Note that any in-line change of the flag `ifnotr12bc(ib,iblk)` requires an appropriate re-initialization of the code because the basic flag array `iflg` has to be reloaded. Therefore, it must be done by editing the subroutine `RUNCTRL` in file `'f10_taskinpt.f'`.

10.4. Parallel translation of the whole mesh in the direction of bulk motion

Another useful opportunity offered by the ALE algorithm is a spatially uniform parallel mesh translation with a prescribed and generally time-dependent translation velocity. This may be a useful option when the region of main interest (the “bulk” of the simulated target) is accelerated in a certain direction and tends to leave the physical volume covered by the computational mesh (either in an Eulerian simulation or in an ALE simulation with fixed in space mesh boundaries).

To activate this option, one has to set the flag `ifmeshslide=.true.`. After that, every time when the subroutine REZONE is called, the mesh is uniformly translated with the 2D velocity $\vec{u}_{tr} = (\text{bulk_ux}, \text{bulk_uy})$. The components `bulk_ux` and `bulk_uy` of the “bulk” velocity are calculated in the subroutine BULKVALS, file ‘f05_remap.f’. If the user wants to change the algorithm for calculation of the mesh translation velocity, he has to edit either steps 0 and/or 6 in the subroutine REZONE (file ‘f05_remap.f’), or the subroutine BULKVALS, or both.

Here, however, caution is advised: the user must ascertain that the mesh translation velocity is not too high, so that the outflow velocity across a translated outflow boundary does not in fact become an inflow velocity.

11. MATERIAL PROPERTIES

Different parts of every mesh block can be filled with different materials. Any mesh cell can contain only one type of material. On the total, up to 30 different materials are foreseen in the RALEF code, whose properties are listed in a rank-2 array `PROPRTY(i,imat) = PROPRTY(1:100,1:30)`. The second index `imat = 1, 2, ..., 30` of this array is the sequential material number, while its first index `i = 1, 2, ..., 100` is used to list all the user defined parameters for a given material `imat`. The meaning of the individual elements of the `PROPRTY(i,imat)` array are explained in Table III.

12. TIME STEP LIMITATION

Under thermal processes we understand all the heating-cooling terms on the right-hand of the energy equation other than the $p dV$ work. In our case we have three such terms due, respectively, to thermal conduction (W_i^T), radiation transport (W_i^r), and possible external energy deposition (W_i^{dep}); see Eq. (6.1) in the RALEF-2D report [?]. Because time discretization of these thermal terms is done by using the symmetric semi-implicit (SSI) method, we need an additional [with respect to the usual Courant-Friedrichs-Lewy (CFL) condition] “thermal” constraint on the value of the time step Δt . This constraint is based on the requirement that the temperature increment $|\tilde{T}_i - T_i|$ in cell i at the SSI phase of the Lagrangian step must not be too large, namely, on the condition

$$\left| \tilde{T}_i - T_i \right| = \left| \frac{W_i \Delta t + \delta_i}{c_{V,i} M_i + D_i \Delta t} \right| \leq \varepsilon_0 (T_i + T_s), \quad (12.83)$$

where $W_i = W_i^{dep} + W_i^T + W_i^r$ is the total thermal heating power of cell i , $c_{V,i} M_i$ its heat capacity, $D_i = -\partial W_i / \partial t$, $\delta_i = \delta_{T,i} + \delta_{r,i}$ is the SSI energy correction, taken from the previous hydro cycle, and ε_0 and T_s are two user-defined free parameters.

Presently there are two versions of the thermal limit on Δt implemented in the RALEF-2D code: a “hard” one and a “soft” one. They are distinguished by the value of the user-defined parameter $c_{1,tst}$: for $c_{1,tst} = 0$ the “hard” version is active, for $c_{1,tst} > 0$ the “soft” one applies.

TABLE III: Array proprty(i,imat) of material properties

EOS #	1	2	3	5	7	8	
type	polytropic	linear	quadratic	HOM	GLT	SESAME	default
i=1	1.0	2.0	3.0	5.0	7.0	8.0	1.0
2	ρ_{00}	ρ_{00}	ρ_{00}	ρ_{00}	ρ_{00}	ρ_{00}	1.0
3	p_{floor}	p_{floor}	p_{floor}	p_{floor}	p_{floor}	p_{floor}	–ceiling
4	$A_s = (\gamma + 1)/2$	A_s	A_s	A_s	A_s	A_s	4/3
5	γ	c_1	c_1	C	GLT #m	SESAME #	5/3
6	c_V	c_2	c_2	S	Maxwl flag	RSCALE	1.0
7	c_p	c_3	c_3	V_{sw}		ESHIFT	1.0
8		c_4	c_4	C_1		A_1	undef
9		c_V	c_5	S_1		A_2	undef
10			c_6	F		A_3	undef
11			c_7	G		IREVFLG	undef
12			c_8	H			undef
13			c_V	I			undef
14				J			undef
...				...			
19	e_{floor}	e_{floor}	e_{floor}	e_{floor}	e_{floor}	e_{floor}	–ceiling
...				...			
27	A_{mol}	A_{mol}	A_{mol}	A_{mol}	A_{mol}	A_{mol}	undef
28	Z_{mol}	Z_{mol}	Z_{mol}	Z_{mol}	Z_{mol}	Z_{mol}	undef
29	z_i (ioniz.deg)	z_i	z_i				1.0
parameters for calculating the conduction coefficient κ and the limiting flux h_l							
model type	$\kappa = \kappa_0 T^n$ $h_l = f_{inh} \rho T^{3/2}$	ad hoc analytic	Spitzer $f_{inh} \frac{\rho z_i T^{3/2}}{A_{mol}}$	Basko-met $f_{inh} \frac{\rho z_i T \sqrt{T_F}}{A_{mol}}$	GLT $f_{inh} \frac{\rho z_i T^{3/2}}{A_{mol}}$		
30	1.0	2.0	3.0	5.0	7.0		undef
31	κ_0				GLT #m		undef
32	n		$(\ln \Lambda)_{min}$				undef
...				...			
37	f_{inh}	f_{inh}	f_{inh}	f_{inh}	f_{inh}		–ceiling
...				...			
parameters for calculating radiation opacities							
model type	$k_R = k_{R,0} \rho^\alpha T^\beta$ $k_P = k_{P,0} \rho^\alpha T^\beta$	ad hoc analytic	Kramers free-free		GLT		
40	1.0	2.0	3.0		7.0		1.0
41	$k_{R,0}$	$k_{R,0}$			GLT #m		1.0
42	$k_{P,0}$	$k_{P,0}$					1.0
43	α	T_0					0.0
44	β						0.0
45							undef
parameters for laser absorption coefficient (dielectric constant)							
model type	$k_{las} = k_0 \rho^\alpha T^\beta$	ad hoc analytic	Kramers free-free	Basko metal	GLT		
46	1.0	2.0	3.0	5.0	7.0		undef
47	k_0	k_0	$z_{ii,min}$	$z_{ii,min}$	GLT #m		undef
48	α	T_0	$z_{ie,min}$	$z_{ie,min}$			undef
49	β						undef
...				...			

