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K-FLUORESCENCE LINES IN SPECTRA OF X-RAY BINARIES

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ABSTRACT

The reflection of the X-rays in K lines of heavy elements from a cold surface is calculated. The radiative transfer is treated rigorously, with the albedo in K lines expressed in terms of the *H*-function for isotropic scattering. This approach is also applied to evaluate the spectral shape of the low-energy wing of the line, formed by the singly scattered K photons. A detailed discussion is given of the main characteristics of K-emission from the X-ray binaries. The prospects of observations of the narrow Kα lines in X-ray binaries are analyzed, with special attention paid to the new information that can be obtained from such observations. The minimum equivalent width which one could expect in the case of the iron Kα line is evaluated for a number of well-known X-ray binaries.

Subject headings: radiative transfer — X-rays: binaries — X-rays: spectra

I. INTRODUCTION

With the construction of new sensitive X-ray instruments it will be soon possible to detect narrow lines in the spectra of cosmic X-ray sources. The existence of such lines is evidenced by a number of recent experiments (Sanford, Mason, and Ives 1975; Serlemitsos *et al.* 1975; Davison, Culhane, and Mitchell 1976; Becker *et al.* 1976; Serlemitsos *et al.* 1977; Pravdo 1976; Pravdo *et al.* 1977). The correct interpretation of the X-ray spectroscopic data—impossible without detailed analysis of the X-ray line-emission mechanisms—will undoubtedly supply us with new valuable information on the nature of X-ray sources (McCray 1977). In this paper a detailed analysis of one of such mechanisms—the K-fluorescence of heavy elements in X-ray binaries—is presented.

In a close binary system a significant fraction ($\sim 10\%$) of X-rays emitted by the compact source is absorbed in relatively cold plasma layers. Such a cold plasma is present in the photosphere of the normal companion, at the outskirts of the accretion disk (if the latter exists), in the gas streams within the system, etc. Under the temperature $T \sim$ few $\times$ 10^4 K, typical for these regions, the atoms of heavy elements (such as S, Ar, Fe, Ni) are in rather low stages of ionization with filled K and L shells. The X-ray photons in such cold gas are absorbed mainly due to the photoeffect on K electrons, and if the ejection of the K electron is followed by the radiative transition from the occupied L shell (Kα fluorescence), a Kα photon is emitted. The transition from the M shell would create a Kβ photon. From the point of view of the transfer theory the most important property of the Kα line is the absence of resonant absorption in the line. Some of the Kα photons get absorbed by the photoionization of heavy elements with smaller values of nuclear charge Z; others escape. Thus, any cold surface (or a cloud of cold gas) in the vicinity of an X-ray source effectively processes by means of K-fluorescence the continuum X-rays into K lines of heavy elements. Note that the scattering of X-rays by free electrons and those bound in the atoms of hydrogen and helium should be also taken into account.

In an earlier paper (Basko, Sunyaev, and Titarchuk 1974) on the reflection effect in X-ray binaries it was mentioned that Kα lines of iron and some other heavy elements should be present in the spectra of X-ray binaries. Only a rough estimate of the equivalent width of the iron Kα line was given there. Felsteiner and Opher (1976) performed Monte Carlo calculations of the reflection of X-rays from the cold plane-parallel atmosphere. Their estimates of the iron Kα emission exceed the results obtained below by $\sim 20\%$, which may be partly due to the difference in the adopted values of heavy element abundances and photoionization cross sections and partly due to the assumption below of monochromatic scattering of the primary X-rays. In the paper by Hatchett and Weaver (1977), which is entirely devoted to the Kα emission calculations, the transfer of both continuum and line radiation is treated in the diffusion approximation. In the framework of this approximation they discuss the basic characteristics of the iron Kα line, the dependence of the line equivalent width on the heavy element abundances, and parameters of the binary system.

In this paper, as contrasted to the work of Hatchett and Weaver (1977), the exact solution of the radiative transfer problem in a plane-parallel atmosphere is used. The intensity of the K line core and of the singly scattered wing is expressed in terms of the Ambartsumyan function (a particular case of the Chandrasekhar *H*-function). Of particularly simple form is the analytic expression (13) for the differential albedo in a spectrally narrow core of the K line, which is the most interesting from the observational point of view. The numerical values of the equivalent width of the iron Kα line are somewhat less than those of Hatchett and Weaver (1977); some of their qualitative conclusions are significantly refined. Besides iron, the equivalent width of the Kα line is also evaluated for sulfur, argon, and nickel. Some general, as well as more specific, problems in the physics of X-ray binaries are outlined that can be resolved by the observations of Kα lines.

II. REFLECTION IN K LINE FROM A PLANE-PARALLEL ATMOSPHERE

Since the mean path length of the X-ray photons is much less than the radius of the normal star, one can calculate the reflection in K lines assuming the atmosphere to be plane-parallel at each point. In this section I calculate the intensity of the K line emerging from a plane-parallel atmosphere irradiated by an external X-ray flux with continuous spectrum. To perform such calculations, one should solve a double transfer problem: first, one should calculate the transfer of the primary X-rays with continuous spectrum; and then one must solve the transfer problem for K line radiation generated by the primary radiation field. For this, the transfer problem should be treated as rigorously as possible, since the flux in the K line is usually formed at optical depths $\tau \lesssim 1$.

a) *Basic Assumptions*

The basic equations (4) and (7) governing the double transfer problem mentioned above are written under the following assumptions.

i) The opacity of the cold atmosphere for both continuum and line radiation is due to two processes: photoabsorption by neutral atoms of heavy elements, and Thomson scattering by free electrons and by electrons bound in atoms of hydrogen and helium (which can be treated as free for photon energies ~ few keV [Gorshkov, Michailov, and Sherman 1973]).

ii) The frequency of X-ray photons does not change by scattering; the scattering cross section is isotropic, $d\sigma_S = \sigma_T d\Omega/4\pi$, where $\sigma_T = 6.652 \times 10^{-25}$ cm^2 is the Thomson cross section. Since the strongest K lines fall in the range 2 keV $\lesssim \epsilon_K \lesssim$ 8 keV and belong to the elements with large Z, assumption (i) is safely satisfied in the atmospheres of normal stars, while assumption (ii) deserves some more detailed discussion.

Let us estimate for the Kα line of iron the typical error introduced by the assumption of monochromatic scattering of the primary X-rays. First of all, note that it always results in an underestimate of the K photon flux because with each scattering the photon energy decreases due to Compton recoil, while the probability of photoabsorption followed by K photon emission increases. It is shown below (see Fig. 1) that the fraction of the primary X-ray flux converted into Kα radiation is approximately proportional to ϵ^{-3}, where ϵ is the energy of primary photons. Then, under the assumption of monochromatic scattering, the total Kα flux is proportional to the integral

$$\int_{\epsilon_{th}}^{\epsilon_{max}} \epsilon^{-3} d\epsilon$$

(for the flat incident spectrum). Suppose now that the photoabsorption cross section corresponds not to the initial photon energy ϵ but to the decreased value

$$\epsilon_f = (\epsilon^{-1} + \bar{N}/m_e c^2)^{-1}, \tag{1}$$

where

$$\bar{N}(\epsilon) \approx [1 + \sigma_T/\sigma_{ph}(\epsilon)]^{1/2} \approx [1 + (\epsilon/9.84 \text{ keV})^3]^{1/2} \tag{2}$$

is the mean number of scatters (Ivanov 1969). The average energy shift by single scattering is taken to be $\Delta\epsilon = -\epsilon^2/m_e c^2$; $\sigma_{ph}(\epsilon)$ is the photoabsorption cross section per electron. The total flux in the Kα line is then proportional to

$$\int_{\epsilon_{th}}^{\epsilon_{max}} \{\epsilon^{-1} + [1 + (\epsilon/9.84 \text{ keV})^3]^{1/2}/m_e c^2\}^3 d\epsilon\,. \tag{3}$$

For $\epsilon_{th} = 7.11$ keV and $\epsilon_{max} = 25$ keV, integral (3) exceeds that for the monochromatic scattering by 13%. Thus, *the error introduced into K line intensity by the assumption of monochromatic scattering of the primary X-rays is always negative and of the order of* 10%. The typical error in the total K line flux due to the assumed isotropy of the scattering cross section is ~1–2%. Assumption (ii), however, cannot be adopted for the secondary radiation when calculating the K line profile; for that purpose the scattering process of K photons should be treated rigorously (see § II*d*).

b) *Equations of Transfer*

Under assumptions (i) and (ii) the reflection of the continuum X-rays in K lines can be calculated separately for each value of the incident photon energy ϵ. The equation of the primary radiation transfer has a well-known form (Chandrasekhar 1960):

$$\mu \frac{dI(\tau, \mu)}{d\tau} = I(\tau, \mu) - \frac{\lambda}{2}\int_{-1}^{+1} I(\tau, \mu')d\mu' - \frac{\lambda}{4\pi} I_0 \exp(-\tau/\mu_0)\,. \tag{4}$$

