RADIAL PULSATIONS OF A WHITE DWARF IN THE CASE OF INHOMOGENEOUS ROTATION

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The energy method is used to estimate the periods of radial pulsations of inhomogeneously rotating white dwarfs near the Chandrasekhar limit. Use is made of the results of [1] (without allowance for neutronization corrections) and [2], in which such corrections are made. It is shown that allowance for neutronization effects appreciably increases the period of radial pulsations, but nevertheless the pulsations periods of rotating stars may be shorter than those of nonrotating stars by a factor of 4-5.

In [1, 2], a simple energy method is used to estimate the periods of radial pulsations of rotating white dwarfs near the Chandrasekhar limit. In both cases it is assumed that the star rotates rigidly. In [2], unlike [1], corrections for matter neutronization as well as general relativity effects are taken into account. This results in an appreciable lengthening of the minimal pulsation periods.

In the present paper we use the energy method to estimate the periods of radial pulsations of white dwarfs when differential rotation is present.

The mass of a star in hydrostatic equilibrium is determined from the equation [3]

$$\left(\frac{\partial E}{\partial \rho_c^{\frac{1}{3}}}\right)_{M,\,K} = 0, \tag{1}$$

where E is the total energy of the star, ρ_c is the density at the center, M is the mass, and K is the total angular momentum of the star. As an unperturbed configuration we take a polytropic sphere with index n = 3, and we allow for the effects of general relativity, neutralization, and rotation as small corrections to the energy of this configuration.

We also take into account the first two corrections to the equation of state of a degenerate ultra-relativistic electron gas. The frequency of the basic mode of radial pulsations is determined from [3];

$$\omega^2 = \left(\frac{\partial^2 E}{\partial (\rho_c^{\frac{1}{3}})^2}\right)_{M,\,K} \Big/ \rho_c^{-\frac{2}{3}}\, 4\pi \int_0^R \rho r^4 dr. \tag{2}$$

Let us consider the correction to the total energy due to the rotation of the star. The expression for the angular velocity of rotation can be written on the basis of dimensional arguments in the form

$$\Omega^2 = G\rho_c \varepsilon^2 \overline{\Omega}^2 (\xi,\ \sin \vartheta) \tag{3}$$

where ε is a dimensionless parameter characterizing the degree of rotation, $\xi = \xi_1(r/R)$ is a dimensionless coordinate, R is the radius of the star, $\xi_1 = 6.897$ for n = 3, ϑ is the polar angle, and $\overline{\Omega}$ is a dimensionless function.

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TABLE 1. Critical Parameters of Inhomogeneously Rotating White Dwarfs

γ	0	0.0033	0.013	0.03	0.053	0.083	0.12
$(M/M_{\odot})_1$	1.21	1.22	1.23	1.25	1.26	1.27	1.29
$(M/M_{\odot})_2$	1.01	1.02	1.04	1.05	1.06	1.03	1.00
$(\rho_c)_1 \cdot 10^{-10}$ g/cm³	3.0	5.6	37	2.9×10^2	1.41×10^3	4.55×10^3	1.14×10^4
$(\rho_c)_2 \cdot 10^{-10}$ g/cm³	0.33	0.37	0.53	1.03	2.7	7.9	22

TABLE 2. Minimal Periods of Radial Pulsations

γ	0	0.0033	0.013	0.03	0.053	0.083	0.12
$T_{1\min}$, sec	1.75	1.5	0.66	0.18	0.064	0.028	0.015
$T_{2\min}$, sec	2.44	2.4	2.17	1.75	1.12	0.60	0.33