The “hard” version of the time-step limit corresponds to the most strict implementation of condition (12.83). Here, however, the main obstacle is the fact that, when δ_i is calculated, the value of Δt for the next hydro cycle (where δ_i must be redeposited) is not known. As a consequence, the “hard” time-step limit is realized by splitting the criterion (12.83) into the following two conditions

$$\left| \frac{W_i \Delta t}{c_{V,ij} M_{ij} + \Delta t D_i} \right| \leq (\varepsilon_0 - \varepsilon_1) (T_i + T_s), \quad \left| \frac{\bar{\delta}_i}{c_{V,i} M_i} \right| \leq \varepsilon_1 (T_i + T_s), \quad (12.84)$$

where ε_1 is an additional free parameter in our criterion. Clearly, one must ensure that $\varepsilon_1 < \varepsilon_0$. The bar over δ_i in Eq. (12.84) means that this is a “postponed” quantity, to be used for calculating $\tilde{T}_i - T_i$ only in the next hydro cycle. Because $\bar{\delta}_i$ is, in its turn, proportional to the current value of Δt (see Eq. (5.36) in Ref. [?] and Eq. (?) in Ref. [?]), we can, by choosing sufficiently small values of $\varepsilon_0 > \varepsilon_1$ and T_s , always keep relative temperature variation $|\tilde{T}_i - T_i|/T_i$ at one time step within desired limits.

The principal drawback of the “hard” limit (12.84) is that in many practical situations, where one has $D_i \Delta t \gg 1$, this constraint turns out to be too restrictive and either unnecessarily slows down the simulation by a significant factor or completely blocks it. Typically it occurs when strong heat-conduction fluxes are present in a tenuous medium with relatively small values of cell heat capacities $c_{V,i} M_i$. To speed up the simulation in such cases, an alternative “soft” version of the criterion (12.83) has been implemented.

The “soft” time-step control is based on the following strategy: we keep track of the relative temperature change

$$\frac{\delta T(\Delta t)}{T} = \max_i \left\{ (T_i + T_s)^{-1} \left| \frac{W_i \Delta t + \delta_i}{c_{V,i} M_i + D_i \Delta t} \right| \right\} \quad (12.85)$$

and make small corrections to Δt in order to keep $\delta T/T$ within a range $0.5\varepsilon_0 < \delta T/T < 0.6\varepsilon_0$. More precisely, the SSI phase of every hydro cycle starts with a trial value of the time step

$$\Delta t_* = \min \{ c_{dtgr} \Delta t_{prev}; \Delta t_{CFL}; \Delta_{ev} \}, \quad (12.86)$$

where $c_{dtgr} > 1$ is a user-defined growth factor (typically, $c_{dtgr} = 1.05$ – 1.1), Δt_{prev} is the value of Δt in the previous hydro cycle, Δt_{CFL} is the value of Δt obtained from the purely hydrodynamic CFL criterion, and Δ_{ev} is some other eventual time step limit. Next, the maximum relative temperature change $\delta T(\Delta t_*)/T$ is calculated for the starting value Δt_* , which is subsequently modified to (a “fine-tuning” correction)

$$\Delta t = \begin{cases} \Delta t_*, & \delta T(\Delta t_*)/T \leq 0.5\varepsilon_0, \\ \min\{\Delta t_{prev}, \Delta t_*\}, & 0.5\varepsilon_0 < \delta T(\Delta t_*)/T \leq 0.6\varepsilon_0, \\ \frac{\min\{\Delta t_{prev}, \Delta t_*\}}{1 + c_{1,tst}(c_{dtgr} - 1)}, & 0.6\varepsilon_0 < \delta T(\Delta t_*)/T \leq 1.2\varepsilon_0. \end{cases} \quad (12.87)$$

This correction is made only once, i.e. the above “fine-tuning” procedure is non-iterative. In Eq. (12.87) one can make use of the values of $c_{1,tst} > 1$ to speed up the reduction of Δt relative to its increase rate by means of the parameter $c_{dtgr} > 1$.

Occasionally the “fine-tuning” procedure (12.87) becomes unstable, and one ends up with a large temperature variation $\delta T(\Delta t_*)/T > 1.2\varepsilon_0$. Such a situation is considered as a failure (or a “crash”) of the “soft” time-step control procedure. Here one has to impose a more

dramatic reduction of Δt to bring down the relative temperature variation $\delta T/T$, and it has to be done in an iterative loop. The problem is that, if we want to conserve energy, we are not allowed to change δ_i as Δt is reduced. As a practical solution, we choose to sacrifice energy conservation and scale down the values of δ_i (in proportion to Δt) in those cells i where $(\delta T/T)_i > 0.5\varepsilon_0$. In other words, when the “fine-tuning” procedure (12.87) crashes, we violate strict energy conservation and bring down the temperature change to a level $\delta T/T < 0.5\varepsilon_0$ by strong reduction of Δt in an iterative loop similar to that in the “hard” version of the thermal time-step control algorithm. If $D_i\Delta t \gg 1$, the “crash” reduction of Δt may be by several orders of magnitude.

For optimal performance, the following values of the user-defined control parameters can be recommended:

- for the “hard” option of the thermal time-step control:

$$c_{1,tst} = 0, \quad \varepsilon_0 = 0.1\text{--}0.2, \quad \varepsilon_1 = 0.5\varepsilon_0; \quad (12.88)$$

- for the “soft” option of the thermal time-step control:

$$c_{1,tst} = 1\text{--}2, \quad \varepsilon_0 = 0.06. \quad (12.89)$$

In the “soft” option, the value of ε_1 is irrelevant. The values of c_{dtgr} are allowed in the range $1 < c_{dtgr} < 2$, recommended are $c_{dtgr} = 1.05\text{--}1.1$. In simulations of radiative hohlraums, the “soft” option of thermal time-step control allowed to reduce the computing time by about a factor 2–3.

Correspondence with the code variables:

$c_{1,tst}$	= <code>c1dtssi</code>	– user-defined parameter which controls application of a “hard” or a “soft” thermal time-step limit; default = 0;
c_{dtgr}	= <code>dtgrow</code>	– user-defined growth factor for increasing the time step Δt ; default = 1.05;
ε_0	= <code>eps0ssi</code>	– principal time-step control parameter at the SSI stage; default = 0.1;
ε_1	= <code>eps1ssi</code>	– secondary time-step control parameter at the SSI stage; default = 0.05;
T_s	= <code>tempsns</code>	– sensitivity threshold for T variation;
T_{flr}	= <code>tempflr</code>	– absolute minimum for T values; default = <code>floor</code> ;
$\delta T/T$	= <code>dtmpssi</code>	– relative temperature change at the SSI phase of the hydro cycle;
Δt_{prev}	= <code>dtprev</code>	– time step in the previous hydro cycle;

13. PARALLELIZATION WITH OPENMP

Clearly, the computational work needed for calculation of the radiative heating powers W_i^r can be easily divided into independent blocks corresponding to different beamlet directions $\vec{\Omega}_L$ and different photon frequencies $[\nu_k, \nu_{k+1}]$ that can be processed in parallel. And though all the frequency groups $[k] = 1, 2, \dots, N_\nu$ are independent from one another, this is not the case for angular directions $\vec{\Omega}_L$, which have to be processed in groups.