Here $I(\tau, \mu)$ [ergs cm^{-2} s^{-1} sr^{-1} erg^{-1}] is the intensity of the diffusive component of the primary X-rays, arccos μ is the angle between the direction of the radiation propagation and the normal to the atmosphere. The optical depth τ and the single scattering albedo λ are defined by

$$\tau = \int_Z^\infty Ne[\sigma_T + \sigma_{ph}(\epsilon)]dz\,, \tag{5}$$

$$\lambda = \sigma_T/[\sigma_T + \sigma_{ph}(\epsilon)]\,, \tag{6}$$

where z is measured along the normal, σ_T is the Thomson cross section, $\sigma_{ph}(\epsilon)$ is the photoabsorption cross section of photons with energy ϵ per electron, N_e is the electron density (electrons bound in hydrogen and helium are also included). The incident flux of X-rays I_0 [ergs cm^{-2} s^{-1} erg^{-1}] is defined so that

$$\mu_0 \int_0^\infty I_0(\epsilon)d\epsilon$$

is the total amount of energy crossing 1 cm^2 of the atmospheric surface per 1 s (arccos μ_0 is the angle of incidence). Note that λ, τ, I, and I_0 depend on ϵ.

Let now $J_\epsilon(\tau_K, \mu)d\epsilon$ [ergs cm^{-2} s^{-1} sr^{-1}] be that fraction of the K line radiation intensity which is created by the primary radiation field originating from the incident flux $I_0 d\epsilon$ ($d\epsilon$ is the energy interval of the incident X-ray photons). Then the equation of transfer of the K line radiation is

$$\mu \frac{dJ_\epsilon(\tau_K, \mu)}{d\tau_K} = J_\epsilon(\tau_K, \mu) - \frac{\lambda_K}{2}\int_{-1}^{+1} J_\epsilon(\tau_K, \mu')d\mu' - \frac{\delta\lambda_K}{\lambda}\frac{\epsilon_K}{\epsilon} S(\tau_K\lambda_K/\lambda)\,, \tag{7}$$

where $\tau_K = \tau(\epsilon_K)$ and $\lambda_K = \lambda(\epsilon_K)$ are the optical depth and the single scattering albedo in the line. The coefficient δ is defined by

$$\delta(\epsilon) = \omega_K \sigma_{ph,K}(\epsilon)/\sigma_T\,, \tag{8}$$

where ω_K is the fluorescence yield, and $\sigma_{ph,K}(\epsilon)$ is the cross section of K shell photoabsorption of the fluorescing element per one electron. The source function of the primary radiation field,

$$S(\tau) \equiv \frac{\lambda}{2}\int_{-1}^{+1} I(\tau, \mu')d\mu' + \frac{\lambda}{4\pi} I_0 \exp(-\tau/\mu_0)\,, \tag{9}$$

is to be found from the solution of equation (4). Of practical interest is the calculation of

$$\int_{\epsilon_{th}}^\infty J_\epsilon(0, \mu)d\epsilon \equiv J(0, \mu) \tag{10}$$

—the total energy flux in the K line emerging from the atmosphere at an angle arccos μ with respect to the normal. Both the calculational procedure and the observational properties warrant the quantity $J(0, \mu)$ to be split into a sum

$$J(0, \mu) = J^{(0)}(0, \mu) + J^{(1)}(0, \mu) + \cdots \tag{11}$$

of individual terms $J^{(n)}(0, \mu)$, each one corresponding to a fixed $(=n)$ number of scatters suffered by K photons. Due to the linear dependence on I_0, the analogous expansion of $J_\epsilon(0, \mu)$ can be written in the form

$$J_\epsilon(0, \mu) = J_\epsilon^{(0)}(0, \mu) + J_\epsilon^{(1)}(0, \mu) + \cdots$$
$$= \frac{1}{\pi} I_0\mu_0\psi(\epsilon, \mu_0, \mu) = \frac{1}{\pi} I_0\mu_0[\psi^{(0)}(\epsilon, \mu_0, \mu) + \psi^{(1)}(\epsilon, \mu_0, \mu) + \cdots]\,. \tag{12}$$

The dimensionless function $\psi(\epsilon, \mu_0, \mu)$, which is determined only by the properties of the atmosphere, will be called the differential albedo in the K line. It characterizes both the effectiveness of the reprocessing of the incident X-rays with photon energy ϵ into K line radiation and the beam pattern of the emergent K line flux. If all the energy of the incident flux $I_0 d\epsilon$ transformed without loss into the energy of K photons, and if the latter came out of the atmosphere isotropically, the function $\psi(\epsilon, \mu_0, \mu)$ would identically equal unity. It is evident from the definition of $\psi(\epsilon, \mu_0, \mu)$ that it must vanish for $\epsilon < \epsilon_{th}$, where ϵ_{th} is the K photoionization threshold. From equations (4) and (7) one obtains:

$$\psi^{(0)}(\epsilon, \mu_0, \mu) = \frac{\delta\lambda_K}{4}\frac{\epsilon_K}{\epsilon} H(\mu_0, \lambda)\frac{H(\mu\lambda_K/\lambda, \lambda)}{\mu_0 + \mu\lambda_K/\lambda}\,, \tag{13}$$

$$\psi^{(1)}(\epsilon, \mu_0, \mu) = \frac{\delta\lambda_K{}^2}{8}\frac{\epsilon_K}{\epsilon} H(\mu_0, \lambda)\left\{\mu\frac{H(\mu\lambda_K/\lambda, \lambda)}{\mu_0 + \mu\lambda_K/\lambda}\ln(1 + 1/\mu) + \int_0^1 \frac{d\mu'}{\mu - \mu'}\left[\mu\frac{H(\mu\lambda_K/\lambda, \lambda)}{\mu_0 + \mu\lambda_K/\lambda} - \mu'\frac{H(\mu'\lambda_K/\lambda, \lambda)}{\mu_0 + \mu'\lambda_K/\lambda}\right]\right\}\,. \tag{14}$$

The derivation of (13) and (14), as well as the approximate formulae for $H(\mu, \lambda)$ and $\psi^{(1)}(\epsilon, \mu_0, \mu)$, are given in the Appendix.

c) Basic Properties of the Plane-Parallel Line Albedo

A number of important conclusions can be drawn from equations (13) and (14). I will illustrate some of them with the example of the iron Kα line, which is very promising from the observational point of view because of the high iron abundance and its large fluorescence yield, $\omega_K = 0.34$ (Bambynek *et al.* 1972). No distinction will be made between Kα_1 ($\epsilon_K = 6.404$ keV) and Kα_2 ($\epsilon_K = 6.391$ keV), and the value $\epsilon_K = 6.40$ keV will be adopted. I normalize the abundance of iron,

$$N_{Fe} = 3.08 \times 10^{-5} Y_{Fe} N_H , \tag{15}$$

so that $Y_{Fe} = 1$ corresponds to the normal chemical composition and the coefficient of Y_{Fe} in equation (18) is unity. The cross section for photoabsorption by the K shell of iron ($\epsilon_{th} = 7.11$ keV) per electron (the number of electrons $= 1.2N_H$) is (Rakavy and Ron 1967):

$$\sigma_{ph,K}(\epsilon) = 3.17 \times 10^{-22}\ \mathrm{cm}^2\,(1\ \mathrm{keV}/\epsilon)^3 Y_{Fe} . \tag{16}$$

For the abundance of heavy elements with $Z < 26$, responsible for the absorption of iron Kα photons, I adopt the values given by Brown and Gould (1970). The photoabsorption cross section at $\epsilon < \epsilon_{th}$ is then

$$\sigma_{ph}(\epsilon) = 3.17 \times 10^{-22}\ \mathrm{cm}^2\,(1\ \mathrm{keV}/\epsilon)^3\, Y , \tag{17}$$

where the factor Y accounts roughly for the possible deviations from the normal chemical composition. Now the definitions of λ, λ_K, and δ become

$$\lambda = [1 + (Y + Y_{Fe})(7.81\ \mathrm{keV}/\epsilon)^3]^{-1} , \tag{18}$$

$$\lambda_K = (1 + 1.81\, Y)^{-1} , \tag{19}$$

$$\delta = Y_{Fe}(5.26\ \mathrm{keV}/\epsilon)^3 . \tag{20}$$

First of all note that *the effectiveness of reprocessing of X-ray photons into K line flux drops rapidly with increasing photon energy* ϵ: if $\lambda \approx 1$ ($\epsilon \gg \epsilon_{th}$), the differential albedo $\psi \propto \epsilon^{-4}$, which immediately follows from (13), (14), and (20); near the threshold $\epsilon \gtrsim \epsilon_{th}$ this behavior is not so steep—the numerical estimates show that approximately $\psi \propto \epsilon^{-3}$. The dependence on ϵ is clearly demonstrated on Figure 1, where the values of the total albedo

$$\alpha(\epsilon, \mu_0) = 2 \int_0^1 \psi(\epsilon, \mu_0, \mu)\mu d\mu \tag{21}$$

are plotted versus ϵ. The values of $\alpha(\epsilon_{th}, \mu_0)$, given in Table 1, show the dependence on μ_0.

