COLLAPSE OF LOW-MASS STARS

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Calculations are made of the loss of stability and the initial phase in the collapse of the iron cores of stars with masses $1.19M_\odot$ and $1.21M_\odot$ resulting from nonequilibrium β processes. Detailed allowance is made for the kinetics of the β processes and electron capture to excited states of the daughter nuclei. It is shown that in the collapse of stars with low mass near the Chandrasekhar limit electron capture to excited states plays an important part in the neutronization of matter, which begins at densities $\rho_c \gtrsim 10^{11}$ g/cm^3.

Introduction

Evolution calculations made for stars with masses on the main sequence in the range $64M_\odot \gtrsim M_{st} \gtrsim 4M_\odot$ [1-3] show that at the end of their evolution cores are formed consisting of elements of the iron group, the cores having masses $3M_\odot \gtrsim M_{cor} \gtrsim 1M_\odot$. The further evolution of such cores takes place almost independently of the existence of the shell of the star. If the resulting core of the star with a definite chemical composition has a mass slightly exceeding the Chandrasekhar limit corresponding to this chemical composition, such cores, contracting slowly, lose mechanical stability because of neutronization of the matter [4] (or, simply because of cooling [5]), whereas the loss of stability in the cores of stars with large masses $M_{cor} \gtrsim 5M_\odot$ results from processes of photodissociation of the iron nuclei into α particles and neutrons [6].

In cores with masses near the Chandrasekhar limit, the central densities in a state of equilibrium reach values of $\rho_c \gtrsim 10^9$ g/cm^3, at which the capture of excited electrons by the nuclei becomes possible. The process of electron capture has much longer characteristic times than the photonuclear reactions and hydrodynamic processes, and determines the characteristic time of loss of mechanical stability by stellar cores with low masses [7, 8].

Taking place independently, the electron captures, and also other β processes, have an influence on the thermodynamic characteristics of the matter, changing the entropy and the temperature, and also on the chemical composition, shifting it to a state in which the nuclei of chemical elements that are overenriched with neutrons predominate.

In the contraction process, the energy of the degenerate electrons reaches such high values that e^- capture to excited states of the nuclei become possible [9]. The opening of the additional channels may considerably increase the total probability of electron capture.

The influence of nonequilibrium β processes on the contraction of the iron core of a star was considered earlier in [9-12]. In [10], Rudzskii and Seidov found the maximal temperature to which cold matter can be heated because of nonequilibrium β processes accompanying the slow contraction of the matter. The kinetics of e^- captures in the case of free-fall contraction was studied in [9, 11]. In [12], a study was made of the self-consistent problem of the loss of stability by a low-mass star as a result of neutronization.

In this paper, we investigate the evolution and collapse of a low-mass star as a result of β processes (in what follows, we shall mean by a star the core of a star, ignoring the shell). We shall take into account more carefully than in [12] the kinetics of the β processes under conditions of "cold neutronization" ($T < 3\cdot10^9$ °K). We shall study in detail the part played by electron capture to excited states of the daughter nuclei, which was not taken into account in [12]. We shall show that in the collapse

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of stars with mass near the Chandrasekhar limit this process plays an important part in the neutronization of matter beginning at densities $\rho \sim 10^{11}$ g/cm^3.

1. Equations Describing the Structure of the Star, the State of the Matter, and the Kinetics of the β Processes

At a high matter density $\rho \gtrsim 10^9$ g/cm^3, the electron gas in a star is relativistic and degenerate. The density profile in such a star can be described fairly accurately by a polytrope with index n = 3 [13] (although to find the radius of the star and the absolute values of the density and the temperature it is necessary to take into account small deviations of the adiabatic exponent from the value $\gamma = 4/3$). Both in the process of evolution and in the hydrodynamic stage, the contraction of the star takes place virtually homologously. As is shown by numerical calculations of the collapse of an iron star with mass $M = 2M_\odot$ [14], there is a departure from homologous contraction only when densities $\rho_c \sim 10^{12}$ g/cm^3 are reached. At lower densities, the method of profiling the density using a polytrope with n = 3 [15] is completely satisfactory.

In this paper, to describe the hydrodynamics of the contraction of the star, we shall use the polytropic approximation in the same way as in [12, 15]. Then the hydrodynamic partial differential equations describing the radial motion of the matter

$$\frac{\partial^2 r}{\partial t^2} = -4\pi r^2 \frac{dP}{\partial m} - \frac{Gm}{r^2}, \tag{1}$$

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi\rho r^2}, \tag{2}$$

reduce to a single ordinary differential equation for the total radius of the star:

$$\ddot{R}(t)\int_0^1 \left[\frac{\psi(q)}{\psi(1)}\right]^2 dq = \frac{3}{R(t)}\int_0^1 \frac{P}{\rho}\,dq - \frac{GM}{R^2(t)}\int_0^1 \frac{\psi(1)}{\psi(q)}\,q\,dq, \tag{3}$$

where q = m/M is the mass fraction of the star, which plays the part of a Lagrangian coordinate, and the function $\psi(q)$ is defined in such a way that $r(q) = R\psi(q)/\psi(1)$ is the radius of the layer of the star with the coordinate q; $\psi(1) = 6.897$.