In the case of Cartesian (x, y) geometry every *independent* angular group combines 4 beamlets from the 4 different octants: these 4 beamlets have the same value of index $l =$

$1, 2, \dots, N_\Omega$, but different values of the octant indices $i_{ox} = \pm 1$, $i_{oy} = \pm 1$. Thus, for $\text{IRADIAL} = 0$ all independent angular groups have the same number of individual beamlets, and there are in total N_Ω independently processable angular groups.

In the axi-symmetric (r, z) geometry the division into independent angular groups is more complex, and different independent groups generally have different number of individual beamlets. To make the distribution of individual beamlets over the independently processable groups more even, we assume that all the angular beamlets can be divided into $N_{\Omega bdl s}$ independently processable Ω -bundles of beamlets. One bundle may, in fact, contain either one big, or two smaller independently processable groups. Generally, $N_{\Omega bdl s} \leq N_\Omega$. For $\text{IRADIAL} = 0$ we always have $N_{\Omega bdl s} = N_\Omega$.

To optimize work distribution among N_{thr} different threads, we combine the $N_{\Omega bdl s}$ Ω -bundles and the N_ν frequencies into a single iteration loop, with the iteration index K running through the values $K = 1, 2, \dots, N_K$; in the simplest case $N_K = N_\nu \times N_{\Omega bdl s}$. Thus, the principal parallelized section of the RALEF-2D code is represented by a combined frequency \times direction do-loop

```

!$OMP DO SCHEDULE(STATIC,nchunk(kOMPass))

do K=1,1,nKloop(kOMPass)

...
enddo

!$OMP END DO

```

(13.90)

In a regular hydrocycle this loop is passed only once, and $\text{kOMPass} = 1$. In rare hydrocycles, where the spectral diagnostics is to be computed and written out, this loop is passed twice: first time with $\text{kOMPass} = 1$ for the main radiation-hydro calculation, and second time with $\text{kOMPass} = 2$ for the diagnostics.

From the point of view of parallelization logic, the total number $N_K = \text{nKloop}(\text{kOMPass})$ of combined iterations is divided into N_{thr} *chunks* of work in a static manner, so that each chunk consists of $N_{ch} = \text{nchunk}(\text{kOMPass})$ sequential iterations performed one after another by a single thread; N_{thr} is the total number of threads. Thus, each thread processes one and only one chunk of work; the last chunk is allowed to have less than N_{ch} iterations.

From the point of view of physics, the total number N_K of combined iterations consists of $N_{\Omega bdl s}$ separate *frequency clusters* because for each Ω -bundle the angular information is calculated only once, and then is used N_ν times by each of the N_ν frequencies. Evidently, the angular information must be calculated anew, firstly, at the start of every chunk and, secondly, by transition from one frequency cluster to another. Because one does not know a priori where in a chunk such transition will occur, it is convenient to emulate calculation of angular information by any such transition as an extra frequency group with a number $[k] = 0$. The latter is justified by the fact that in our case the CPU time required to recalculate the angular information is approximately equal to the CPU time needed to process one frequency. Thus, in iterations with $[k] = 0$ the index $i_{\Omega bdl}$ of the Ω -bundle is incremented by 1, the angular information is calculated anew, and no frequency is processed. Then, in subsequent iterations with $[k] = 1, 2, \dots$ no angular information is calculated, and one frequency group $[k] \geq 1$ is processed at a time.

Finally, we arrive at the following values of the parameters governing the parallelization process. The

$$N_K = \begin{cases} N_{\Omega bdl s} \times N_{\nu cl}, & N_{\nu cl} = N_\nu, \\ (N_{\Omega bdl s} \times N_{\nu cl}) - 1, & N_{\nu cl} = N_\nu + 1, \end{cases} \quad (13.91)$$

TABLE V: Distribution of work among $N_{thr} = 4$ threads by OpenMP parallelization for the case of $N_{\Omega bdl_s} = 5$, $N_\nu = 3$, $N_{\nu cl} = 4$, $N_{ch} = 5$.

thread	0					1					2					3			
K	1, 2, 3, 4, 5	6, 7, 8, 9, 10	11, 12, 13, 14, 15	16, 17, 18, 19															
[k]	1, 2, 3, 0, 1	2, 3, 0, 1, 2	3, 0, 1, 2, 3	0, 1, 2, 3															
$i_{\Omega bdl}$	1, 1, 1, 2, 2	2, 2, 3, 3, 3	3, 4, 4, 4, 4	5, 5, 5, 5															

TABLE VI: Distribution of work among $N_{thr} = 4$ threads by OpenMP parallelization for the case of $N_{\Omega bdl_s} = 4$, $N_{\nu cl} = N_\nu = 4$, $N_{ch} = 4$.

thread	0				1				2				3			
K	1, 2, 3, 4	5, 6, 7, 8	9, 10, 11, 12	13, 14, 15, 16												
[k]	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4												
$i_{\Omega bdl}$	1, 1, 1, 1	2, 2, 2, 2	3, 3, 3, 3	4, 4, 4, 4												

TABLE VII: Distribution of work among $N_{thr} = 4$ threads by OpenMP parallelization for the case of $N_{\Omega bdl_s} = 3$, $N_\nu = 6$, $N_{\nu cl} = 7$, $N_{ch} = 5$.

thread	0					1					2					3				
K	1, 2, 3, 4, 5	6, 7, 8, 9, 10	11, 12, 13, 14, 15	16, 17, 18, 19, 20																
[k]	1, 2, 3, 4, 5	6, 0, 1, 2, 3	4, 5, 6, 0, 1	2, 3, 4, 5, 6																
$i_{\Omega bdl}$	1, 1, 1, 1, 1	1, 2, 2, 2, 2	2, 2, 2, 3, 3	3, 3, 3, 3, 3																

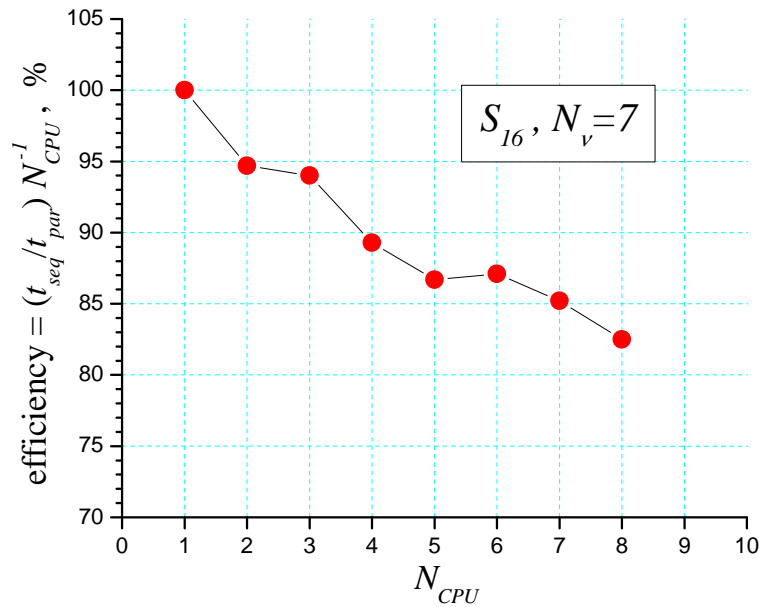


FIG. 13.23: Relative efficiency (in percentage points) of the present parallelization scheme, obtained for $N_{\Omega bdl_s} = 36$, $N_\nu = 7$ with $N_{thr} \leq 8$ threads.