From (13) and (14) it follows also that the K line flux is rather sensitive to the chemical composition of the atmosphere, and especially to the abundance of the fluorescing element. If, for example, $Y \sim Y_{Fe} \sim 1$, the intensity of the iron Kα line is almost directly proportional to Y_{Fe} (with increasing Y_{Fe} this dependence weakens) and almost inversely proportional to Y; at $Y \sim Y_{Fe} \ll 1$ it is still proportional to Y_{Fe} and almost independent of Y—see Figure 2. Hatchett and Weaver (1977) chose to describe both the abundance of iron and of all other heavy elements with a single parameter A, and thus obtained only a weak dependence on A. With two parameters we have a much more detailed picture. If, for instance, we know Y and can measure $\psi^{(0)}$, we are able to make a good estimate of the iron abundance Y_{Fe}. If one can measure both $\psi^{(0)}$ and $\psi^{(1)}$, then both parameters Y and Y_{Fe} can be found, since the ratio $\psi^{(1)}/\psi^{(0)} \propto \lambda_K$ is rather sensitive to Y. On the other hand, the value of Y might be estimated by some other method; and since it characterizes the combined photoabsorption by a number of heavy elements

TABLE 1

MONOENERGETIC ALBEDOS IN THE IRON Kα LINE OF THE PLANE-PARALLEL ATMOSPHERE AT DIFFERENT ANGLES OF INCIDENCE

μ_0	$\alpha^{(0)}(\epsilon_{th}, \mu_0)$	$\alpha^{(1)}(\epsilon_{th}, \mu_0)$
0...........	5.44×10^{-2}	5.26×10^{-3}
0.2..........	3.99×10^{-2}	6.32×10^{-3}
0.4..........	3.29×10^{-2}	6.04×10^{-3}
0.6..........	2.81×10^{-2}	5.63×10^{-3}
0.8..........	2.47×10^{-2}	5.22×10^{-3}
1...........	2.20×10^{-2}	4.85×10^{-3}

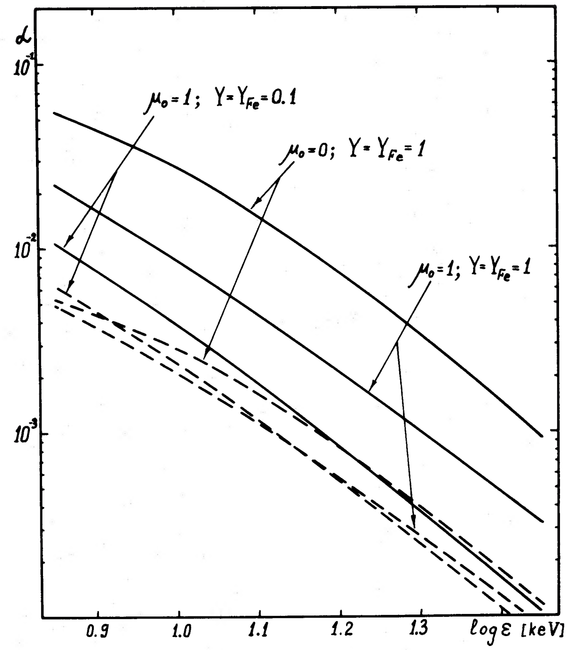

FIG. 1.—The K-line albedo of the plane-parallel atmosphere as a function of the photon energy ϵ of primary X-rays at different values of the cosine μ_0 of the incidence angle ($\mu_0 = 1$, normal incidence) and of heavy element abundances Y and Y_{Fe}. *Solid curves*, $\alpha^{(0)}(\epsilon, \mu_0)$; *dashed curves*, $\alpha^{(1)}(\epsilon, \mu_0)$.

(the contribution of each individual element being not very large), the error in the value of Y may be considerably less than the errors in the abundances of individual elements.

d) *The Profile of the* Kα *Line*

Of all the K series, the most interesting is the Kα line which, as was mentioned above, consists of two components Kα_1 and Kα_2. I will not discuss Kβ fluorescence here since the ratio of the probability to emit a Kβ photon to the sum of the Kα_1 and Kα_2 fluorescence yields is 17:150 for iron (Kikoin 1976). However, the intensity of the Kβ

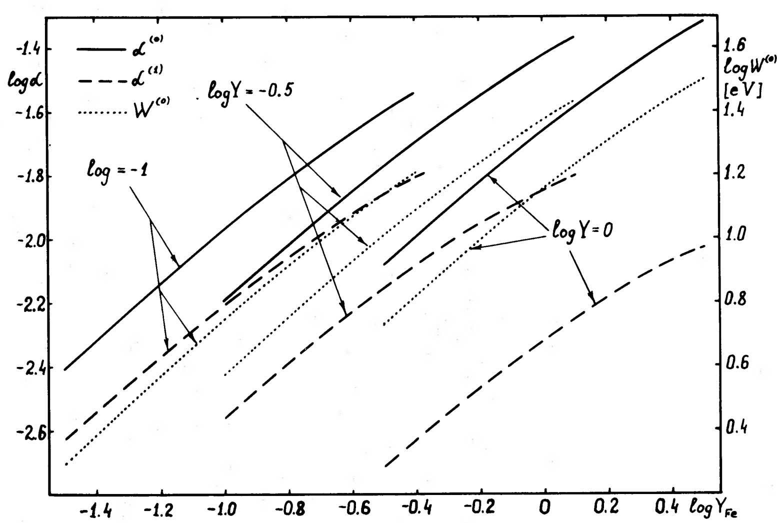

FIG. 2.—The dependence of the iron Kα line intensity on the chemical composition of the atmosphere. Solid and dashed curves show the plane albedos $\alpha^{(0)}(\epsilon_{th}, 1)$ and $\alpha^{(1)}(\epsilon_{th}, 1)$ (*left-hand ordinates*) of the normally incident primary flux with photon energy $\epsilon = \epsilon_{th} = 7.11$ keV. *Dotted curves*, the equivalent width of the Kα line core (*right-hand ordinates*) at the maximum of the optical light of HZ Her/Her X-1 system. The abscissae are the relative (with respect to the normal) abundances of iron Y_{Fe}; Y is the relative abundance of heavy elements with $Z < 26$.

line can be easily calculated from (13) and (14) if needed. The natural widths of Kα_1 and Kα_2 are rather large (the FWHM $= 2\epsilon_0 \approx 3.5$ eV for iron) and comparable to the separation between them (~ 13 eV). The probability of Kα_1 emission is twice that of Kα_2. The broadening of the Kα line due to the thermal motions of emitting atoms is completely negligible, $\Delta\epsilon \approx \epsilon_K(kT/Am_pc^2)^{1/2} \approx 0.05$ eV for $T = 2 \times 10^4$ K, which implies that the core of the Kα line $J^{(0)}(0, \mu)$ has a damping profile. In the case of the iron Kα line all the energy $J^{(0)}(0, \mu)$ is concentrated within the narrow interval $\Delta\epsilon \lesssim 30$ eV.

The recoil upon electron scattering shifts Kα photons downward in energy. After the first scattering, the initially monoenergetic beam spreads over $\Delta\lambda = 2\lambda_c = 2h/m_ec = 0.0485$ Å, which corresponds to $\Delta\epsilon = 156$ eV for the iron Kα line. The Doppler broadening due to the thermal motions of free electrons is small, $\Delta\epsilon \approx \epsilon_K(kT/m_ec^2)^{1/2} = 12$ eV for $T = 2 \times 10^4$ K, and will be neglected below. Thus, due to Compton scattering, an extended wing forms on the "red" side of the narrow Kα core. Below I calculate that part of the "red" wing which is formed by the singly scattered Kα photons.

Suppose that Kα photons are being created within an infinitely narrow line with wavelength $\lambda = \lambda_K$. To describe the spectral distribution of the singly scattered Kα photons, I introduce the function $\psi^{(1)}(\epsilon, \mu_0, \mu, x)$ such that

$$\frac{dJ_\epsilon^{(1)}(0, \mu)}{d\lambda} d\epsilon = \frac{1}{\pi\lambda_c} \mu_0 I_0 d\epsilon \psi^{(1)}(\epsilon, \mu_0, \mu, x) \quad (22)$$

is the intensity of the singly scattered Kα radiation at the boundary of the plane-parallel atmosphere, generated by the primary X-ray flux $I_0 d\epsilon$. Here $x = (\lambda - \lambda_K)/\lambda_c$ is the dimensionless wavelength of the outgoing Kα radiation. Equations (22) and (12) imply that

$$\int_0^2 \psi^{(1)}(\epsilon, \mu_0, \mu, x) dx = \psi^{(1)}(\epsilon, \mu_0, \mu) . \quad (23)$$

The function $\psi^{(1)}(\epsilon, \mu_0, \mu, x)$ can be evaluated by direct integration of the primary source (9) over all the atmosphere depth. Having introduced the polar coordinates (arccos μ', φ) with the axis along the propagation direction of the scattered Kα photons and using equation (A1) in the Appendix, one can derive

$$\psi^{(1)}(\epsilon, \mu_0, \mu, x) = \frac{\delta\lambda_K^2}{8} \frac{\epsilon_K}{\epsilon} H(\mu_0, \lambda) \tfrac{3}{4}[1 + (1 - x)^2]$$
$$\times \left\{\mu \frac{H(\mu\lambda_K/\lambda, \lambda)}{\mu_0 + \mu\lambda_K/\lambda} \frac{1}{\pi} \int_0^\pi \frac{d\varphi}{\mu - \mu'} - \frac{1}{\pi} \int_{\varphi^*}^\pi \frac{\mu'}{\mu - \mu'} \frac{H(\mu'\lambda_K/\lambda, \lambda)}{\mu_0 + \mu'\lambda_K/\lambda} d\varphi \right\} , \quad (24)$$

where

$$\mu' = (1 - x)\mu - [x(2 - x)(1 - \mu^2)]^{1/2} \cos\varphi , \quad (25)$$

$$\begin{aligned} \varphi^* &= 0, \quad 0 < x < 1 - (1 - \mu^2)^{1/2} , \\ &= \pi/2 - \arctan\{\mu(1 - x)[x(2 - x) - \mu^2]^{-1/2}\} , \quad 1 - (1 - \mu^2)^{1/2} < x < 1 + (1 - \mu^2)^{1/2} , \\ &= \pi , \quad 1 + (1 - \mu^2)^{1/2} < x < 2 . \end{aligned} \quad (26)$$

In deriving (24) the exact formula

$$x \equiv (\lambda - \lambda_K)/\lambda_c = 1 - \cos\theta \quad (27)$$

for the Compton energy shift and the Thomson differential cross section

$$d\sigma_s = \tfrac{3}{4}\sigma_T(1 + \cos^2\theta) \frac{d\Omega}{4\pi} \quad (28)$$

were used. The plots of $\psi^{(1)}(\epsilon, \mu_0, \mu, x)$ versus x for $Y = Y_{Fe} = 1$, $\epsilon = \epsilon_{th} = 7.11$ keV, $\mu_0 = 1$, and three different values of μ are given in Figure 3. An interesting feature is the discontinuity of the first derivative at $x = 1 \pm (1 - \mu^2)^{1/2}$, which is nothing else but the effect of the atmospheric boundary. In reality both this feature and the discontinuity at $\lambda = \lambda_K + 2\lambda_c$ are smeared out due to the finite natural width of the line and the Doppler broadening of thermal electrons. For comparison the real width of the Kα line core is also shown on this figure. The line profile $\psi^{(1)}(\epsilon, \mu_0, \mu, x)$ is in general agreement with that of Hatchett and Weaver (1977). It is worth noting that, in cold atmospheres with a large fraction of neutral hydrogen and helium, the simple formulae (27) and (28) are not valid at $x \lesssim \alpha^2(m_ec^2/\epsilon_K)^2 \approx 0.34$, and one must invoke more sophisticated expressions for the scattering on bound electrons ($\alpha = 1/137$ is the fine structure constant).

In this paper I do not consider the contribution of multiply scattered K photons. First of all, their contribution is difficult to calculate with the method employed here, and it would apparently be more reasonable to use the diffusion approximation. Second, they seem to be irrelevant from the observational point of view, since their

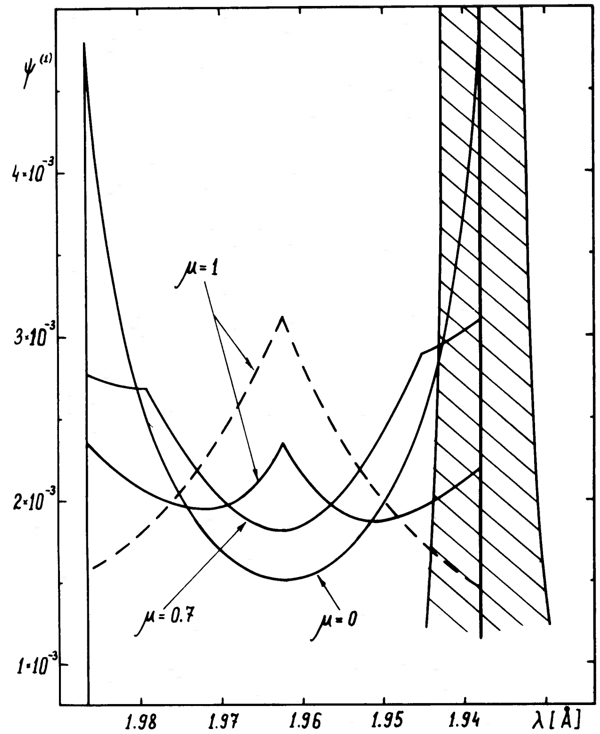

FIG. 3.—The profiles of the singly scattered wing of the iron Kα line for different values of μ, the cosine of the exit angle, under the assumptions of infinitely thin line core $\lambda = \lambda_K = 1.938$ Å and zero electron temperature. The primary flux falls along the normal and is monochromatic, $\epsilon = \epsilon_{th} = 7.11$ keV. Solid curves correspond to the Thomson scattering phase function (28); the dashed curve corresponds to the isotropic scattering phase function. Hatched is the real profile of the Kα core including two components, $\lambda_1 = 1.936$ Å and $\lambda_2 = 1.940$ Å.

contribution at normal chemical abundances is $\lesssim 10\%$ (for the iron Kα line), and they are spread over a broad spectral interval, raising the continuum level by no more than few percent. The fraction of the multiply scattered K photons which escape increases, however, with decreasing abundance of heavy elements. If, for instance, the normal companion of the X-ray source has 10 times less heavy elements than adopted for the normal composition, more than the half of the iron Kα photons will be spread over the broad interval 6 keV $\lesssim \epsilon < 6.4$ keV, while the narrow line core will still be noticeable (see Fig. 2).

III. Kα-EMISSION FROM X-RAY BINARIES

There can be a number of regions in the binary system which contribute to the Kα fluorescence: the surface of the normal star, the accretion disk, the plasma of the stellar wind, the gas streams, etc. The results of calculations of Kα emission from the surface of the normal companion are presented below; for other regions, only order-of-magnitude estimates are given. The normal star is assumed to be a perfect sphere, and the compact X-ray source is supposed to radiate isotropically.

To find the flux toward the observer in the Kα line, one should integrate $J_\epsilon(0, \mu)$ over the spectrum of the X-ray source and over that fraction of the visible part of the stellar surface which is illuminated by the X-ray source. Having divided the result by the spectral flux in the X-ray continuum, one obtains the equivalent width of the Kα line:

$$W(\Theta) = \frac{\mu^{*2}}{1+\beta}\int_{\epsilon_{th}}^{\infty}\frac{L(\epsilon)}{L(\epsilon_K)}\,d\epsilon\,\frac{2}{\pi}\int_{\mu^*}^{\mu_1}d\mu'\int_0^{\varphi_0}\frac{\mu\mu_0\psi(\epsilon, \mu_0, \mu)}{1+\mu^{*2}-2\mu^*\mu'}\,d\varphi\,, \tag{29}$$

where

$$\mu_0 = (\mu' - \mu^*)(1 + \mu^{*2} - 2\mu^*\mu')^{-1/2}\,,$$
$$\mu = \mu'\cos\Theta + (1 - \mu'^2)^{1/2}\sin\Theta\cos\varphi\,; \tag{30}$$

$$\mu_1 = 1,\quad 0 \le \Theta \le \pi/2\,,$$
$$= \sin\Theta\,,\quad \pi/2 < \Theta < \pi - \arcsin\mu^*\,; \tag{31}$$

$$\varphi_0 = \pi\,, \quad \sin\Theta \leq \mu'\,,$$
$$= \pi/2 + \arctan\left[\mu' \cos\Theta(\sin^2\Theta - \mu'^2)^{-1/2}\right]\,, \quad \mu' < \sin\Theta\,. \tag{32}$$

Here 2 arcsin μ^* is the X-ray eclipse angle in the orbital plane; Θ is the angle between two rays emerging from the center of the normal star, one pointing to the X-ray source, and the other to the observer;

$$\int_0^\infty L(\epsilon)d\epsilon$$

is the total luminosity of the X-ray source. The quantity $\beta \ll 1$ accounts for the reflected X-ray continuum with photon energy $\epsilon = \epsilon_K$; its values typically do not exceed a few percent.

From (29) one easily finds that the equivalent width of Kα line is determined by three basic quantities: (i) the spectrum of the X-ray source $L(\epsilon)$, (ii) the chemical composition of the atmosphere of the normal star, and (iii) the solid angle $\Omega^* = 2\pi[1 - (1 - \mu^{*2})^{1/2}]$ subtended by the normal star from the X-ray source. As for the dependence on Ω^*, it is quite simple and in complete agreement with the results of Hatchett and Weaver (1977): the deviations from the direct proportionality of the equivalent widths $W^{(0)}$ and $W^{(1)}$ to Ω^* do not exceed $\pm 5\%$ in the range $0.3 < \mu^* < 1$. The dependence of $W(\Theta)$ on the abundance of heavy elements is discussed in the next section. The steeper the drop of the spectrum above the threshold $\epsilon > \epsilon_{th}$, the less is the value of the Kα equivalent width $W(\Theta)$. If $W(\Theta)$ is known for one X-ray spectrum, it can be approximately renormalized to any other with the aid of the following proportionality law:

$$W(\Theta) \propto \int_{\epsilon_{th}}^\infty \frac{L(\epsilon)}{L(\epsilon_K)} \frac{d\epsilon}{\epsilon^3}\,. \tag{33}$$

The typical error of this procedure is $\sim 5\%$.

In Table 2 and Figure 4 the values of $W^{(0)}(\Theta)$ and of $W^{(1)}(\Theta)$ are given for the iron Kα line in the HZ Her/Her X-1 system [$\mu^* = 0.429$, $Y = Y_{Fe} = 1$, $L(\epsilon) =$ const. for 6.4 keV $< \epsilon <$ 25 keV (Giacconi *et al.* 1973; Holt *et al.* 1974)]. Note that the eclipse in the Kα line is much broader than that in the primary X-rays. From Table 2 one can calculate the Kα light curve for any value of the inclination angle other than 90°, making use of the geometrical relationship between the angle Θ and the orbital phase ϕ:

$$\cos\Theta = -\sin i \cos 2\pi\phi\,. \tag{34}$$

In Table 3 the values of $W^{(0)}(0)$ and $W^{(1)}(0)$ are listed for Kα lines of nickel ($\epsilon_K = 7.47$ keV), argon ($\epsilon_K = 2.957$ keV), and sulfur ($\epsilon_K = 2.308$ keV). The abundance of nickel adopted is $N_{Ni} = 3.2 \times 10^{-6} N_H$ (Allen 1955); the cross section of its photoionization is taken from McGuire (1968). The abundances of argon and sulfur and their photoionization cross sections are those used by Brown and Gould (1970).

To make a comparison with the calculations of Hatchett and Weaver (1977), the equivalent width of the iron Kα line was evaluated for $\mu^* = 0.5$, $Y = Y_{Fe} = 1$ and $L(\epsilon) = \exp(-\epsilon/20\text{ keV})$, the result being $W^{(0)}(0) + W^{(1)}(0) = 20$ eV. Even if we increase this value by some 10–15% (the correction for the high-order scatters), it will be still 1.5 times less than the value $W = 33$ eV obtained by Hatchett and Weaver for the same values of all parameters. The difference should originate mainly from the double use by the latter authors of the diffusion approximation.

All the basic conclusions of § II*d* on the profile of the Kα line hold also for the Kα emission from the binary system, since the typical macroscopic velocities in close X-ray binaries are $\sim$100–300 km s^{-1}, and they broaden the line by no more than the order of magnitude of its natural width.

TABLE 2

Equivalent Widths of Unscattered and Singly Scattered Iron Kα Lines in the Hercules X-1 System

θ (degrees)	$W^{(0)}$ [eV]	$W^{(1)}$ [eV]
0	14.3	3.45
18	13.8	3.30
36	12.1	2.85
54	9.38	2.14
72	6.24	1.36
90	3.44	0.70
108	1.47	0.27
126	0.41	0.06

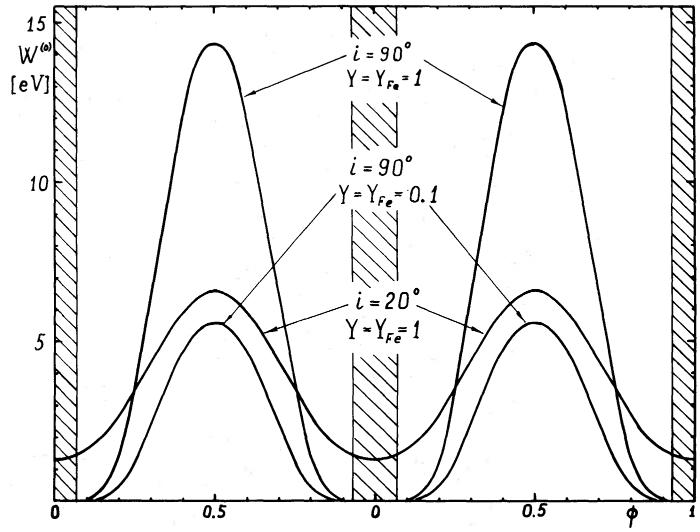

FIG. 4.—The equivalent width of the iron Kα core reflected from the surface of HZ Her as a function of the binary phase ϕ for two values of the inclination angle i and heavy element abundances Y, $Y_{\rm Fe}$. Hatched is the eclipse duration in primary X-rays.

If the primary X-rays illuminating the normal star show regular pulsations with a period p, similar pulsations should be observed in Kα lines too. If the primary flux is deeply modulated and the period p is not too large ($\Lambda = R/cp > 1$, where R is the radius of the normal star and c is the speed of light), the amplitude of regular pulsations in the Kα line can be evaluated with the method developed by Basko, Sunyaev, and Titarchuk (1974) for the reflected X-ray continuum. But one should keep in mind that the definitions of the amplitude of pulsations encountered in the literature sometimes differ. If one defines it as the ratio of the difference between the maximum and minimum flux values to its average, then the pulsation amplitude of that part of Kα emission which is reflected from the surface of the normal star in HZ Her/Her X-1 system ($\Lambda \approx 7$) should be ~7–15%. But if one thinks of it as the ratio of the coefficient of the first Fourier harmonic to the average signal level, in the same example it should be ~2–5%. In other X-ray pulsars the amplitude of pulsations in Kα lines can be estimated using the fact that it is inversely proportional to Λ at $\Lambda > 1$ (Basko, Sunyaev, and Titarchuk 1974) and coincides with the amplitude of the primary flux pulsations at $\Lambda \ll 1$.

Besides the surface of the normal star, a relatively cool atmosphere can be present at the outer parts of an accretion disk around the X-ray source. According to theoretical calculations, the disk must have a concave (saucer-like) shape (Shakura and Sunyaev 1973), with outer parts intercepting some fraction of the X-ray flux. Observational information on the disk parameters in X-ray binaries is rather scarce, and I will give only an order-of-magnitude estimate of its Kα emission based on the phenomenological model of Gerend and Boynton (1976). Since the observer can see only one side of the disk at a time, the solid angle subtended by this part of its surface from the X-ray source is

$$\Delta\Omega \approx 2\pi \sin\left[(\Delta\theta_1 + \Delta\theta_2)/4\right] \approx 2\pi \sin 14^\circ . \quad (35)$$

Assuming that the Kα radiation is emitted isotropically, one arrives at the following estimate of the Kα equivalent width:

$$W_d \approx \frac{\Delta\Omega}{2\pi} \int_{\epsilon_{\rm th}}^{\infty} \frac{L(\epsilon)}{L(\epsilon_{\rm K})}\, \alpha(\epsilon, \mu_0) d\epsilon \approx \frac{\Delta\Omega}{2\pi}\, \alpha(\epsilon_{\rm th}, \mu_0) \int_{\epsilon_{\rm th}}^{\infty} \frac{L(\epsilon)}{L(\epsilon_{\rm K})} \left(\frac{\epsilon_{\rm th}}{\epsilon}\right)^3 d\epsilon . \quad (36)$$

Here $\alpha(\epsilon_{\rm th}, \mu_0)$ is the plane albedo in the Kα line at the threshold of K-shell ionization (see eq. [21] and Table 1). Taking $\mu_0 = 0.2$, $L(\epsilon) = $ const. up to $\epsilon = 25$ keV and assuming normal chemical abundances, we obtain for the

TABLE 3

EQUIVALENT WIDTHS OF Kα LINES OF NICKEL, ARGON, AND SULFUR IN THE HERCULES X-1 SYSTEM

Element	$W^{(0)}(0)$ [eV]	$W^{(1)}(0)$ [eV]
Ni...........	2.55	0.54
Ar...........	0.29	0.012
S............	0.44	0.010

iron Kα line $W_d^{(0)} \approx 30$ eV and $W_d^{(1)} \approx 5$ eV. The relative contribution of the disk as compared to the numbers in Table 2 may be somewhat underestimated due to the limb brightening effect, or to smaller values of the inclination angle. Thus, *the disk surface can be* 2–3 *times more luminous in the* Kα *line than the surface of the normal star.*

Iron ions with a filled L shell may also be present in a tenuous outflowing corona with temperature $T \sim 10^6$ K, surrounding the binary system. The existence of such a corona is evidenced by theoretical calculations (Basko and Sunyaev 1973; Milgrom and Salpeter 1975) and by the X-ray observations of Her X-1 in the OFF state of its 35 day cycle (Becker *et al.* 1977). Since the removal of M and N electrons only slightly changes the energy of the Kα transition (House 1969), the coronal iron ions should be also taken into account when estimating Kα emission at $\epsilon = 6.40$ keV. Under the assumption of spherical symmetry,

$$W_{\rm cor} \lesssim \tau_{T,\rm cor}\omega_{\rm K} \int_{\epsilon_{\rm th}}^{\infty} \frac{L(\epsilon)}{L(\epsilon_{\rm K})} \frac{\epsilon_{\rm K}}{\epsilon} \frac{\sigma_{\rm ph,K}(\epsilon)}{\sigma_{\rm T}}\, d\epsilon\,. \quad (37)$$

With the scattering optical depth of the corona $\tau_{T,\rm cor} = 0.05$ (Becker *et al.* 1977) one obtains $W_{\rm cor} \lesssim 43$ eV for Her X-1. Thus, the estimates for Her X-1 show that the relative contribution of the surface of the normal star in Kα emission may not be large, but that does not mean that it is worthless trying to single it out. The point is that Kα emission from the disk and corona, and Kα emission from the normal star, behave quite differently with the orbital phase ϕ. The contribution of the normal star to the equivalent width rapidly drops as the orbital phase shifts away from the moment $\phi = 0.5$ (see Fig. 4), and practically vanishes at $\phi = 0.2$ and $\phi = 0.8$, while the contribution of the disk and corona must be almost constant within the interval $0.15 \lesssim \phi \lesssim 0.85$, decrease a little before the X-ray eclipse ($\phi \geq 0.07$ and $\phi \leq 0.93$), rise sharply just after the X-ray eclipse ($\phi < 0.07$ and $\phi > 0.93$) due to the drop in X-ray continuum, and then decrease toward the moment $\phi = 0$. In order to calculate this behavior numerically, one should adopt a detailed physical model of the disk and of the corona.

IV. NEW INFORMATION THAT CAN BE OBTAINED FROM Kα OBSERVATIONS

Let us first estimate whether it is realistic to expect detection of Kα lines of iron in the spectra of the known X-ray binaries with currently feasible instrumentation. For this the optimum spectral range of observations must be outlined. Let $\mathscr{L}_l^{(0)}$ [ergs s^{-1} sr^{-1}] be the energy flux per unit solid angle in the Kα line core; $\epsilon_0 \approx 1.7$ eV, the natural half-width of the line; D, the distance of the X-ray source; and S, the effective surface area of the X-ray counter. Then the total number of line photons N_l detected by the counter within the time interval t in the spectral range $\epsilon_{\rm K} - \Delta\epsilon < \epsilon < \epsilon_{\rm K} + \Delta\epsilon$ ($\Delta\epsilon \ll \epsilon_{\rm K}$) for the Lorentz line profile is

$$N_l = \frac{St}{D^2} \frac{\mathscr{L}_l^{(0)}}{\pi\epsilon_0\epsilon_{\rm K}} \int_{-\Delta\epsilon}^{+\Delta\epsilon} \frac{d\epsilon}{1 + (\epsilon/\epsilon_0)^2}\,. \quad (38)$$

The number of continuum photons detected in the same time and spectral intervals is

$$N_c = \frac{St}{D^2} \frac{\mathscr{L}_c(\epsilon_{\rm K}) 2\Delta\epsilon}{\epsilon_{\rm K}}\,, \quad (39)$$

where $\mathscr{L}_c(\epsilon)$ [ergs s^{-1} sr^{-1} erg^{-1}] is the spectral flux per unit solid angle in the X-ray continuum. To determine the time of integration necessary to detect the line, the following condition should be imposed on the statistical error:

$$(N_l + N_c)^{1/2} \lesssim \frac{1}{n} N_l\,, \quad (40)$$

which means that we want the line to be detected at the level $n\sigma$; now we have

$$t \gtrsim n^2 \frac{D^2}{S\mathscr{L}_c(\epsilon_{\rm K})} \frac{\epsilon_{\rm K}}{W^{(0)}} \frac{\pi}{2} \left[\frac{\pi\epsilon_0}{W^{(0)}} \frac{\Delta\epsilon}{\epsilon_0} + \arctan\frac{\Delta\epsilon}{\epsilon_0}\right] \arctan^{-2}\frac{\Delta\epsilon}{\epsilon_0}\,. \quad (41)$$

The function

$$f(\beta, x) = (\beta x + \arctan x) \arctan^{-2} x$$

has a minimum at $x = x^*(\beta)$. The typical values of $\beta = \pi\epsilon_0/W^{(0)}$ for Her X-1 are $\beta \approx 0.1$–0.3, and we have $x^*(0.1) = 3.85$, $x^*(0.3) = 2.57$; $f(0.1, 3.85) = 0.98$, $f(0.3, 2.57) = 1.37$. Thus, to have a minimum integration time, one should observe in the narrow spectral interval centered on $\epsilon_{\rm K}$ and $\sim(6$–$8)\epsilon_0$ wide. Since the two components of the iron Kα line are only 13 eV $\approx 8\epsilon_0$ apart, the optimum spectral interval must apparently include both of them and be equal to $2\Delta\epsilon \approx 30$ eV. Substituting $n = 3$, $S = 100$ cm^2, $2\Delta\epsilon = 30$ eV, $D = 5$ kpc, $\mathscr{L}_c(\epsilon_{\rm K}) = 5 \times 10^{43}$ ergs s^{-1} sr^{-1} erg^{-1}, $W^{(0)} = 14$ eV into (41), we find that the integration time in the case of Her X-1 should be $t \approx 600$ s. The analogous estimate for $W^{(1)}$ gives $t \sim 10^5$ s. Thus, *the most promising from an* *observational point of view is the narrow core $W^{(0)}$ of the iron* Kα *line which can be detected in bright X-ray binaries with currently feasible instrumentation.*

Turn now to the new information that can be gained from the observations of the narrow Kα line core reflected by the surface of the normal star.

i) First of all, the observations of Kα lines will enable us to establish the binary nature of many compact galactic X-ray sources that show no other evidence of being binary. There should be plenty of binary sources that do not eclipse because of small inclination angles, $i \leqslant 60°$. The regular component in the X-ray light variation due to the orbital motion is in such cases so small that it is completely indistinguishable against the random flux fluctuations. In this case the equivalent width of Kα line has a great advantage: it does not depend on the absolute value of the X-ray flux and clearly shows periodic variations with the binary period even at small inclinations, $i \leqslant 10°$. Note, however, that it is worthless to observe the line with time resolution better than R/c, since all the fluctuations with smaller time scales smooth down by the reflection from the normal star. The observations of the Kα line could, for instance, reconfirm the optical identification of Cyg X-1 and the binary nature of Sco X-1, and reveal the binary period of Cyg X-2.

ii) The relative amplitude of regular variations in the Kα equivalent width over the orbital phase ϕ is rather insensitive to the chemical composition and to the primary X-ray spectrum, and is determined by two parameters, i and μ^*, the latter depending on the mass ratio of binary components and the degree of Roche-lobe filling by the normal star. Having determined this amplitude from the observations, one obtains an extra equation relating the parameters of the binary system. This relationship is especially useful at small inclination angles, when other methods fail to give a reliable estimate of i.

iii) When i, μ^*, and the X-ray spectrum are known, the absolute amplitude of regular variations of $W^{(0)}$ enables one to estimate approximately the chemical composition of the normal star. If one measures just $W^{(0)}$, this gives one equation for two parameters Y and $Y_{\rm Fe}$. To find both, one has to measure $W^{(1)}$ too. Note that $W^{(0)}$ depends mainly on the relative abundance of the fluorescing element $Y_{\rm Fe}/Y$ and slowly changes if $Y_{\rm Fe}$ is proportional to Y; see Figure 2. I want to emphasize again that to obtain the above information one must separate the contribution to Kα emission by the normal star from that of other regions, which may not be easily accomplished.

Besides general information, the observations of Kα lines will help us to solve a number of specific problems concerning some particular X-ray sources.

a) *Hercules X-1*

The numerical values of the equivalent width of the iron Kα line for HZ Her/Her X-1 system were given in § III. Note that Kα emission from the surface of HZ Her may be somewhat less than according to Table 2 due to the shadow cast by the disk. Present calculations convincingly demonstrate that the X-ray line $\epsilon = 6.4 \pm 0.5$ keV discovered from the *OSO 8* satellite (Pravdo 1976; Pravdo *et al.* 1977) is not the iron Kα line reflected from the surface of HZ Her because (i) the typical equivalent width of the latter must be $\sim$20 eV, and (ii) it must show a clear correlation with the orbital phase (see Fig. 4), which is not observed. The equivalent width of the observed line is $\sim$100–400 eV, which exceeds even the contribution of the disk and corona estimated above. The major portion of this emission may originate from the vicinity of the neutron star as a complex of emission lines of highly ionized iron ions (Pravdo *et al.* 1977; Ross, Weaver, and McCray 1978). Note that if one wants to locate the region of line emission, observations near X-ray eclipse are most important.

The Kα line observations may provide a severe test for 35 day cycle models of Her X-1. For example, in the framework of the model proposed by Gerend and Boynton (1976) the Kα flux from the disk surface must be strictly correlated with the phase of the 35 day cycle. This correlation is easy to calculate once the disk geometry is fixed. The absolute Kα flux in this model does not change significantly from the ON to the OFF state, and the equivalent width of the narrow core of the iron Kα line in the OFF state should amount to a few hundred eV. In the model of Bisnovatyi-Kogan and Komberg (1975), on the other hand, the absolute Kα flux must achieve maximum in the ON state, when the X-ray source is buried within the dense gas stream, and drop significantly in the OFF state, when the stream occupies but a small solid angle from Her X-1.

b) *Scorpius X-1*

Sco X-1 is the brightest X-ray source and the easiest—one would think—from which to discover Kα emission. But in this case we have the two unfavorable circumstances of a comparatively soft X-ray spectrum and a rather small inclination angle (there are no X-ray eclipses). Since almost nothing is known of the parameters of the system, it is difficult to give a reliable estimate of the iron Kα equivalent width. If, for instance, $i = 0°$, $\mu^* = 0.429$, $Y = Y_{\rm Fe} = 1$, and $L(\epsilon) = \exp(-\epsilon/5 \text{ keV})$ (Laros and Singer 1976), then $W^{(0)} = 1.7$ eV, which could have been already detected with an experiment analogous to that of Stockman *et al.* (1973), but tuned to $\epsilon_{\rm K} = 6.40$ keV instead of $\epsilon_{\rm K} = 6.70 \pm 0.04$ keV. But surely $i \neq 0$, and for $i = 20°$ we have $W^{(0)} = 3.2$ eV at the orbital phase $\phi = 0.5$ corresponding to the maximum in the regular component of optical light variations (Wright, Gottlieb, and Liller 1975). On the other hand, the value of μ^* is completely unknown; and if we take $\mu^* = 0.3$, which corresponds to $M_x/M_{\rm opt} = 2$ with the normal star filling its Roche lobe, then $W^{(0)} = 0.9$ eV for $i = 0$.

c) *Centaurus X-3 and 3U 1700−37*

For these X-ray sources one would expect rather large values of $W^{(0)}$ because of large inclination and eclipse angles. Assuming $\mu^* = 0.64$ and $L(\epsilon) = \epsilon \exp(-\epsilon/5.6 \text{ keV})$ (Swank *et al.* 1976) for Cen X-3, and $\mu^* = 0.73$ (Avni and Bahcall 1976) and $L(\epsilon) = \text{const.}$ up to $\epsilon = 25$ keV for 3U 1700−37, we obtain $W^{(0)} = 27$ eV and $W^{(0)} = 49$ eV, respectively. The fact that the normal components of these X-ray sources are young giant stars with high abundances of heavy elements makes the above estimates more reliable.

d) *Cygnus X-1*

The normal companion of this X-ray source should also have a chemical composition close to the normal one. Taking $i = 30°$, $\mu^* = 0.44$ (for the optical star filling its Roche lobe and having a mass $M_{\text{opt}} = 2M_x$) and $L(\epsilon) = \epsilon^{-0.55}$ (Rothschild *et al.* 1976), we obtain $W^{(0)}(\phi = 0.5) = 7.1$ eV. Since Cyg X-1 is one of the brightest X-ray sources on the sky (~12 times brighter than Her X-1), it apparently will not take long before the iron Kα line is detected in its X-ray spectrum. Note that according to paragraph (ii) the Kα light curve will give us additional information on the system parameters, which may be of importance for the ultimate determination of whether Cyg X-1 is a black hole or not.

e) *Cygnus X-3*

A number of theoretical models have been proposed for this X-ray source, some of them better and some worse in accounting for its observational properties. The observations of the narrow core of the iron Kα line would be very useful when deciding among these models. It is impossible to estimate the values of $W^{(0)}$ that should be expected when parameters of the system are completely unknown. If, for instance, the model of Basko, Sunyaev, and Titarchuk (1974) is valid, one should expect rather large value of $W^{(0)}$—tens, or even hundreds, of eV (because of the diminution of the X-ray continuum by reflection)—that must be *constant* over the binary phase. In the model of Milgrom and Pines (1978) for $i = 90°$, $\mu^* = 0.95$, $L(\epsilon) = \epsilon^3 \exp(-\epsilon/2 \text{ keV})$ the contribution of the normal star to the iron Kα emission at $\phi = 0.5$ should be $W^{(0)} \approx 60$ eV. The observed equivalent width must be 2–3 times less because of Compton scattering in the cocoon with $\tau_T \sim 1$ (by scattering Kα photons escape from the narrow line core) and sharply peaked at the orbital phase $\phi = 0.5$. To evaluate the Kα emission of the cocoon itself, one should adopt its rather detailed physical model. Note that the cocoon Kα emission can be separated from that of the normal star by its weak dependence on ϕ.

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APPENDIX

The solution of the radiative transfer equation (4) in the case of diffusive reflection of the primary flux I_0 from the plane-parallel atmosphere is

$$I(0, \mu) = \frac{1}{\mu} \int_0^\infty e^{-\tau/\mu} S(\tau) d\tau = \frac{\lambda}{4\pi} I_0 \mu_0 \frac{H(\mu_0, \lambda) H(\mu, \lambda)}{\mu_0 + \mu} \tag{A1}$$

(Chandrasekhar 1960). Here $S(\tau)$ is the source function (9), $H(\mu, \lambda)$ is the Ambartsumyan function (a particular case of the Chandrasekhar H-function for isotropic scattering) satisfying the integral equation

$$H(\mu, \lambda) = 1 + \frac{\lambda}{2} \mu H(\mu, \lambda) \int_0^1 \frac{H(\mu', \lambda)}{\mu + \mu'} d\mu' , \tag{A2}$$

from which one can, among other things, deduce the following identity:

$$\int_0^1 H(\mu, \lambda) d\mu = 2\lambda^{-1}[1 - (1 - \lambda)^{1/2}] . \tag{A3}$$

The function $H(\mu, \lambda)$ has explicit integral representations (Ivanov 1969). One can estimate it from the approximate expression

$$H(\mu, \lambda) \approx \frac{1 + 3^{1/2}\mu}{1 + [3(1 - \lambda)]^{1/2}\mu} . \tag{A4}$$

The error of (A4) does not exceed 8% in the region $[0 \leq \lambda \leq 1; 0 \leq \mu < \infty]$, and this formula gives correct asymptotic behavior at $\mu \to \infty$. In this paper the function $H(\mu, \lambda)$ has been evaluated from

$$H(\mu, \lambda) = \left\{(1-\lambda)^{1/2} + \frac{\lambda}{2}\int_0^1 \frac{\mu'}{\mu+\mu'} g(\mu', \lambda) d\mu'\right\}^{-1}, \tag{A5}$$

where

$$g(\mu, \lambda) = \frac{1+3^{1/2}\mu}{1+[3(1-\lambda)]^{1/2}\lambda}\left\{1 - \frac{\lambda}{4}(1+\lambda^3)\mu[\ln\mu + 1.33 - 1.458\mu^{0.62}]\right\}. \tag{A6}$$

Note that the substitution of $H(\mu', \lambda)$ instead of $g(\mu', \lambda)$ transforms (A5) into an integral equation for $H(\mu, \lambda)$ which is equivalent to (A2).

The values of $g(\mu, \lambda)$ calculated from (A6) approximate $H(\mu, \lambda)$ in the square $[0 \leq \lambda \leq 1; 0 \leq \mu \leq 1]$ with an error $\lesssim 0.8\%$, while the error of the formula (A5) does not exceed 0.2% in the whole band $[0 \leq \lambda \leq 1, 0 \leq \mu < \infty]$.

The solution of equation (7) can be written in the form

$$J_\epsilon(0, \mu) = \frac{1}{\mu}\int_0^\infty e^{-t/\mu} R(t) dt, \tag{A7}$$

where the function

$$R(t) = \frac{\lambda_K}{2}\int_{-1}^{+1} J_\epsilon(t, \mu') d\mu' + \frac{\delta\lambda_K}{\lambda}\frac{\epsilon_K}{\epsilon} S(t\lambda_K/\lambda) \tag{A8}$$

satisfies the integral equation

$$R(t) = \frac{\lambda_K}{2}\int_0^\infty E_1(|t-t'|) R(t') dt' + \frac{\delta\lambda_K}{\lambda}\frac{\epsilon_K}{\epsilon} S(t\lambda_K/\lambda). \tag{A9}$$

Here

$$E_1(x) = \int_1^\infty t^{-1} e^{-tx} dt.$$

Now expand $R(t)$ into a sum of terms, each one corresponding to a fixed number of scatters of Kα photons:

$$R(t) = R^{(0)}(t) + R^{(1)}(t) + \cdots, \tag{A10}$$

$$R^{(0)}(t) = \frac{\delta\lambda_K}{\lambda}\frac{\epsilon_K}{\epsilon} S(t\lambda_K/\lambda), \tag{A11}$$

$$R^{(1)}(t) = \frac{\lambda_K}{2}\int_0^\infty E_1(|t-t'|) R^{(0)}(t') dt'. \tag{A12}$$

Substituting (A11) and (A12) into (A7) and making use of (A1), one readily arrives at (13) and (14). When numerically evaluating $\psi^{(1)}(\epsilon, \mu_0, \mu)$ according to (14), the approximation (A4) was used in the integrand, the result reading

$$\begin{aligned}\psi^{(1)}(\epsilon, \mu_0, \mu) = {} & \frac{\delta\lambda_K^2}{8}\frac{\epsilon_K}{\epsilon} H(\mu_0, \lambda) \\ & \times \left[\!\left[\mu \frac{H(\mu\lambda_K/\lambda, \lambda)}{\mu_0 + \mu\lambda_K/\lambda}\ln(1+1/\mu) + \frac{\mu_0\lambda/\lambda_K}{\mu_0+\mu\lambda_K/\lambda}\frac{1-3^{1/2}\mu_0}{1-[3(1-\lambda)]^{1/2}\mu_0}\ln\left(1+\frac{\lambda_K}{\mu_0\lambda}\right)\right.\right. \\ & \left.\left. + \frac{\lambda/\lambda_K(1-\lambda)^{1/2}}{1+\mu\lambda_K[3(1-\lambda)]^{1/2}/\lambda}\frac{1-(1-\lambda)^{1/2}}{1-\mu_0[3(1-\lambda)]^{1/2}}\ln\left\{1+\frac{\lambda_K}{\lambda}[3(1-\lambda)]^{1/2}\right\}\right]\!\right].\end{aligned} \tag{A13}$$

The typical error introduced by such an approximation into the values of albedo $\alpha^{(1)}$ and equivalent width $W^{(1)}$ is ~1–2%.

REFERENCES

Allen, C. W. 1955, *Astrophysical Quantities* (London: Academic Press).

Avni, Y., and Bahcall, J. 1976, *X-Ray Binaries* (NASA SP-389), p. 615.

Bambynek, W., Craseman, B., Fink, R. W., Freund, H. U., Mark, H., Swift, C. D., Price, R. E., and Rao, P. V. 1972, *Rev. Mod. Phys.*, **44**, 716.

Basko, M. M., and Sunyaev, R. A. 1973, *Ap. Space Sci.*, **23**, 117.

Basko, M. M., Sunyaev, R. A., and Titarchuk, L. G. 1974, *Astr. Ap.*, **31**, 249.

Becker, R. H., Boldt, E. A., Holt, S. S., Pravdo, S. H., Rothschild, R. E., Serlemitsos, P. J., and Swank, J. H. 1976, *Ap. J.* (*Letters*), **209**, L65.

Becker, R. H., Boldt, E. A., Holt, S. S., Pravdo, S. H., Rothschild, R. E., Serlemitsos, P. J., Smith, B. W., and Swank, J. H. 1977, *Ap. J.*, **214**, 879.
Bisnovatyi-Kogan, G. S., and Komberg, B. V. 1975, *Astr. Zh.*, **52**, 457 (Eng. transl. *Soviet Astr.—AJ*, **18**, 217).
Brown, R. L., and Gould, R. J. 1970, *Phys. Rev. D.*, **1**, 2252.
Chandrasekhar, S. 1960, *Radiative Transfer* (New York: Dover).
Davison, P. J. N., Culhane, J. L., and Mitchell, R. J. 1976, *Ap. J. (Letters)*, **206**, L37.
Felsteiner, J., and Opher, R. 1976, *Astr. Ap.*, **46**, 189.
Gerend, D., and Boynton, P. E. 1976, *Ap. J.*, **209**, 562.
Giacconi, R., Gursky, H., Kellogg, E., Levinson, R., Schreier, E., and Tananbaum, H. 1973, *Ap. J. (Letters)*, **184**, L227.
Gorshkov, V. G., Michailov, A. I., and Sherman, S. G. 1973, *J. Exp. Theor. Phys.*, **64**, 1128 (Eng. transl. *Soviet Phys.—JETP*, **37**, 572).
Hatchett, S., and Weaver, R. 1977, *Ap. J.*, **215**, 285; erratum, **218**, 931.
Holt, S. S., Boldt, E. A., Rothschild, R. E., and Serlemitsos, P. J. 1974, *Ap. J. (Letters)*, **190**, L109.
House, L. L. 1969, *Ap. J. Suppl.*, **18**, 21.
Ivanov, V. V. 1969, *Radiative Transfer and Spectra of Celestial Bodies* (Moscow: Nauka; Eng. transl. 1973, *Transfer of Radiation in Spectral Lines* [NBS SP-385]).
Kikoin, I. K. (ed.) 1976, *Tables of Physical Quantities* (Moscow: Atomizdat).
Laros, J. G., and Singer, S. 1976, *Ap. J.*, **205**, 550.
McCray, R. A. 1977, *Comments Ap.*, **7**, 13.
McGuire, E. J. 1968, *Phys. Rev.*, **175**, 20.
Milgrom, M., and Pines, D. 1978, *Ap. J.*, **220**, 272.
Milgrom, M., and Salpeter, E. E. 1975, *Ap. J.*, **196**, 583.
Pravdo, S. H. 1976, Ph.D. thesis, University of Maryland.
Pravdo, S. H., Becker, R. H., Boldt, E. A., Holt, S. S., Serlemitsos, P. J., and Swank, J. H. 1977, *Ap. J. (Letters)*, **215**, L61.
Rakavy, G., and Ron, A. 1967, *Phys. Rev.*, **159**, 50.
Ross, R. R., Weaver, R., and McCray, R. 1978, *Ap. J.*, **219**, 292.
Rothschild, R. E., Boldt, E. A., Holt, S. S., and Serlemitsos, P. J. 1976, *X-Ray Binaries* (NASA SP-389), p. 443.
Sanford, P. W., Mason, K. O., and Ives, J. 1975, *M.N.R.A.S.*, **173**, 9P.
Serlemitsos, P. J., Boldt, E. A., Holt, S. S., Rothschild, R. E., and Saba, J. L. R. 1975, *Ap. J. (Letters)*, **201**, L9.
Serlemitsos, P. J., Smith, B. W., Boldt, E. A., Holt, S. S., and Swank, J. H. 1977, *Ap. J. (Letters)*, **211**, L63.
Shakura, N. I., and Sunyaev, R. A. 1973, *Astr. Ap.*, **24**, 337.
Stockman, H. S., Angel, J. R. R., Novick, R., and Woodgate, B. E. 1973, *Ap. J. (Letters)*, **183**, L63.
Swank, J. H., Becker, R., Boldt, E., Holt, S., Pravdo, S., Rothschild, R., and Serlemitsos, P. J. 1976, *X-Ray Binaries* (NASA SP-389), p. 207.
Wright, E. L., Gottlieb, E. W., and Liller, W. 1975, *Ap. J.*, **200**, 171.

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