

II. THERMODYNAMICS

The principal thermodynamic identity

$$\left(\frac{\partial E}{\partial V}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_V - P = T^2 \left[\frac{\partial}{\partial T} \left(\frac{P}{T}\right)\right]_V, \quad \text{or} \quad \rho^2 \left(\frac{\partial \epsilon}{\partial \rho}\right)_T = P - T \left(\frac{\partial P}{\partial T}\right)_\rho. \quad (1)$$

Adiabatic sound velocity

$$c_s^2 \equiv \left(\frac{\partial P}{\partial \rho}\right)_S = \left(\frac{\partial P}{\partial \rho}\right)_T + \frac{T}{\rho^2} \frac{(\partial P / \partial T)_\rho^2}{(\partial \epsilon / \partial T)_\rho}. \quad (2)$$

If the pressure is given as $P = P(\rho, \epsilon)$, then

$$c_s^2 = \left(\frac{\partial P}{\partial \rho}\right)_\epsilon + \frac{P}{\rho^2} \left(\frac{\partial P}{\partial \epsilon}\right)_\rho. \quad (3)$$

Heat capacity

$$c_v = \left(\frac{\partial \epsilon}{\partial T}\right)_\rho > 0; \quad c_p = c_v + \frac{T}{\rho^2} \frac{(\partial P / \partial T)_\rho^2}{(\partial P / \partial \rho)_T}. \quad (4)$$

III. APPROXIMATE FORMULAE FOR THE IDEAL FERMI-GAS OF FREE ELECTRONS

Let n_e be the number of free electrons per unit volume. Then, the Fermi energy of the ideal free-electron gas is given by

$$E_F = \frac{1}{2} (3\pi^2)^{2/3} \frac{\hbar^2}{m_e} n_e^{2/3} = 26.0 \left(\frac{\rho Z}{A}\right)^{2/3} [\text{eV}]. \quad (5)$$

If we define the dimensionless electron temperature as

$$\theta_e = \frac{T_e}{E_F}, \quad (6)$$

then the specific (per one electron) Helmholtz free energy of this gas can to a good accuracy be evaluated as

$$f_e = \frac{3}{5} E_F - \frac{3}{2} T_e \ln(1 + a\theta_e), \quad (7)$$

where a is a fitting parameter. The exact cold asymptotics $f_e \rightarrow \frac{3}{5} E_F$ as $\theta_e \rightarrow 0$ is ensured for any value of a . The exact Maxwellian asymptotics in the limit of $\theta_e \rightarrow \infty$ is recovered for

$$a = \frac{3}{2} \left(\frac{\pi e^2}{6}\right)^{1/3} = 2.3547987\dots, \quad (8)$$

where $e = 2.71828\dots$ is Euler's e number. From Eq. (7) one calculates the pressure

$$p_e = - \left(\frac{\partial f_e}{\partial n_e^{-1}}\right) = n_e \left(\frac{2}{5} E_F + T_e \frac{a\theta_e}{1 + a\theta_e}\right) \quad (9)$$

and the specific (per one electron) internal energy

$$\epsilon_e = f_e - T_e \left(\frac{\partial f_e}{\partial T_e}\right) = \frac{3}{2} \frac{p_e}{n_e} = \frac{3}{5} E_F + \frac{3}{2} T_e \frac{a\theta_e}{1 + a\theta_e}. \quad (10)$$

The chemical potential μ_e and the specific (per one electron) entropy s_e are given by

$$\mu_e = f_e + \frac{p_e}{n_e} = E_F + T_e \left[\frac{a\theta_e}{1 + a\theta_e} - \frac{3}{2} \ln(1 + a\theta_e)\right], \quad (11)$$

$$s_e = \frac{\epsilon_e - f_e}{T_e} = \frac{3}{2} \left[\frac{a\theta_e}{1 + a\theta_e} + \ln(1 + a\theta_e)\right]. \quad (12)$$

For $a = 2.35479\dots$ the relative error of Eq. (9) (compared to exact Fermi integrals) never exceeds 2.6%; if the exact Maxwellian limit for f_e and μ_e is not of principal importance, the accuracy of Eq. (9) can be improved to 1.5% by setting $a = 5/2$.

IV. PLASMA PHYSICS

Plasma frequency

$$\omega_{pe}^2 = \frac{4\pi e^2 n_e}{m_e} = \dots, \quad (13)$$

$$\hbar\omega_{pe} = 3.713307 \times 10^{-11} \sqrt{n_e} \text{ [eV]}; \quad n_e \text{ is in cm}^{-3}. \quad (14)$$

Plasma critical density

$$n_{e,cr} = \pi \frac{m_e c^2}{\lambda_{\mu m}^2 e^2} = 1.114854 \times 10^{21} \lambda_{\mu m}^{-2} \text{ [cm}^{-3}\text{]}, \quad (15)$$

where $\lambda_{\mu m}$ is the wavelength of the electromagnetic wave in micrometers.

V. ISOTOPES

Mass = M (in atomic units), includes the mass of atomic electrons; natural abundance = δ .

H = ^1H :	$M = 1.007\ 825\ 035(12)$	
D = ^2H :	$M = 2.014\ 101\ 779(24)$	$\delta = 1.15(70) \times 10^{-4}$
T = ^3H :	$M = 3.016\ 049\ 27(4)$	$t_{1/2} = 12.33\text{y}$
^3He :	$M = 3.016\ 029\ 31(4)$	$\delta = 1.37(3) \times 10^{-6}$
^4He :	$M = 4.002\ 603\ 24(5)$	
^6Li :	$M = 6.015\ 121\ 4(7)$	$\delta = 7.59(4)\%$
^7Li :	$M = 7.016\ 003\ 0(9)$	$\delta = 92.41(4)\%$
^9Be :	$M = 9.012\ 181\ 2(4)$	$\delta = 100\%$
^{10}B :	$M = 10.012\ 936\ 9(3)$	$\delta = 19.9(7)\%$
^{11}B :	$M = 11.009\ 305\ 4(4)$	$\delta = 80.1(7)\%$