combined frequency \times direction iterations consist of $N_{\Omega bdl_s}$ frequency clusters; every frequency cluster has

$$N_{\nu cl} = \begin{cases} N_{\nu}, & N_{thr} \text{ is a multiple of } N_{\Omega bdl_s}, \text{ or } N_{\Omega bdl_s} \text{ is a multiple of } N_{thr}, \\ N_{\nu} + 1, & \text{otherwise,} \end{cases} \quad (13.92)$$

frequencies (or frequency-equivalent elements); the N_K combined iterations are divided into N_{thr} chunks of work, each chunk comprising

$$N_{ch} = \begin{cases} \text{INT}(N_K/N_{thr}), & N_K = N_{thr} \times \text{INT}(N_K/N_{thr}), \\ \text{INT}(N_K/N_{thr}) + 1, & N_K > N_{thr} \times \text{INT}(N_K/N_{thr}) \end{cases} \quad (13.93)$$

successive iterations to be performed by one parallel thread. The current index of the Ω -bundle can be calculated as

$$i_{\Omega bdl} = \begin{cases} \text{INT}[(K-1)/N_{\nu cl}] + 1, & N_{\nu cl} = N_{\nu}, \\ \text{INT}(K/N_{\nu cl}) + 1, & N_{\nu cl} = N_{\nu} + 1. \end{cases} \quad (13.94)$$

The described parallelization scheme is illustrated on three characteristic examples in Tables V–VII. Figure 13.23 shows the efficiency of parallel simulation for $N_{\Omega bdl_s} = 36$, $N_{\nu} = 7$ and $N_{thr} = 1, 2, \dots, 8$.

Correspondence with the code variables:

$N_{\Omega bdl_s}$	= <code>n0mbndl</code>	number of Ω -bundles;
$i_{\Omega bdl}$	= <code>i0mbndl</code>	sequential number of the current Ω -bundle;
$N_{\nu cl}$	= <code>nnuclust(kOMPAss)</code>	number of frequencies (frequency-element positions) in a frequency cluster;
N_{ch}	= <code>nchunk(kOMPAss)</code>	number of combined frequency \times direction iterations in one chunk;
N_K	= <code>nKloop(kOMPAss)</code>	total number of combined frequency \times direction iterations;

14. RAM REQUIREMENTS

Principal RAM requirements for simulations with RALEF-2D are associated with the field arrays (for such variables like fluid density, flow velocity, etc.) allocated in `module COMDECK1`. The basic dimension unit for these arrays is given by the parameter `msize`, which represents the dimension of an array for a scalar physical variable on the entire 2D mesh. Here we use the memory block needed for a single scalar `real(8)` array of size `msize` as a convenient unit of RAM and call it an *r8-block*. As an example of an advanced high-resolution simulation, we may consider the case of `msize = 1 000 000`, where the r8-block is equal to 8 MB of RAM.

Module `COMDECK1` contains two types of field arrays:

- “sovereign” arrays are once and for all associated with the corresponding physical variables (like temperature `temp(msize)`, for example);
- working arrays (like `w1w01(msize)`, `w2w01(2*msize)` and so on) can be used for different physical variables in different parts of the code; association with physical variables is done by using corresponding pointers.

There is a clear naming convention for the working arrays.

The RAM requirements for the code version **RAD-XY-2010.11** are as follows

- 34 basic r8-blocks for “sovereign” arrays needed for hydrodynamics and thermal conduction;
- 6 additional r8-blocks for “sovereign” arrays needed for radiation transport and laser energy deposition;
- 38 basic r8-blocks for working arrays needed for hydrodynamics and thermal conduction;
- 32 additional r8-blocks for working arrays needed for radiation transport and laser energy deposition;

Thus, the total RAM requirement is 110 r8-blocks, which amounts to about 1 GB of RAM for a high-resolution simulation. However, such an estimate will only be correct for a sequential simulation without parallelization. If an OpenMP version of the code is used with N_{thr} parallel threads, then the actual RAM requirement will be

$$110 + 56(N_{thr} - 1) \quad (14.95)$$

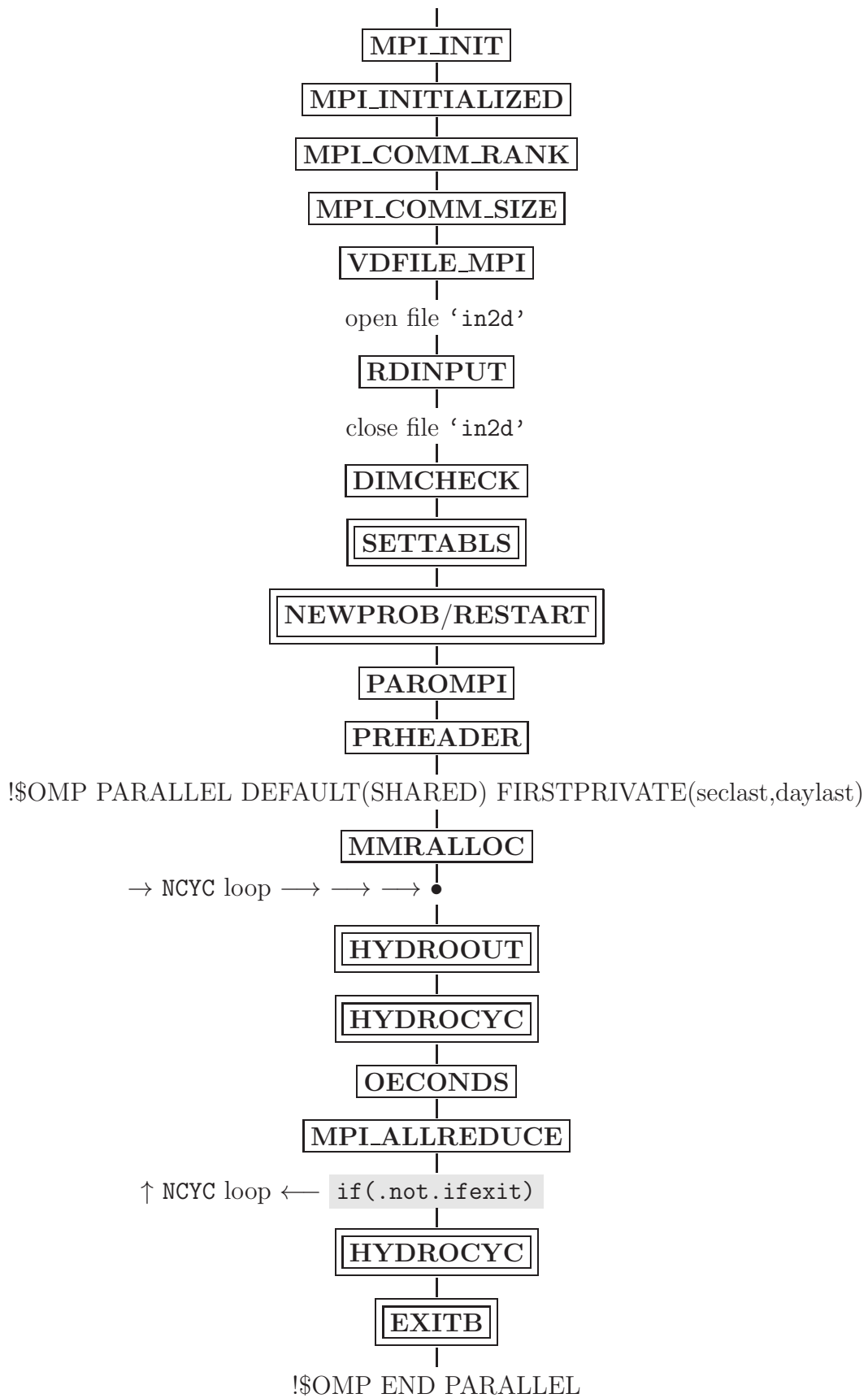
r8-blocks, because 56 r8-blocks must be allocated to thread-private working arrays. Then, a high-resolution simulation with 8 threads will require about 4 GB of RAM.