For the complete description of the motion of the matter and the structure of the star we consider besides (3) the equation that describes the change in the internal energy E of the matter:

$$\frac{\partial E}{\partial t} = \frac{P}{\rho^2}\frac{\partial \rho}{\partial t} - \varepsilon_\nu, \tag{4}$$

where ε_ν is the rate of energy loss due to neutrino emission (in our case, photon emission and heat conduction can be ignored), and also the system of equations describing the kinetics of the β processes.

Taking into account the strong dependence of the rates of the nuclear reactions on the temperature, we assume that for $T < 3\cdot10^9$ °K nuclear reactions do not take place at all and there are only successive captures of electrons leading to the formation of nuclei with mass number A = 56 that are overenriched with neutrons (cold neutronization). The necessary data on the nuclei that are formed, which are unstable under terrestrial conditions, were taken from [16, 17]. The reactions of $e^\pm$ decay and e^+ capture for $T < 3\cdot10^9$ °K are not taken into account, since they are much less probable than e^- capture. The e^- capture reactions take place in a state of disequilibrium, and to describe them we use kinetic equations in the same manner as in [9, 11]:

$$\frac{d\chi_{AZ}}{dt} = -\chi_{AZ}w_{AZ}^{-c} + \chi_{AZ+1}w_{AZ+1}^{-c}. \tag{5}$$

Here, χ_{AZ} is the number of nuclei with atomic weight A and charge Z per nucleon, w_{AZ}^{-c} is the probability of e^- capture by the nucleus (A, Z). In our case for A = 56 it is sufficient to consider five equations of the type (5), since, as the calculations show, the temperature increases to values greater than $3\cdot10^9$ °K before an appreciable number of nuclei with $Z \leqslant 21$ is produced.

At temperature $T > 3\cdot 10^9\ ^\circ K$, the rates of the majority of nuclear reactions increase so much that statistical equilibrium with respect to the nuclei is established almost instantaneously [18]. In the approximation of statistical equilibrium, the concentrations of the nuclei, χ_{AZ}, the protons, χ_p, and neutrons, χ_n, are single-valued functions of T, ρ, and μ_e, where

$$1/\mu_e = \sum_{AZ} Z\chi_{AZ} + \chi_p \tag{6}$$

is the difference between the number of electrons and positrons per nucleon (including nucleons in nuclei). To calculate χ_{AZ}, χ_p, χ_n in this paper, we use the same procedure as in [12].

The change in μ_e is determined by the kinetics of the β processes, among which (in contrast to the case of cold neutronization) it is now necessary to include $e^{\pm}$ decays and e^+ capture. Assuming that the chemical composition is adjusted instantaneously to the given values of T, ρ, and μ_e, we have instead of (5)

$$\frac{d}{dt}\left(\frac{1}{\mu_e}\right) = \sum_{AZ} w_{AZ}\chi_{AZ} + w_p\chi_p + w_n\chi_n, \tag{7}$$

where

$$w_{AZ} = w_{AZ}^{-d}(T, \rho, \mu_e) + w_{AZ}^{+c}(T, \rho, \mu_e) - w_{AZ}^{-c}(T, \rho, \mu_e) - w_{AZ}^{+d}(T, \rho, \mu_e). \tag{8}$$

In (8), w_{AZ}^{-d}, w_{AZ}^{-c} are the rates of electron (w_{AZ}^{+d} and w_{AZ}^{+c} positron) decay and capture. More details about the rates of the β processes will be given in Sec. 2.

The internal energy E in Eq. (4) was calculated with allowance for the contribution of the nuclei and the free nucleons E_n, the electrons and positrons E_e, and the radiation E_γ, the energy of the nuclei being measured from the rest energy of the neutrons. The expressions for these components have the form

$$E_n = \frac{1}{m_p}\left[\frac{3}{2}\left(\sum_{AZ}\chi_{AZ} + \chi_p + \chi_n\right)kT - \sum_{AZ}E_{AZ}\chi_{AZ} - 2.53\, m_e c^2/\mu_e\right], \tag{9}$$

$$E_e = 8\pi\,\frac{m_e c^2}{\rho}\left(\frac{m_e c}{2\pi\hbar}\right)^3\int_1^\infty \varepsilon^2(\varepsilon^2-1)^{1/2}\left\{\frac{1}{1+\exp[(\varepsilon-\varphi_e)/\lambda]} + \frac{1}{1+\exp[(\varepsilon+\varphi_e)/\lambda]}\right\}d\varepsilon, \tag{10}$$

$$E_\gamma = \frac{\pi^2 k^4}{15\hbar^3 c^3}\,\frac{T^4}{\rho}. \tag{11}$$

The same components were taken into account in the calculation of the pressure:

$$P_n = \left(\sum_{AZ}\chi_{AZ} + \chi_p + \chi_n\right)\frac{kT\rho}{m_p}, \tag{12}$$

$$P_e = \frac{8\pi}{3}\,m_e c^2\left(\frac{m_e c}{2\pi\hbar}\right)^3\int_1^\infty (\varepsilon^2-1)^{3/2}\left\{\frac{1}{1+\exp[(\varepsilon-\varphi_e)/\lambda]} + \frac{1}{1+\exp[(\varepsilon+\varphi_e)/\lambda]}\right\}d\varepsilon, \tag{13}$$

$$P_\gamma = \frac{\pi^2 k^4}{45\hbar^3 c^3}\,T^4. \tag{14}$$

In the expressions (10) and (13), $\lambda = kT/m_e c^2$, $m_e c^2\varphi_e$ is the chemical potential of the electrons with allowance for their rest mass determined by the equation

$$\rho = \mu_e m_p 8\pi\left(\frac{m_e c}{2\pi\hbar}\right)^3\int_1^\infty \varepsilon(\varepsilon^2-1)^{1/2}\left\{\frac{1}{1+\exp[(\varepsilon-\varphi_e)/\lambda]} - \frac{1}{1+\exp[(\varepsilon+\varphi_e)/\lambda]}\right\}d\varepsilon. \tag{15}$$

The integrals in Eqs. (10), (13), and (15), and also in the expressions (18)-(20) used below to calculate the entropy were determined in accordance with Laguerre's formula with ten points.

In the rate of loss of the energy ε_ν on neutrino emission (which are assumed to leave the star freely) we included the losses $\varepsilon_{\nu u}$ due to the universal Fermi interaction, which are taken into account in accordance with [19], and the losses $\varepsilon_{\nu\beta}$ associated with β processes.

As a measure of the disequilibrium of the β processes, it is convenient to use the change ΔS in the entropy S per nucleon. In the present work, the values of S were calculated with allowance for the contribution of the nuclei and free nucleons

$$S_n = k\sum_{AZ}\chi_{AZ}\left\{\frac{5}{2}+\ln\left[\frac{\omega_{AZ}m_p}{\rho\chi_{AZ}}\left(\frac{m_p kTA}{2\pi\hbar^2}\right)^{3/2}\right]\right\} \tag{16}$$

(here the sum is taken over the nuclei and the free nucleons), the electrons, and positrons

$$S_e = \frac{8\pi}{3}\frac{\lambda^3 m_p}{\rho}k\left(\frac{m_e c}{2\pi\hbar}\right)^3\left\{\left(3G_3-3\frac{\varphi_e}{\lambda}G_1+G_2\right)+\left(3G_3'-3\frac{\varphi_e}{\lambda}G_1'+G_2\right)\right\}, \tag{17}$$

where

$$G_1=\int_0^\infty\frac{(u+1/\lambda)\sqrt{u(u+2/\lambda)}}{1+\exp(u-\psi)}\,du, \tag{18}$$

$$G_2=\int_0^\infty\frac{u(u+2/\lambda)\sqrt{u(u+2/\lambda)}}{1+\exp(u-\psi)}\,du, \tag{19}$$

$$G_3=\int_0^\infty\frac{(u+1/\lambda)^2\sqrt{u(u+2/\lambda)}}{1+\exp(u-\psi)}\,du, \tag{20}$$

$$G_i'=G_i(-\psi+2/\lambda),\quad \psi=(\varphi_e-1)/\lambda; \tag{21}$$

and the radiation

$$S_\gamma=\frac{4}{45}\frac{\pi^2k^4}{\hbar^3c^3}\frac{T^3m_p}{\rho}. \tag{22}$$

Thus, for $T < 3\cdot10^9\,^\circ$K, when the nuclear reactions can be assumed to be "frozen", the evolution of the star was calculated in accordance with the system of equations (3)-(5). For $T > 3\cdot10^9\,^\circ$K, the calculation was in accordance with the system (3), (4), and (7), in which the concentrations $\chi_{AZ}(T, \rho, \mu_e)$ of the nuclei and $\chi_p(T, \rho, \mu_e)$, $\chi_n(T, \rho, \mu_e)$ of the free nucleons were found from the system of transcendental equations of the statistical equilibrium with respect to the nuclei [12, 8]. The thermodynamic quantities were calculated in accordance with Eqs. (9)-(22).

2. Rates of β Processes

In the calculation of the rates of the β processes, we distinguish two cases. 1) For nuclei of each species only one state participates in the β transformations; this is either the ground state or an excited state with low excitation energy and for which the considered β transition is allowed. The probabilities of the processes of this type were calculated in accordance with the well-known formulas, and for nuclei with unknown values of ft this quantity was taken equal to 10^5 sec [12]. 2) All possible states of the nuclei participate in the β transformations, these including the highly excited states with excitation energy up to tens of MeV.

The excited states must be taken into account only in e^- capture reactions, since the degenerate electron gas has a high Fermi energy E_F, whereas the energy of the positrons is $\sim kT \ll E_F$; in addition, their number is small because of the strong degeneracy of the electrons. Under our conditions, β decay to excited states of the daughter nuclei are also improbable.