Further,

$$K = 2\pi \int_0^R \int_0^\pi \Omega \rho r^4 \sin^3 \vartheta \, dr d\vartheta = \varepsilon G^{\frac{1}{2}} \rho_c^{-\frac{1}{6}} M^{\frac{5}{3}} k_1, \tag{4}$$

$$E_{\mathrm{rot}} = \pi \int_0^R \int_0^\pi \Omega^2 \rho r^4 \sin^3 \vartheta \, dr d\vartheta = \varepsilon^2 G \rho_c^{\frac{1}{3}} M^{\frac{5}{3}} k_2, \tag{5}$$

$$k_1 = (4\pi \cdot 2.018)^{-\frac{5}{3}} 2\pi \int_0^{\xi_1} \int_0^\pi \overline{\Omega} (\xi, \sin\vartheta)\, \theta^3(\xi) \xi^4 \sin^3 \vartheta \, d\xi d\vartheta, \tag{6}$$

$$k_2 = (4\pi \cdot 2.018)^{-\frac{5}{3}} \pi \int_0^{\xi_1} \int_0^\pi \overline{\Omega}^2 (\xi, \sin\vartheta)\, \theta^3(\xi) \xi^4 \sin^3 \vartheta \, d\xi d\vartheta. \tag{7}$$

Here $\theta(\xi)$ is the Lane–Emden function for n = 3. We denote the ratio of the rotational to the gravitational energy by

$$\left| \frac{E_{\mathrm{rot}}}{E_{\mathrm{gr}}} \right| = \varepsilon^2 \frac{k_2}{0.639} = \gamma. \tag{8}$$

We then readily obtain

$$\left(\frac{\partial E_{\mathrm{rot}}}{\partial \rho_c^{\frac{1}{3}}} \right)_{M,K} = 1.278 \gamma G M^{\frac{5}{3}}, \tag{9}$$

$$\left(\frac{\partial^2 E_{\mathrm{rot}}}{\partial (\rho_c^{\frac{1}{3}})^2} \right)_{M,K} = 1.278 \gamma G M^{\frac{5}{3}} \rho_c^{-\frac{1}{3}}. \tag{10}$$

It can be seen that the terms in (1) and (2) that take into account the rotation differ from the corresponding terms in [1, 2] only by their coefficients. As a parameter characterizing the rotation it is convenient to take $\gamma = |E_{\mathrm{rot}}/E_{\mathrm{gr}}|$, since this enables one to estimate the frequencies of radial pulsations without particularizing the law of differential rotation. For rigid-body rotation, $\gamma_{\max} = 2.52 \cdot 10^{-2}$ [1]. For differential rotation, γ can be several times larger [4]. The results of calculations are given in Tables 1 and 2. In the calculations we assumed $\mu_e = 56/26$. The subscript 1 is appended to the white-dwarf parameters calculated in accordance with the formulas of [1], i.e., without allowance for neutronization, and the subscript 2 is appended to the parameters calculated from the formulas of [2] (with allowance for neutronization).

Note that allowance for neutronization appreciably decreases the critical densities and increases the pulsation periods. When $\gamma = 0.1$ it is already necessary to take into account the neutron pressure.

In [4] it is shown that inhomogeneously rotating white dwarfs with large γ can exist, although the oblateness of the stars then becomes appreciable. Since we allow for rotation as a small correction to the spherical configuration, our results become more and more unreliable with increasing γ. However, our estimates suggest that differential rotation could reduce the minimal periods of pulsations by a factor of 4-5 compared with those of nonrotating configurations.

Since the energy method overestimates the frequency, we can conclude that it is hardly possible for rotating white dwarfs to exist with periods of radial pulsations less than a few tenths of a second.

When choosing the actual distribution of the angular velocities over the star, it is necessary to remember that the requirement of stability imposes two necessary conditions [3, 5]: a) Ω must depend only on $s = r \sin \vartheta$; b) $d(\Omega s^2)/ds > 0$.

The existence of an ionic crystal lattice [6, 7] in the case of electron degeneracy means that the white dwarf could have a dense nucleus rotating rapidly with constant angular velocity, outside which Ω decreases toward the periphery. But with all these restrictions one can choose a distribution of angular velocities for which γ in any case is of order 0.1.

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