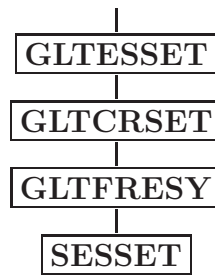
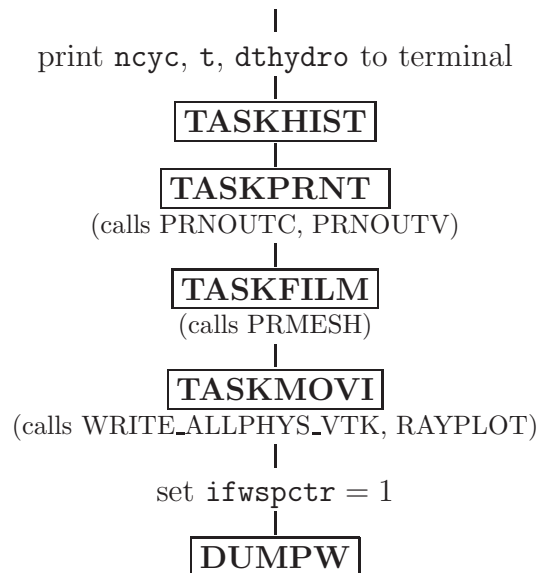
From the above data one infers also that in simulations without radiation transport and laser deposition one can save about 30% of RAM by commenting out the corresponding memory allocation statements in module COMDECK1 for these processes, and by using the dummy versions of the files "f09_rad.f" and "f11_taskdepo.f".