Allowance for excited states of nuclei in β transformations may be important for two reasons: first, it leads to an increase in the total probability of e^- capture; second, the fraction of the energy of the captured electron carried away by a neutrino decreases. The excited nucleus goes over to the ground state with the emission of γ rays, and if the excitation energy is high ($\gtrsim$8-10 MeV), then neutrons are also emitted [9]. Thus, e^- captures to excited levels lead to an additional growth of the entropy and the temperature of the matter.

The probabilities of e^- captures with allowance for excited states of the nuclei were calculated in [20-24]. The study of this process is difficult because of our incomplete knowledge of the structure of nuclei, especially those far from the β stability region — which must be taken into account in the collapse calculations. Since the reliability of the estimates of the rates of e^- capture to excited states is poor, to calculate them it is sensible to use one of the simplest models of the nucleus — the model in which the nucleus is regarded as a mixture of Fermi gases of noninteracting protons and neutrons.

Such a calculation was made in [24], in which it was shown that the total probability of e^- capture to all possible excited states of the daughter nucleus can be represented in the form

$$w_{ESAZ}^{-c} = |H|^2 \frac{3Z}{8\pi^3\hbar^7} \frac{m}{p_0^3} (J'_e + J''_e). \tag{23}$$

Here, H is the matrix element, taken to be independent of the excitation energy. The function J'_e for the region of electron Fermi energies $p_{e0}c \lesssim E_{F,e} \lesssim 100$ MeV has the form

$$J'_e = \frac{8}{3} \frac{mcx^2}{(1+x)^3} p_{ke}^4 \left[\frac{1}{6} p_{ke}^2 + \frac{1}{5} p_{ke} p_{e0} (2-x) + \frac{1}{4} p_{e0}^2 (1-x)\right]. \tag{24}$$

For the correction J''_e, which takes into account the nonzero temperature of the electrons, we readily obtain the expression

$$J''_e = \left(\frac{kT}{c}\right)^2 \frac{\pi^2}{6} \frac{8mcx^2}{3(1+x)^3} [5p_{ke}^4 + 4p_{ke}^3 p_{e0} (2-x) + 2p_{ke}^2 p_{e0}^2 (1-x)]. \tag{25}$$

The notation used in (23)-(25) is as follows: p_0 and q_0 are the Fermi momenta of the protons and neutrons in the nucleus, $p_{e0} = (q_0 - p_0)/2 + (q_0^2 - p_0^2)/2mc$ is the threshold momentum for e^- capture in the employed model of the nucleus, $m \approx 0.5m_p$ is the effective mass of a nucleon in the nucleus, $p_{ke} = p_{F,e} - p_{e0}$, $x = q_0/mc$; $p_{F,e}$ and T are the Fermi momentum and the temperature of the electron gas.

As in the case of (23), we can write down an expression for the rate of loss of energy carried away by the neutrino emission per nucleus:

$$\varepsilon_{\nu ESAZ}^{-c} = |H|^2 \frac{3Z}{8\pi^3\hbar^7} \frac{m}{p_0^3} (I'_e + I''_e), \tag{26}$$

where

$$I'_e = \frac{8}{3} \frac{mc^2x^2p_{ke}^4}{(1+x)^4} \Bigg[\left(\frac{1}{7} + \frac{x^2}{35}\right) p_{ke}^3 + \left(\frac{1}{2} - \frac{x}{3} + \frac{x^2}{30}\right) p_{e0} p_{ke}^2 + \left(\frac{3}{5} - \frac{4}{5} x + \frac{x^2}{5}\right) p_{e0}^2 p_{ke} + \frac{1}{4} (1-x)^2 p_{e0}^3\Bigg], \tag{27}$$

$$I''_e = \left(\frac{kT}{c}\right)^2 \frac{\pi^2}{6} \frac{8mc^2x^2}{3(1+x)^4} \Bigg[6p_{ke}^5 \left(1 + \frac{x^2}{5}\right) + 5p_{ke}^4 \left(3 - \frac{x}{2} + \frac{x^2}{5}\right) p_{e0} + 4p_{ke}^3 (3 - 4x + x^2) p_{e0}^2 + 3p_{ke}^2 (1-x)^2 p_{e0}^3\Bigg]. \tag{28}$$

The square of the modulus of the matrix element can be estimated if in the same model and for the same nucleus we calculate the probability of μ^- capture and use the

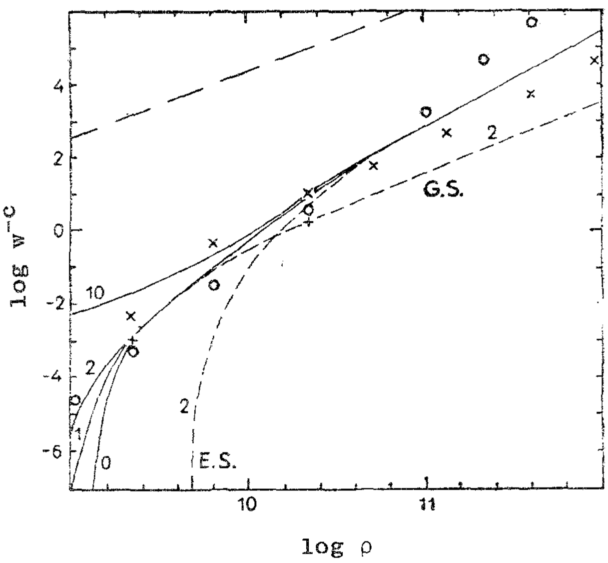

Fig. 1. Dependence of the rate of electron capture on the density and temperature for Fe^{56}. The continuous curves show the behavior of log w^{-c} calculated with allowance for the contribution of the excited states of Mn^{56}. The broken curves show the behavior of log $w^{-c}_{G.S}$ separately for transitions to the ground state, it being assumed that ft for this transition is 10^5 sec, and log $w^{-c}_{E.S.}$ for capture only on excited states (E.S.). The numbers next to the curves give the temperature in 10^9 °K. Also plotted are the values of log w^{-c} for the temperature $2\cdot10^9$ °K obtained in the following papers: [20] (open circles), [22] (the x's), and [21] (the two vertical crosses). The chain curve [sic] shows the logarithm of the rate of e^- capture by the 26 free protons for $\mu_e = 56/26$ and T = 0.

experimental data on this process (disregarding the small difference in the values of the matrix elements for electron and muon capture).

However, for the majority of the nuclei that we take into account there are no data on the probability of μ^- capture. In these cases, for $|H|^2$ we used the expression

$$|H|^2 = \frac{1}{2}(G_V^2 + 3G_A^2). \tag{29}$$

Here, $G_V = G_W = 1.43\cdot10^{-49}$ erg/cm^3, $G_A = 1.23G_W$, where G_W is the universal coupling constant of the weak interaction. Using (29), we represent the expressions (23) and (26) in the form

$$w^{-c}_{ESAZ} = 1.50\cdot10^{11} Z\left(\frac{mc}{p_0}\right)^3 \frac{J'_e + J''_e}{(mc)^7}\ \text{sec}^{-1}, \tag{30}$$

$$\frac{\varepsilon^{-c}_{ESAZ}}{m_e c^2} = 1.38\cdot10^{11} Z\left(\frac{mc}{p_0}\right)^3 \frac{I'_e + I''_e}{mc^2(mc)^7}\ \text{sec}^{-1}. \tag{31}$$

The reaction rate w^{-c} for the nucleus Fe^{56} calculated in accordance with (30) is shown by the broken curve in Fig. 1 labeled E.S.. Note that Eqs. (30) and (31) describe the contribution of only the excited levels of the daughter nucleus, and to them it is necessary to add the contribution of the ground state, which for Fe^{56} is shown in Fig. 1 by the broken curve labeled G.S..

3. Results of Calculations and Main Conclusions

We calculated two star models with masses $1.19M_\odot$ and $1.21M_\odot$. The initial chemical composition was chosen in the form of pure iron Fe^{56}, and for initial values of the density and temperature in the center (log ρ_c, log T_c) we took the values (9.25; 8) and (9.5; 8), respectively. The initial distribution of the temperature through the star was specified in the form $T/T_c = (\rho/\rho_c)^\alpha$. Then for fixed values of M, ρ_c, and T_c the

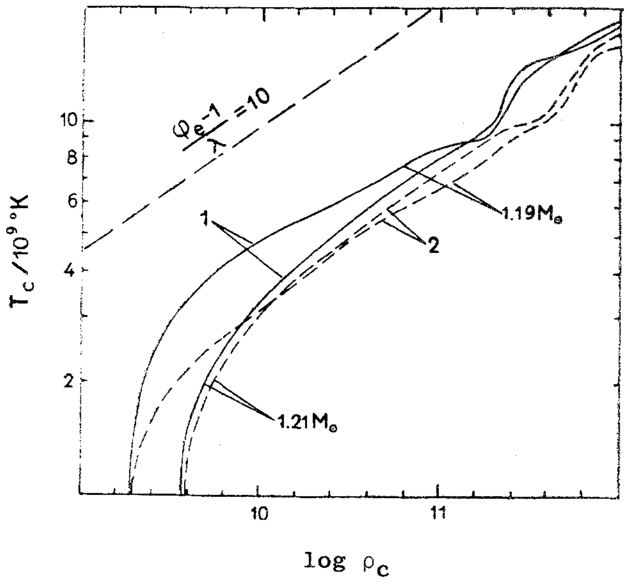

Fig. 2. Change in the temperature at the center of the calculated models during the contraction. We have indicated the line on which the degeneracy parameter of the electron gas satisfies $(\varphi_e - 1)/\lambda = 10$ for $\mu_e = 56/26$.

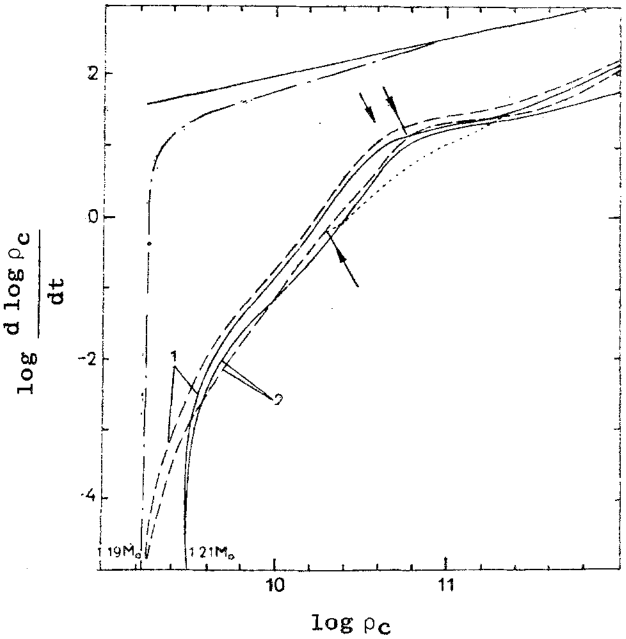

Fig. 3. Dependence of the rate of contraction $d \log \rho_c/dt$ on the central density. The points show the behavior of the contraction rate for the model with $M = 1.19 M_\odot$ without allowance for excited states with earlier inclusion of the hydrodynamic program. The arrows show the positions at which the calculation in accordance with the hydrodynamic part of the program was begun for all variants with mass $1.19 M_\odot$. For comparison, we also show the characteristic velocity of free fall with zero initial density (continuous curves) and with initial density $\log \rho_c = 9.25$ (chain curve).

condition of hydrostatic equilibrium distinguishes a unique value α, which in our models was ≈ 0.1.

In the considered stage of evolution, the dynamics of the contraction of the star is completely determined by β processes. The results of the present work confirm the conclusion of the authors in [12] about there being a smooth acceleration of the evolution contraction, the reason for this being the loss of pressure due to the increase of μ_e during the process of neutronization with subsequent transition to free fall to the center, i.e., collapse. The evolution tracks of the centers of the stars on the plane ($\log\rho_c$; $\log T_c$) are plotted in Fig. 2, and the rate of contraction is shown in Fig. 3. All calculations were continued to the density $\rho_c = 10^{12}$ g/cm^3, at which it is still not necessary to take into account neutrino opacity [14].

It can be seen from Figs. 4 and 5, in which we have plotted the change in the entropy and μ_e in the center of the star during the contraction, that in this section of the evolution and collapse three characteristic stages can be distinguished: 1) The initial stage, during which the density is almost constant and the star is heated because the initial density at the center exceeds the threshold density for the reaction $Fe^{56} + e^- \rightarrow Mn^{56} + \nu_e$ and for e^- captures by Mn^{56} nuclei [7].

2) The intermediate stage in the density range 10^{10} g/cm$^3 \lesssim \rho_c \lesssim 10^{11}$ g/cm^3, in which the entropy and μ_e in the center hardly change. The reason for the weak variation of S and μ_e is that in this stage there is a rapid acceleration of the quasiequilibrium contraction, which goes ahead of the increase in the rate of the β processes decisively. Indeed, with increasing density ρ_c the rate of the β processes, even with allowance for the excited states, increases not faster than ρ_c^2 [24], whereas in accordance with Fig. 3 the rate of contraction $d\log\rho_c/dt$ in this stage changes much more rapidly. Note that despite the small values of $d\mu_e/d\log\rho_c$, e^- captures are predominant in the β processes, and the matter of the star is far from the state of kinetic equilibrium with respect to the β processes considered in [25].

3) For $\rho_c \gtrsim 5\cdot10^{10}$ g/cm^3, the quasistatic contraction of the star is replaced by the hydrodynamic stage of free fall to the center. In the free-fall regime, the rate of contraction increases only as $\rho_c^{1/2}$, and for $\rho_c \gtrsim 2\cdot10^{11}$ g/cm^3 the third stage commences; this is the stage of rapid neutronization, in which the Fermi energy of the degenerate electrons appreciably exceeds the threshold of e^- capture by nuclei and the

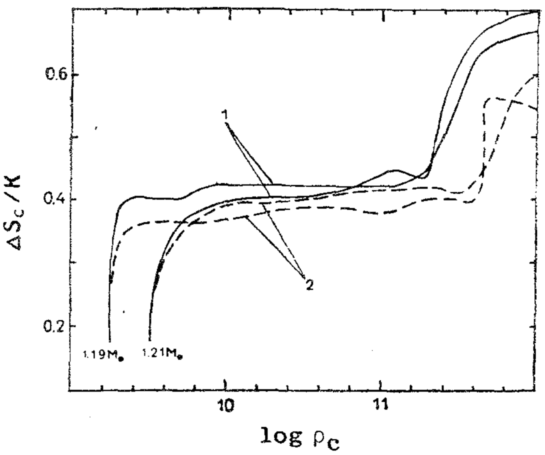

Fig. 4. Change in the specific entropy (per 1 nucleon) at the center during the contraction.

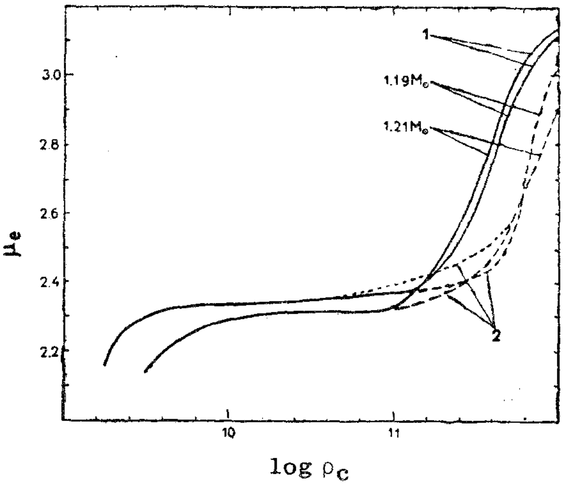

Fig. 5. Change in the value of μ_e (the ratio of the total number of nucleons to the total number of protons) at the center of the calculated models. The points show the result of calculation for the model M = 1.19$M_\odot$ with early inclusion of the hydrodynamic part of the program for the variant without allowance for excited states of the nuclei.

rate of the β processes overtakes the contraction rate.

The entire interval of evolution and collapse calculated in the present paper is characterized by a strong degeneracy of the electron gas. In this respect, the results of the present paper differ strongly from the results for $M = 1.21M_{\odot}$ in [12], in which an error found its way into the computer calculations for this variant and led to strongly overestimated (by ~2 times) values of the temperature. As a result, the β processes on free nucleons become predominant only when $\rho_c \gtrsim 10^{12}$ g/cm^3, and not at $\rho_c \gtrsim 10^{10}$ g/cm^3, as was found in [12].

One of the aims of the present paper was to elucidate the part played by e^- capture to excited states of daughter nuclei, and for both masses we calculated the evolution with allowance for these states (these are the variants 1 in Fig. 2-5) and without them (2 in Figs. 2-5). The graphs in Figs. 2, 4, and 5 show that it is important to take into account the excited states in the third stage in the density range $2 \cdot 10^{11}$ g/cm$^3 \lesssim \rho \lesssim 10^{12}$ g/cm^3, when electron capture to these states significantly accelerates the increase in the entropy (Fig. 4) and μ_e (Fig. 5) during the collapse process. At higher densities $\rho_c > 10^{12}$ g/cm^3, the situation is less definite, since it is necessary to take into account neutrino opacity and neutrino degeneracy.

We now discuss the part played by excited states during the initial stage when $T_c < 3 \cdot 10^9$ °K and the nuclear reactions are "frozen", and the Fermi energy of the electron gas only slightly exceeds the threshold of e^- capture by Fe^{56} nuclei. Our calculations showed that in the initial stage and for the same values of the density in the center of a star with mass $1.19M_{\odot}$ allowance for the excited states leads to an increase in the temperature by more than 1.5 times (see Fig. 2), whereas for the mass $1.21M_{\odot}$ this difference is much less. The reason for this is as follows. The threshold for neutronization of Fe^{56} is twice the analogous threshold for the next isobar Mn^{56}. As a result, in the initial stage of "cold" neutronization in the reaction $Fe^{56} + e^- \rightarrow Mn^{56} + \nu_e$ one need take into account only one excited state of Mn^{56} with spin and parity 1^+ and energy 109 keV, whereas in the reaction $Mn^{56} + e^- \rightarrow Cr^{56} + \nu_e$ it is important to take into account all the excited states. However, Cr^{56} is an even—even nucleus, and its first excited states are separated by appreciable energy gaps ~1 MeV from each other and from the ground state. Under these conditions, when the Fermi energy of the electrons is close to the threshold of capture to the first excited states of Cr^{56}, our model of a nucleus as a mixture of Fermi gases of protons and neutrons is not good. Therefore, it could be that a more realistic model of the Cr^{56} nucleus would decrease the part played by its excited states in the initial stage of neutronization. In the $M = 121M_{\odot}$ model, the initial stage takes place much more rapidly than in the $M = 1.19M_{\odot}$ model; at the same time, smaller values of μ_e are reached (see Fig. 5), and capture to the excited states of Cr^{56} simply do not raise the entropy in the center significantly.

It is interesting that during the collapse process the distribution of the temperature over the star in our case become nonmonotonic with a minimum near the center of the star, allowance for the excited states enhancing this nonmonotonicity. A nonmonotonic variation of the temperature over the star was obtained and discussed in detail in [26].

Besides what we have discussed above, for the $M = 1.21M_{\odot}$ model we also calculated variants with $ft = 10^6$ sec (for nuclei with unknown ft) and with temperature of the transition from cold to hot neutronization of $T = 6 \cdot 10^9$ °K (cf $T = 3 \cdot 10^9$ °K); these revealed a weak dependence of the main characteristics of the evolution and the collapse on these parameters.

Finally, we note that the total energy released in the form of neutrinos in the considered stage of the collapse is for our models $(0.8\text{-}1.4) \cdot 10^{51}$ erg; the mean energy of a neutrino is $E_{\nu} \sim 7\text{-}10$ MeV.

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LITERATURE CITED

1. G. Rakavy, G. Shaviv, and Z. Zinamon, Astrophys. J., 150, 131 (1967).
2. S. Ikeuchi, K. Nakazawa, T. Murai, R. Hoshi, and C. Hayashi, Prog. Theor. Phys., 46,

1713 (1971); 48, 1870 (1972).
3. V. I. Varshavskii and A. V. Tutukov, Nauchn. Inform. Astrosoveta AN SSSR, 23, 47 (1972); 26, 35 (1973).
4. A. Finzi and R. A. Wolf, Astrophys. J., 150, 115 (1967).
5. G. S. Bisnovatyi-Kogan and Z. F. Seidov, Astrofizika, 5, 243 (1969).
6. F. Houle and W. Fowler, Astrophys. J., 132, 565 (1960).
7. G. S. Bisnovatyi-Kogan and Z. F. Seidov, Astron. Zh., 47, 139 (1970).
8. V. S. Imshennik and V. M. Chechetkin, Astron. Zh., 47, 929 (1970).
9. M. A. Rudzskii and Z. F. Seidov, Astron. Zh., 51, 936 (1974).
10. M. A. Rudzskii and Z. F. Seidov, Izv. Akad. Nauk Az. SSR, Ser. Fiz.-Tekh. Mat. Nauk, No. 4, 98 (1974).
11. K. Nakazawa, Prog. Theor. Phys. 49, 1931 (1973).
12. M. M. Basko and V. S. Imshennik, Astron. Zh., 52, 469 (1975).
13. Ya. B. Zel'dovich and I. D. Novikov, Theory of Gravitation and Evolution of Stars [in Russian], Nauka, Moscow (1971).
14. D. K. Nadyozhin, Astrophys. Space Sci., 51, 283 (1977).
15. G. S. Bisnovatyi-Kogan, Astron. Zh., 45, 74 (1968).
16. P. A. Seeger, in: Proc. of Internat. Conf. on Properties of Nuclei Far from Region of Beta-Stability, Geneva (1970).
17. W. D. Myers and W. J. Swiatecki, Nucl. Phys., 81, 1 (1966).
18. V. S. Imshennik and D. K. Nadezhin, Astron. Zh., 42, 1154 (1965).
19. G. Beaudet, V. Petrosian, and E. E. Salpeter, Astrophys. J., 150, 979 (1967).
20. C. J. Hansen, Astrophys. Space Sci., 1, 449 (1968).
21. T. J. Mazurek, J. W. Truran, and A. G. W. Cameron, Astrophys. Space Sci., 27, 261 (1974).
22. Y. Egawa, K. Yokoi, and M. Yamada, Prog. Theor. Phys., 54, 1339 (1975).
23. K. Takahashi, M. F. El Eid, and W. Hillebrandt, Astron. Astrophys., 67, 185 (1978).
24. G. S. Bisnovatyi-Kogan, M. A. Rudzskii, and Z. F. Seidov, Zh. Eksp. Teor. Fiz., 67, 1621 (1974).
25. V. S. Imshennik, D. K. Nadezhin, and V. S. Pinaev, Astron. Zh., 43, 1215 (1966); 44, 768 (1967).
26. L. N. Ivanova, V. S. Imshennik, and V. M. Chechetkin, Astron. Zh., 54, 1009 (1977).

FORMATION OF BLACK HOLES IN THE EARLY UNIVERSE

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One of the mechanisms of formation of primordial black holes in the Universe is considered. It is shown that for a sufficiently high level of the initial inhomogeneities primordial black holes can be formed by statistical clustering of black holes of minimal mass. The mass spectrum of the resulting objects is calculated. The cosmological consequences of the existence of primordial black holes in a "cold" Universe are discussed.

Modern ideas about the physical processes and dynamics of the expansion of the Universe lead to the hypothesis that in the past there existed small inhomogeneities superimposed on the Friedmann expansion of the Universe.

In the early stages of the evolution, the collapse of the initial inhomogeneities could have led to the formation of primordial black holes [1, 2]. As numerical calculations show (see, for example, [3-5]), the parameters of the mass spectrum of these black holes are determined not only by the characteristics of the spectrum of the initial adiabatic perturbations but also to a considerable extent depend on the equation of state of the matter in the period of formation of the primordial black holes. In accordance with [5, 6], the mass spectrum of the black holes is critically dependent on the exponent in the spectrum of the initial inhomogeneities. For a flat spectrum

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