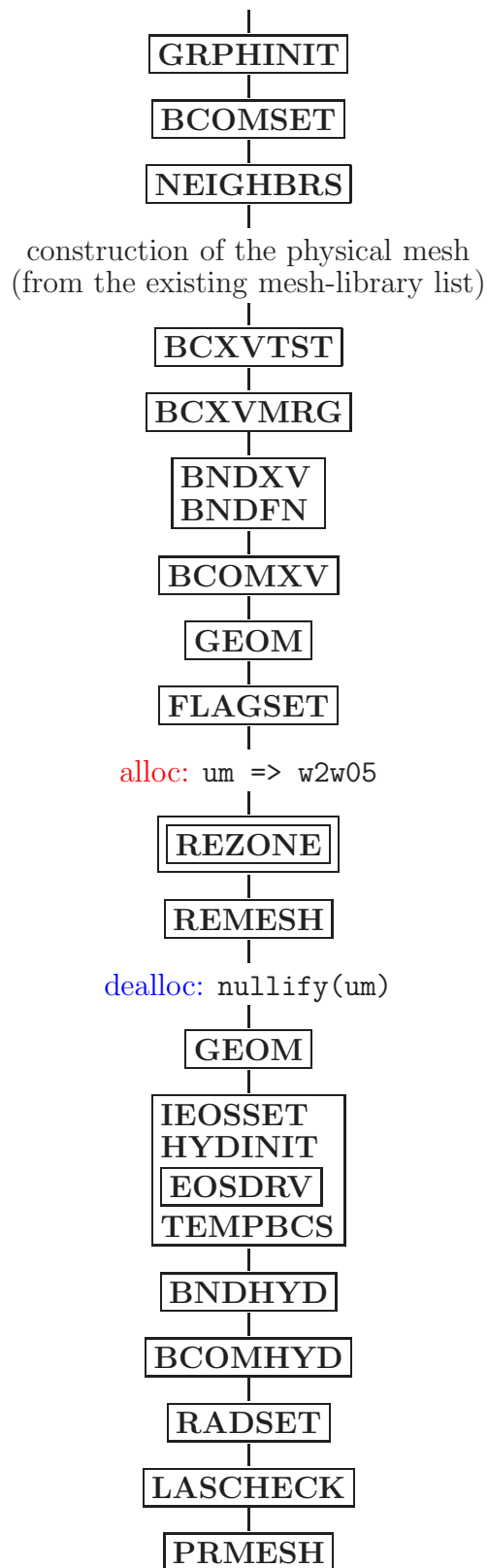
APPENDIX A: FLOWCHARTS OF SUBROUTINE CALLS AND MEMORY ALLOCATION

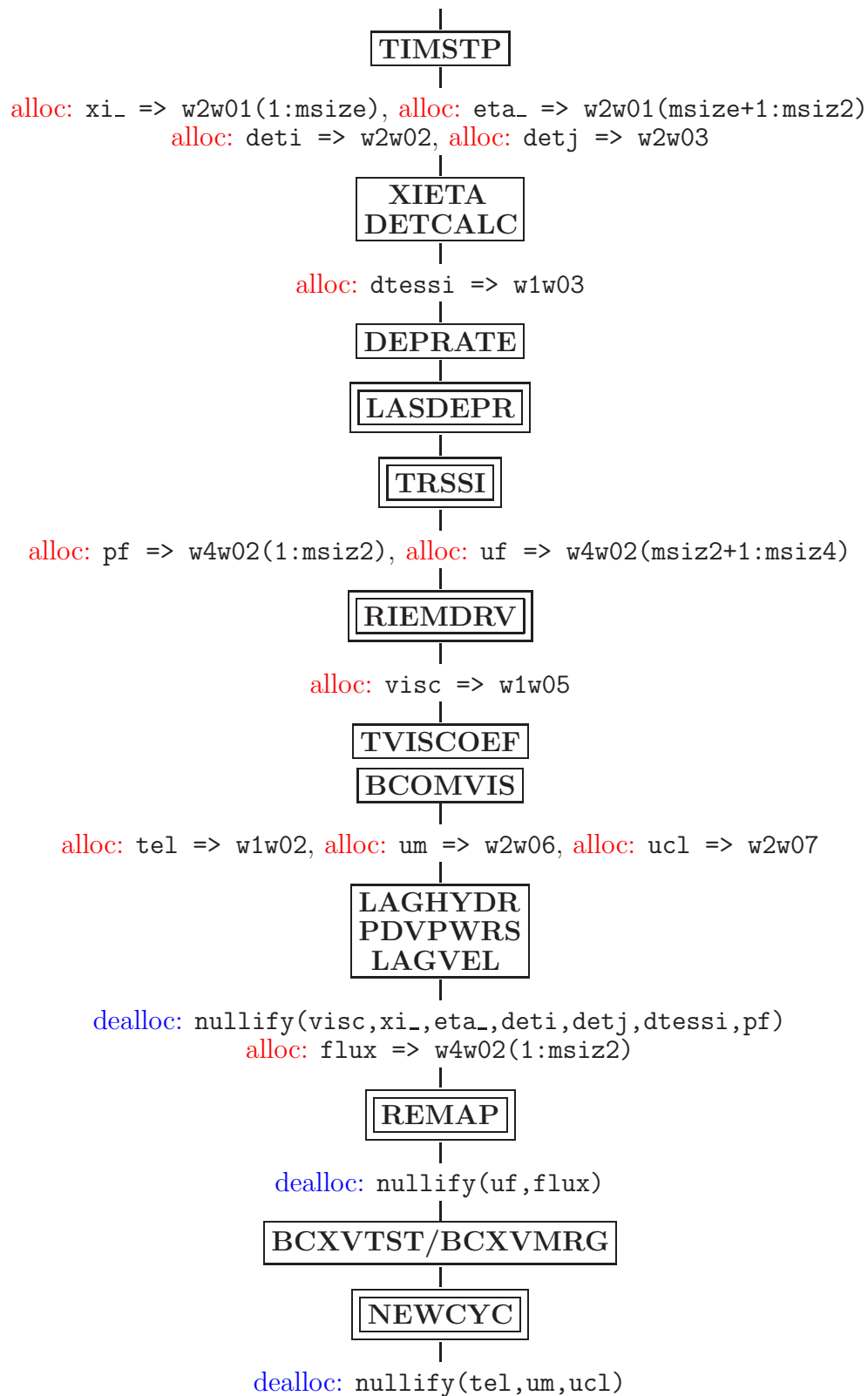
For complex subroutines put in a double frame, like ABC, separate flowcharts of their own are presented. When several subroutine names are combined in a single frame, it means that they are called in a loop over mesh blocks (in an `iblk` loop).

Program RALEF:

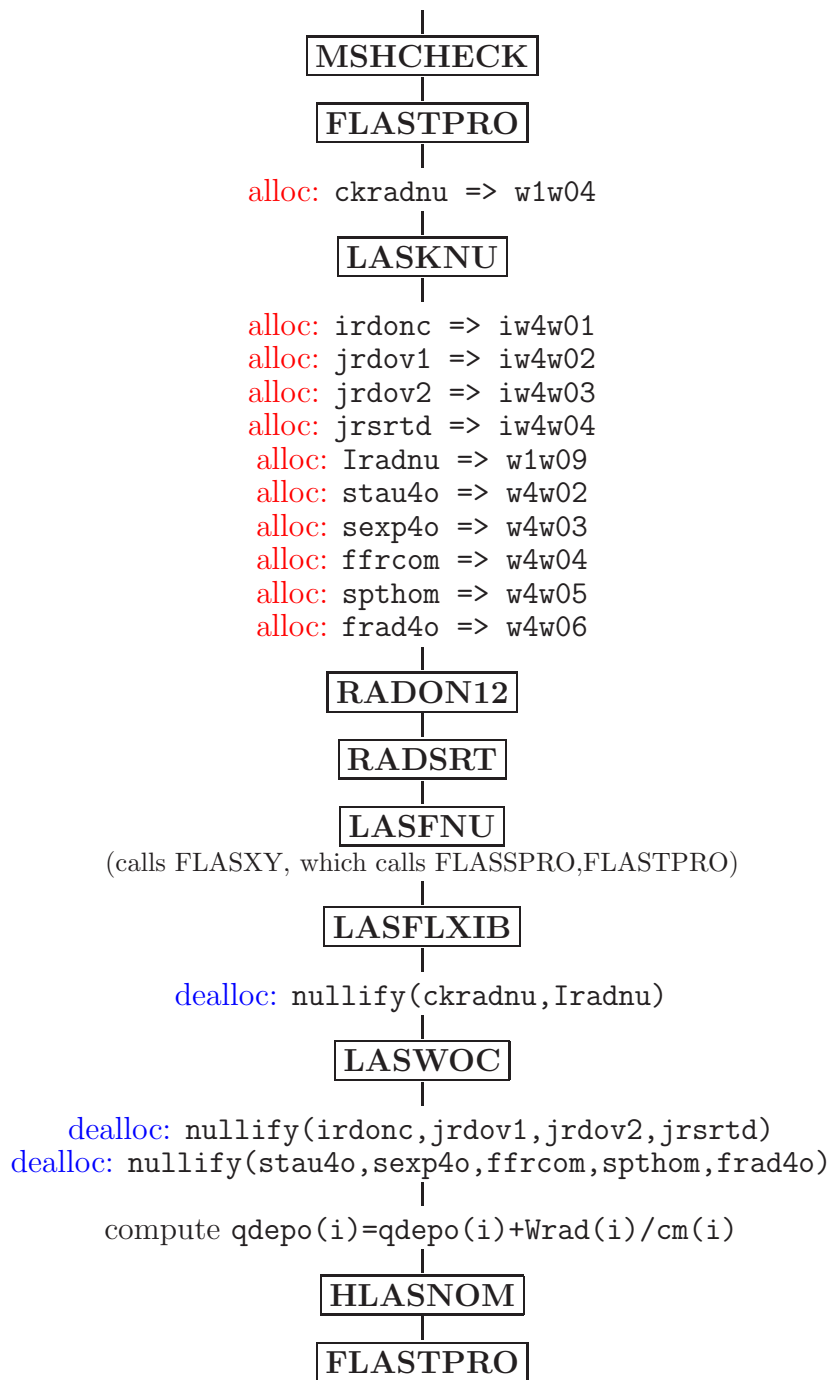


Subroutine SETTABLES:**Subroutine HYDROOUT:**

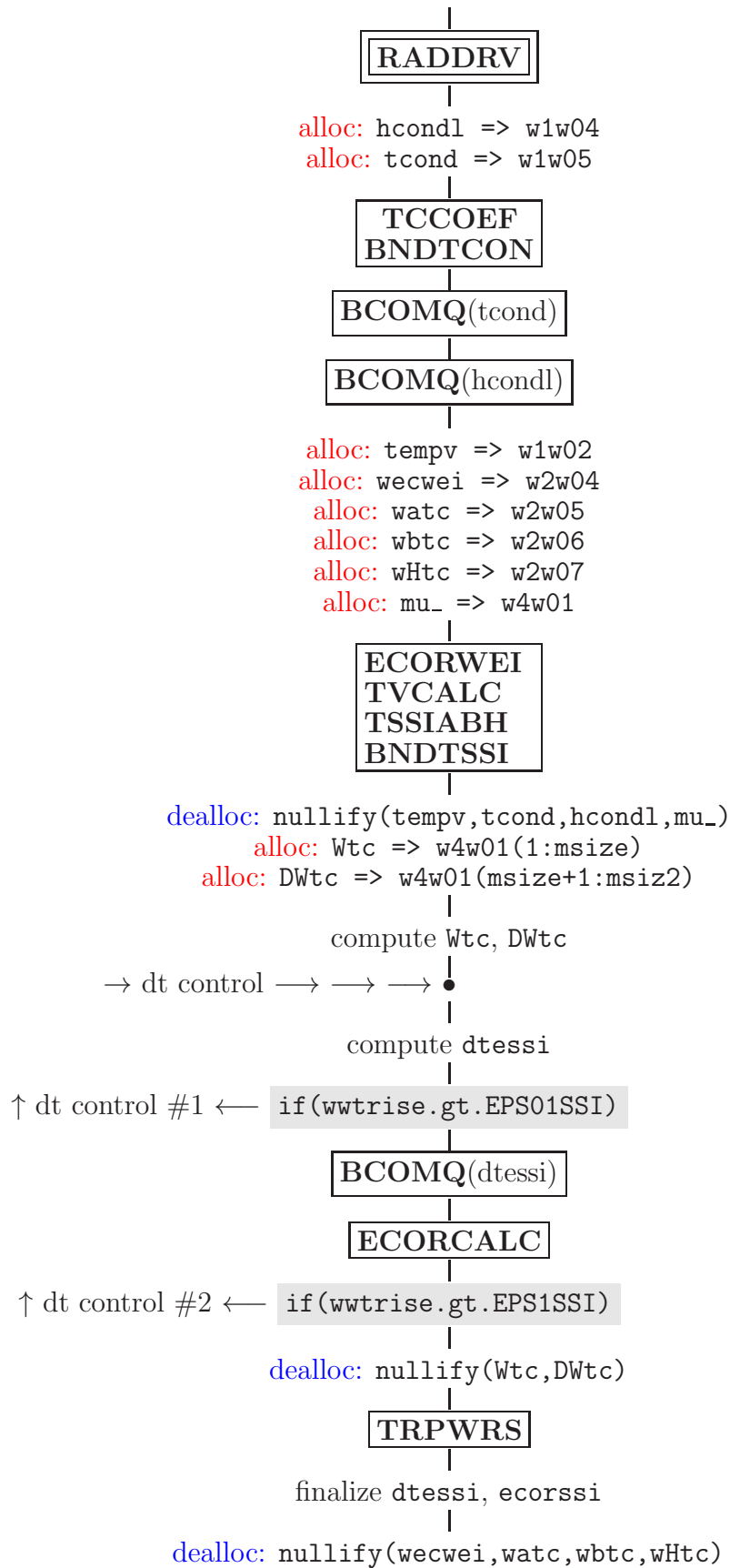
Subroutine NEWPROB:

Subroutine HYDROCYC:

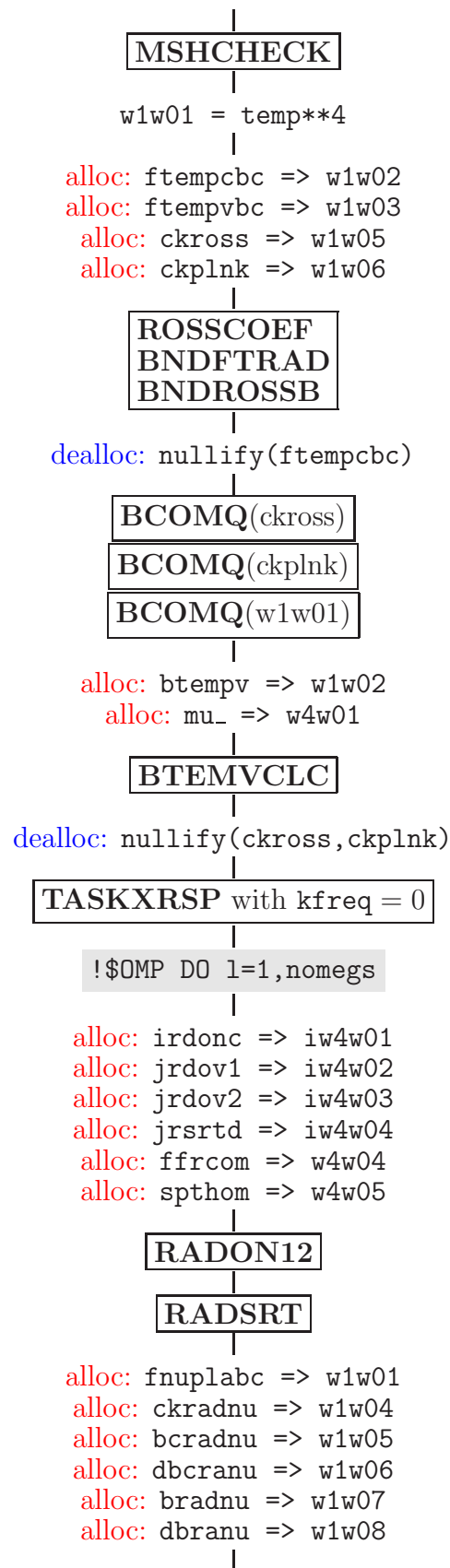
Subroutine LASDEPR:

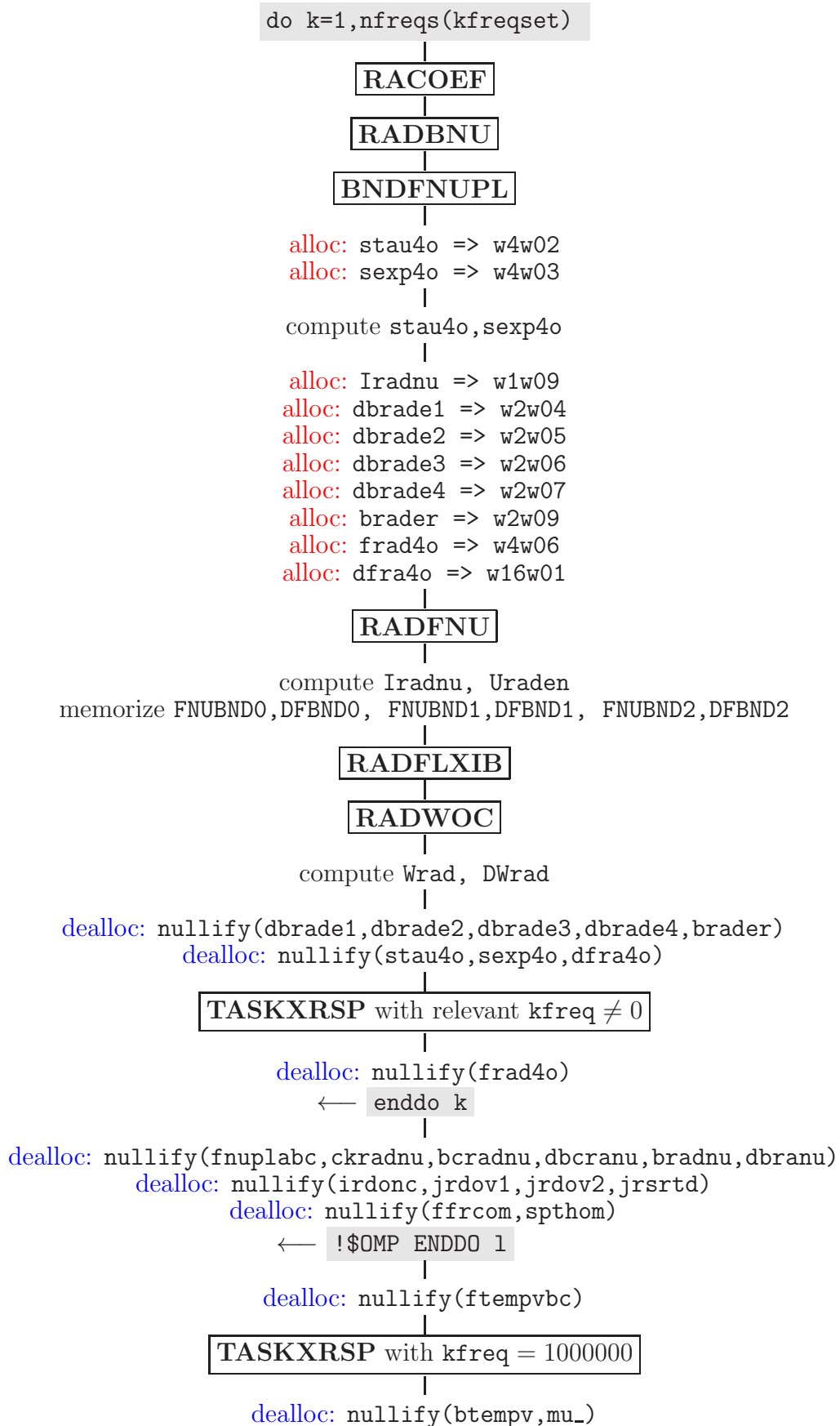


Subroutine TRSSI:



Subroutine RADDRV:

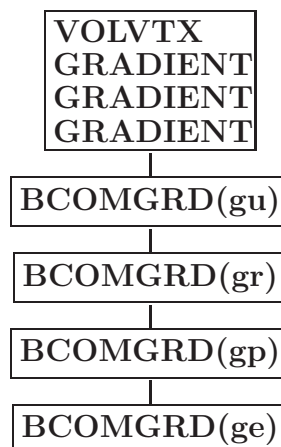


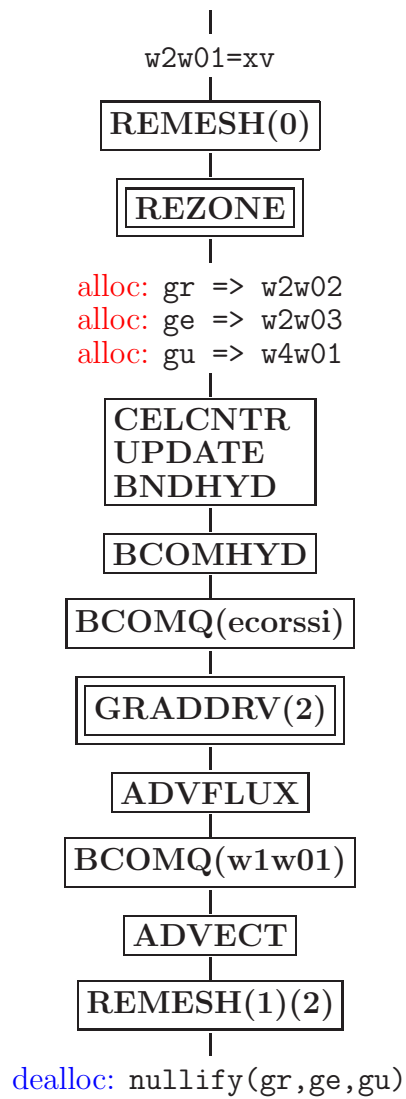


Subroutine RIEMDRV:

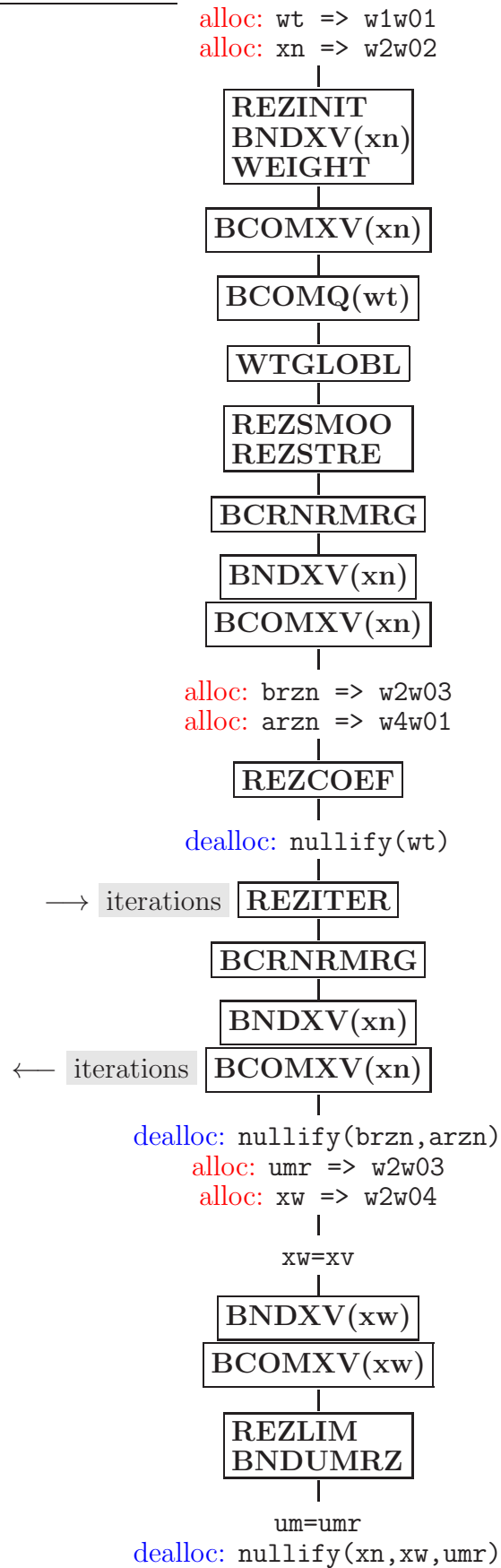


Subroutine GRADDRV:



Subroutine REMAP:

Subroutine REZONE:



Subroutine NEWCYC: