

well to the explanation offered in Ref. 1. The basic effects stem from structural features of the shock wave in the plasma, primarily the discontinuity against the background of a continuous change in the electron temperature. A more detailed analysis of the data available will require a substantial extension of the theory for the structure of the shock wave. In particular, it will be necessary to incorporate the time variation and contribution of ionization and recombination processes to the energy balance.

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Stopping of fast ions in a dense plasma

M. M. Basko

Institute of Theoretical and Experimental Physics

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A theoretical model is proposed for the Coulomb energy loss of fast ions ($0.1 \text{ MeV/amu} \lesssim E_1 \lesssim 1 \text{ GeV/amu}$) over broad ranges of the temperature and density of the stopping medium ($0 < T < 300 \text{ keV}$, $0 < \rho < 10^6 \text{ g/cm}^3$). Both free and bound electrons are taken into account, as is the scattering by ions of the medium. The expressions derived for the stopping power are accurate within $\sim 10\text{--}20\%$ at ion energies $E_1 \sim 0.1\text{--}1 \text{ MeV/amu}$, improving to $\sim 5\%$ at $E_1 > 10 \text{ MeV/amu}$. With increasing temperature in a dense medium ($\rho/m_e A_2 \gtrsim 10^{22} \text{ cm}^{-3}$) the Coulomb mean free path of the fast particles initially increases by $10\text{--}40\%$ before it begins to decrease.

1. INTRODUCTION

In this paper we propose some simple expressions for the rate at which fast ions are stopped in a medium with thermodynamic parameters over broad ranges: $0 < \rho < 10^6 \text{ g/cm}^3$ and $0 < T < 300 \text{ keV}$. The practical need for such expressions arises from the design of high-current light-ion accelerators¹ and the development of the concept of a fusion reactor using beams of light ions.^{2,3} The stopping of heavy ions (and probably that of light ions also), even in the most intense beams presently capable of being produced by the best accelerators, can be described in the one-particle approximation,⁴ i.e., in the limit of vanishingly small density of beam particles. In this approximation, each ion of the beam, with atomic mass A_1 and atomic number Z_1 , gives up its energy individually - in a manner unaffected by the presence of other fast particles - in a Coulomb interaction with the electrons and ions of the medium. "Fast" particles are those with kinetic energies $0.1 \text{ MeV/amu} \lesssim E_1 \lesssim 1 \text{ GeV/amu}$.

Strictly speaking, in order to calculate the stopping power we need to know the complete spectrum of energy excitations of the medium, and we need to be able to calculate the charge of the fast ion, $eZ_{1\text{eff}}$, and the probabilities for transitions between various energy states of the medium. The fact that the fast ion is not a point charge but is instead surrounded by a cloud of electrons must be taken into account; these electrons themselves may undergo transitions from some energy states to others. Since the problem in this general formulation is not going to be solved in the near future, we take a simplified approach here, working from a semiempirical formula for $Z_{1\text{eff}}$ and in the average-ion approximation⁵ in the description of the properties of the medium. This

study differs fundamentally from all earlier studies in using the correct ionization equation for a dense plasma and a more accurate procedure for calculating the average excitation energy $\bar{h}\bar{\omega}$ of the bound electrons. As a result we find an interesting effect, characteristic of a dense plasma and not seen in similar calculations by other investigators^{6,7}: As the temperature of the medium is raised at a fixed density, the stopping power of the medium initially decreases by $10\text{--}40\%$ and only then begins to increase.

2. STOPPING BY BOUND ELECTRONS

We will derive the stopping power $S = -\rho^{-1}dE_1/dx$ for a medium which, for simplicity, is assumed to consist of atoms of a single species with an atomic mass A_2 and an atomic number Z_2 . The medium is assumed to be at a statistical equilibrium characterized in general by equal electron and ion temperatures T_e and T_i . For given values of T_e and ρ , the degree of ionization $y = y(\rho, T_e)$ (the number of free electrons per atomic cell) is found from the ionization equation

$$\mu_e(y/V, T_e) + I(y) = b(Z_2/V)^\beta (1 + \mu T_e^\sigma V^\sigma)^{-1}, \quad (1)$$

where V is the volume of the atomic cell, $I(y)$ is a smoothed interpolation of the discrete ionization potentials, μ_e is the chemical potential of an ideal Fermi gas of electrons, and $b, \beta, \mu,$ and σ are constants which characterize the individual properties of the elements. All the quantities in Eq. (1) are expressed in atomic units. Values of the constants in this equation for several elements are listed in Table I (see Ref. 5 for more details).

We write the overall stopping power as the sum of

TABLE I

	β	b	σ	$10^4 \mu$	ν_0	ϵ_0
Be	0.6	11.63	1.8	5.7	2.20	2.10
C	0.6	11.73	1.6	20.4	3.09	2.30
Al	0.59	10.54	1.72	4.97	3.42	2.06
Ti	0.67	5.5	1.8	3.73	3.30	1.62
Fe	2/3	5.7	1.82	3.2	4.17	2.05
Ag	2/3	5.7	1.8	2.79	4.64	1.73
Au	2/3	5.7	1.85	1.37	6.85	1.61
Pb	2/3	5.7	1.85	1.19	5.35	1.47
U	0.68	5.05	1.8	1.39	6.98	1.20

four terms,

$$-\frac{1}{\rho} \frac{dE_1}{dx} = S = S_{be} + S_{Ie} + S_{II} + S_{nu}, \quad (2)$$

which describe the contributions of bound and free electrons, free plasma ions, and bare nuclei, respectively. The general expression for S_{be} is⁸

$$S_{be} = \frac{4\pi e^4 Z_{Ief}^2}{m_e v_1^2} \left(\frac{Z_2 - y}{A_2 m_A} \right) \left[L_{be} - \ln \left(1 - \frac{v_1^2}{c^2} \right) - \frac{v_1^2}{c^2} \right], \quad (3)$$

where eZ_{Ief} is the charge of a fast ion moving at velocity v_1 , $(Z_2 - y)$ is the number of bound electrons per atom of the medium, m_A is the atomic mass unit, and L_{be} is the Coulomb logarithm. For simplicity we separate the relativistic corrections from the Coulomb logarithm; then in the calculation of this quantity below we can deal with nonrelativistic momenta and energies.

Under the condition $L_{be} \gg 1$, the Coulomb logarithm is conveniently written in the form

$$L_{be} = \ln(p_{max}/p_{min}), \quad (4)$$

where p_{max} and p_{min} are, respectively, the maximum and minimum changes in the momentum of a fast ion as a result of a collision with a field particle (a bound electron in this case). The expression for p_{max} is obvious:

$$p_{max} = 2m_e v_1. \quad (5)$$

The correct values of p_{min} in the classical limit ($\hbar v_1 \ll Z_{Ief} e^2$) and the quantum limit ($\hbar v_1 \gg Z_{Ief} e^2$) were first calculated by Bohr⁹ and Bethe,¹⁰ Bloch¹¹ has studied the transition region. The Bohr-Bethe-Bloch results can be described very accurately by the simple expression

$$p_{min} = [(\gamma Z_{Ief} e^2 \bar{\omega} / v_1)^2 + (\hbar \bar{\omega} / v_1)^2]^{1/2}, \quad (6)$$

where $\bar{\omega}$ is the average excitation frequency of the atomic electrons, and $\ln \gamma = 0.577$ is the Euler constant.

Bethe was able to calculate $\bar{\omega}$ only for an isolated hydrogen atom, for which $\hbar \bar{\omega} = 1.105 \text{ Ry}$. For heavy atoms, no theoretical values of $\bar{\omega}$ have been derived so far. Experimental values of $\bar{\omega}$ have been found for all elements up to uranium¹² - but only at room temperature and atmospheric pressure. We need an approximate procedure for estimating $\bar{\omega}$ which is applicable to both isolated ions of arbitrary charge in a tenuous plasma and the compressed atomic cells in a dense medium. The following two effects must be described accurately:

1) the sequential "turning off" of atomic shells as the velocity of the slowing ion, v_1 , becomes smaller than the atomic electron velocities in the corresponding shells;

2) the increase in the atomic frequencies ω with the thermal ionization of the atom, accompanied by the separation of the outer electrons which are partially screening the nuclear charge.

For this purpose we break up the product $(Z_2 - y)L_{be}$ in (3) into a sum over the individual subshells in the Hartree-Fock-Dirac model of the atom,

$$(Z_2 - y)L_{be} = \sum_{n,l,j} n_{n,l,j} H_{be}(\Lambda_{n,l,j}) \quad (7)$$

and we assume that the quantities

$$\Lambda_{n,l,j} = (p_{max}/p_{min})_{n,l,j} \quad (8)$$

are again given by (5) and (6), in which we are to substitute the appropriate value of the average excitation frequency, $\bar{\omega}_{n,l,j}$, for each subshell. The summation in (7) runs over only those subshells in which there are bound electrons. In our average-ion model, the number of electrons in the outermost filled subshell may be a fraction.

The sequential turning off of the inner subshells for $\Lambda_{n,l,j} \ll 1$ is taken into account by means of a universal dependence $H_{be}(\Lambda)$. Clearly, we have $H_{be}(\Lambda) \approx \ln \Lambda$ at $\Lambda \gg 1$, while this function vanishes at $\Lambda = 0$. The particular way in which $H_{be}(\Lambda)$ decays at $\Lambda \ll 1$ is determined by the shape of the excitation spectrum at frequencies $\omega \ll \bar{\omega}$. Thinking of the more general case of a quasi-continuous spectrum, to which Firsov's result¹³ $H_{be}(\Lambda) \propto \Lambda^{3/2}$ applies, we use the simple expression

$$H_{be}(\Lambda) = \ln [1 + \Lambda / (1 + 3.5/\Lambda^{1/2})], \quad (9)$$

which agrees reasonably well with experimental data¹⁴ on the stopping of protons (Fig. 2).

Describing the exclusion of the shells by the function $H_{be}(\Lambda)$, we may assume that the average frequencies $\bar{\omega}_{n,l,j}$ are independent of the velocity of the fast ion, v_1 , and we need consider their dependence on only the degree of ionization y . Clearly, we have $\hbar \bar{\omega}_{n,l,j} \sim e_{n,l,j}$ in order of magnitude, where $e_{n,l,j} = e_{n,l,j}(y)$ is the detachment energy (energy of the transition to the continuum) of an electron in subshell n, l, j . We set

$$\hbar \bar{\omega}_{n,l,j} = g(y) e_{n,l,j}(y), \quad (10)$$

where the excitation coefficient $g(y)$ is the same for all subshells of a given ion. The accuracy of this approximation can be judged by comparing the values calculated for $g_0 = g(y_0)$ for various substances under standard conditions [$T_e = T_0 = 300 \text{ K}$, $P = 760 \text{ torr}$, $\rho = \rho_0$, $y_0 = y(\rho_0, T_0)$] from experimental values of $\bar{\omega} = \bar{\omega}_{exp}$. Since the

Bethe formula is used to determine $\bar{\omega}_{\text{exp}}$ in the high-energy limit, $E \gg 100$ MeV/amu, at which the contributions of S_{fi} and S_{nu} can be ignored, the values of g_0 can be found from

$$(Z_2 - y_0)L_{\text{be}} + y_0L_{\text{fe}} = Z_2 \ln(2m_e v_0^2 / \hbar \bar{\omega}_{\text{exp}}), \quad (11)$$

where it is assumed that $T_e = T_0$, $\rho = \rho_0$, $Z_1 = A_1 = 1$, $E_1 = m_p c^2$. Values of $\hbar \bar{\omega}_{\text{exp}}$ are tabulated in the review by Ahlen¹²; electron binding energies for isolated atoms are given in Ref. 15; and expressions for L_{fe} are given below (in Sec. 3). We see from Table I, which lists values of y_0 and g_0 for several elements, that the scatter in the values of g_0 from element to element is comparatively small, $1.2 \leq g_0 \leq 2.3$. Using the extreme values $g(y_0) = g_0$ and $g(Z_2 - 1) = g_{\text{H}} = 1.105$, we find the following simple interpolation formula for $g(y)$:

$$g(y) = \begin{cases} g_0, & 0 < y \leq y_c, \\ g_0 + (g_{\text{H}} - g_0)(y - y_c)/(Z_2 - 1 - y_c), & y_c < y < Z_2 - 1, \\ g_{\text{H}}, & Z_2 - 1 \leq y < Z_2, \end{cases} \quad (12)$$

where $y_c = y(\rho, T_0)$ is the low-temperature ionization state.

The increase in the detachment energies $\varepsilon_{n, l, j}(y)$ as the higher stages of ionization are approached is incorporated in the approximation

$$\varepsilon_{n, l, j}(y) = \varepsilon_{n, l, j}(0) + \Delta\varepsilon(y), \quad (13)$$

where the correction $\Delta\varepsilon(y)$, for removal of the screening by the external electrons, does not depend on the quantum numbers n, l, j :

$$\Delta\varepsilon(y) = \max\{0; I_{y+1} - \varepsilon_{z_1-y} - 2 \text{Ry } b(Z_2/V)^b(1 + \mu T_e V^0)^{-1}\}. \quad (14)$$

Here I_{y+1} is the potential of the ionization $y \rightarrow y+1$, and ε_{z_1-y} is the binding energy of the $(Z_2 - y)$ th electron in the isolated neutral atom. Expression (14) gives a fairly accurate description of the limit of a tenuous plasma ($V \rightarrow \infty$), in which we have $\Delta\varepsilon(y) \approx I_{y+1} - \varepsilon_{z_1-y}$ (Ref. 16). In a dense, nonideal plasma, the electrons continue to partially screen the nuclear charge even after they undergo transitions to the continuum. This effect is incorporated in (14) by means of the same correction to $I(y)$ as in ionization equation (1). Clearly, this correction gives a qualitatively correct description of the decrease in $\Delta\varepsilon(y)$ with increasing density. It is hard to judge just how good its quantitative accuracy is, since we have neither theoretical calculations nor experimental data on $\bar{\omega}$ in a highly compressed substance. For non-integer values of y , the quantity $\varepsilon_{n, l, j}(y)$ is found from the formulas for a linear interpolation between the adjacent integer values.

Figure 1 compares the procedure for calculating $\bar{\omega}(y)$ described above with atomic calculations^{17,18} for isolated aluminum and gold ions. The agreement is seen to be far better than in models of other investigators,¹⁹ especially after we assign aluminum the value $g_0 = 1.323$, calculated from the theoretical result¹⁷ $\hbar \bar{\omega}|_{y=0} = 120.7$ eV, which is quite different from the value $\hbar \bar{\omega}_{\text{exp}} = 164$ eV.

3. STOPPING BY FREE ELECTRONS

To calculate the stopping power of the electrons of the continuous spectrum we use the formulas derived in the theory of a collisionless plasma. Incorporating the re-

sults of several theoretical studies,²⁰⁻²² we propose the following algorithm for calculating S_{fe} :

$$S_{\text{fe}} = \frac{4\pi e^4 Z_{\text{ief}}^2 y G(x_e)}{m_e v_i^2 A_2 m_A} \left[L_{\text{fe}} - \ln\left(1 - \frac{v_i^2}{c^2}\right) - \frac{v_i^2}{c^2} \right], \quad (15)$$

$$G(x_e) = \frac{2}{\gamma \pi} \int_0^{x_e} \exp(-t^2) dt - x_e \exp(-x_e^2) \approx (1 + 1.33x_e^{-3})^{-1}, \quad (16)$$

$$x_e = (m_e v_i^2 / 2kT_{\text{ef}})^{1/2}, \quad x_i = (m_A A_2 v_i^2 / 2kT_i)^{1/2}, \quad (17)$$

$$kT_{\text{ef}} = [(kT_e)^2 + (2^{1/2} \pi \hbar^2 n_e^{1/2} / m_e)^2]^{1/2}, \quad (18)$$

$$L_{\text{fe}} = \ln[1 + \Lambda_{\text{fe}} / (1 + 0.5/\Lambda_{\text{fe}}^{1/2})], \quad (19)$$

$$\Lambda_{\text{fe}} = 2m_e v_{\text{ef},e}^2 r_{\text{ef}}^* / (\hbar^2 v_{\text{ef},e}^2 + \gamma^2 Z_{\text{ief}}^2 e^4)^{1/2}, \quad (20)$$

$$m_e v_{\text{ef},e}^2 = 2kT_{\text{ef}} \eta(x_e), \quad m_A A_2 v_{\text{ef},i}^2 = 2kT_i \eta(x_i), \quad (21)$$

$$\eta(x) = 0.353 + x^2(2.34 + x^3)/(11 + x^3), \quad (22)$$

$$r_{\text{ef}}^* = \max\left\{r_{\text{ef}}; \left(\frac{3}{4\pi} \frac{Z_{\text{ief}}}{n_e}\right)^{1/2}\right\}, \quad (23)$$

$$r_{\text{ef}}^{-2} = 4\pi n_e e^2 / m_e v_{\text{ef},e}^2 + 4\pi n_e y e^2 / A_2 m_A v_{\text{ef},i}^2. \quad (24)$$

The quantity $n_e = \rho y / A_2 m_A$ is the volume density of free electrons. Expressions (15)-(24) are actually obtained by matching all the asymptotic expressions differing in the values of the four dimensionless parameters which characterize the stopping of a test charge in an ideal plasma: The degeneracy parameter $\psi = \varepsilon_{\text{F}} / kT_e$ (ε_{F} is the Fermi energy of an electron gas of density n_e), the Bohr parameter $\lambda = e^2 Z_{\text{ief}} / \hbar v_{\text{ef}}$, and the two "acoustic" parameters x_e and x_i . The transition from "supersonic" stopping at $x_e \gg 1$ to "subsonic" stopping at $x_e \ll 1$ calculated for a Boltzmann ($\psi \ll 1$) plasma in Ref. 21, for example, is described well by expression (22): The absolute error in the value of L_{fe} is less than or equal to 0.1. The Boltzmann ($\psi \ll 1$) and Fermi ($\psi \gg 1$) asymptotic expressions have been joined by means of simple expression (18), which gives the correct coefficient of the logarithm in the limit $\psi \gg 1$, $x_e \ll 1$ (Ref. 22). On the other hand, a comparison with the results of a rigorous integration of the dielectric constant of a degenerate plasma²³ shows that the simple expression (18), combined with (22), can result in an error $\sim 50\%$ in the values of S_{fe} at $x_e \sim 1$, while outside the interval $0.4 \leq x_e \leq 3$ this error is always $\leq 40\%$. Just how accurate this scheme is in the intermediate region $\psi \sim 1$ is not clear at this point. In expression (24) for the effective radius of the screening by free charges, r_{ef} , the ions are "turned on" at $x_i \sim 1$ by means of the same formula, (22), as is used to turn on the electrons at $x_e \sim 1$. Expression (3) sets a lower limit on the possible values of r_{ef} : the "strong"-screening radius $(3Z_{\text{ief}}/4\pi n_e)^{1/3}$ (Ref. 24).

4. STOPPING BY THE IONS OF THE MEDIUM

Stopping due to collisions with ions can become important at the end of the range in either a hot plasma with $x_e \ll 1$ or a cold medium if $E_1 \leq 0.5$ MeV/amu and $Z_1 \gg 1$, $Z_2 \gg 1$. Collisions with ions of the medium can be classified somewhat arbitrarily as distant and close; we use an impact parameter $r = r_{\text{S}}$ as the boundary between the two types. We denote by S_{fi} and S_{nu} the corresponding contributions to the total stopping power. If $r \gg r_{\text{S}}$, the beam ions and those of the target interact as if they were point charges eZ_{ief} and $e y$, while if $r \ll r_{\text{S}}$ the nuclei of the colliding ions penetrate into the screen-

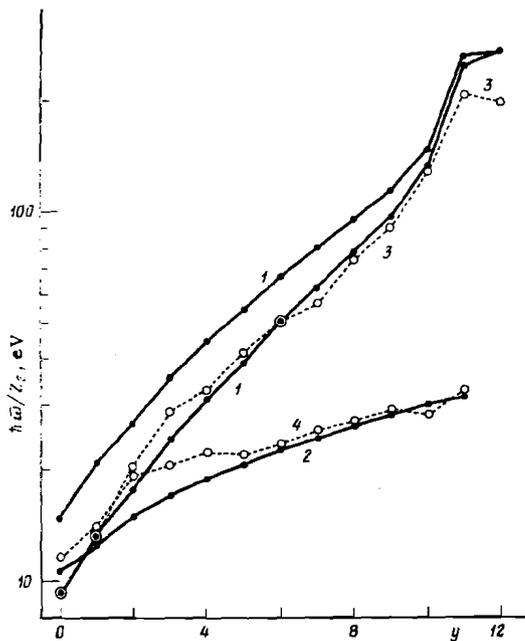


FIG. 1. Average excitation energy $\hbar\bar{\omega}$ of isolated ions versus the ion charge y . 1, 3) Aluminum; 2, 4) gold. 1, 2) Calculated from the formulas of the present study; 3) data from Ref. 17; 4) from Ref. 18.

ing cloud of bound electrons and are repelled as point charges eZ_1 and eZ_2 .

To describe distant collisions we use the results from the theory of Debye screening, as has been done for collisions with free electrons. As a result we find the following expression for $\Lambda_{fi} = P_{max}/P_{min}$:

$$\Lambda_{fi} = \min \left\{ \frac{2M_0 v_{ef,i}^2 r_{ef}^*}{(\hbar^2 v_{ef,i}^2 + \gamma^2 Z_1^2 Z_2^2 e^4)^{1/2}}; \frac{r_{ef}^*}{r_s} \right\}, \quad (25)$$

where $M_0 = m_A A_1 A_2 / (A_1 + A_2)$ is the reduced mass of the colliding ions, and $v_{ef,i}$ is the effective velocity of a test ion, from (21).

Before we write the final expression for S_{fi} we need to refine the concept of a range. As long as we were dealing with the stopping of ions by light electrons we tacitly assumed that the fast ions lose all their energy while moving along a straight line, and we drew no distinction between the reduced mass $m_e/(1 + m_e/m_A A_1)$ and the electron mass m_e . In the case of scattering by heavy particles of mass $m_A A_2$, the length of the rectilinear part of the trajectory, $R_\theta \propto M_0$, may be very different from the length of the trajectory $R_E \propto m_A A_2$, of the total energy loss (the condition $R_\theta < R_E$ always holds). If $R_\theta \ll R_E$, the relaxation of a fast ion becomes diffusive in nature. In this situation we are interested in the projection range $R_p \approx (R_\theta R_E)^{1/2}$, equal to the average displacement of a fast ion along its initial direction. We accordingly assume

$$S_{fi} = \frac{4\pi e^4 Z_1^2 Z_2^2 (1 + A_2/A_1)^{1/2}}{m_A A_2 v_i^2} G(x_i) L_{fi}, \quad (26)$$

where, by analogy with (19),

$$L_{fi} = \ln[1 + \Lambda_{fi}/(1 + 0.5/\Lambda_{fi}^{1/2})]. \quad (27)$$

Going through the corresponding arguments for close collisions, we find

$$S_{nu} = \frac{4\pi e^4 Z_1^2 Z_2^2 (1 + A_2/A_1)^{1/2}}{m_A A_2 v_i^2} G(x_i) L_{nu}, \quad (28)$$

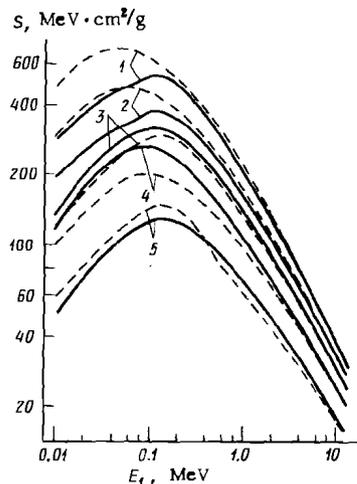


FIG. 2. Stopping powers of various metals under standard conditions for protons, plotted against the proton energy. 1) Be; 2) Al; 3) Fe; 4) Ag; 5) U. Solid lines) Present calculations; dashed lines) compilation of experimental data from Ref. 14.

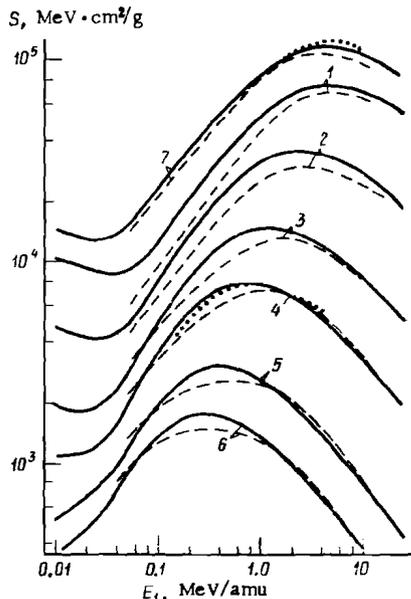


FIG. 3. 1-6) Stopping power of silver under standard conditions; 1) U ions; 2) Ag ions; 3) Ti ions; 4) Al ions; 5) C ions; 6) Be ions; 7) stopping power of aluminum for uranium ions. The stopping powers are plotted against the ion energy. Solid lines) Present calculations; dashed lines) electron stopping powers from Ref. 30; dotted lines) experimental data from Ref. 27 for Al → Ag and from Ref. 29 for U → Al.

$$\Lambda_{nu} = \frac{2M_0 v_{ef,i}^2 r_s}{(\hbar^2 v_{ef,i}^2 + \gamma^2 Z_1^2 Z_2^2 e^4)^{1/2}}. \quad (29)$$

Comparing (28) with the results derived by Lindhard et al.²⁵ in the Thomas-Fermi model, we find the following values for the numerical coefficients in the expressions r_s and L_{nu} :

$$r_s = 0.51 a_0 [Z_1^2 (\max\{Z_1 - Z_1 e_j, 1\})^{-1/2} + Z_2^2 (\max\{Z_2 - y, 1\})^{-1/2}]^{-1/2}, \quad (30)$$

$$L_{nu} = \ln[1 + \Lambda_{nu}/(1 + 0.35/\Lambda_{nu}^{1/2})] \quad (31)$$

(a_0 is the first Bohr radius). In deriving (30) we used the method proposed in Ref. 26.

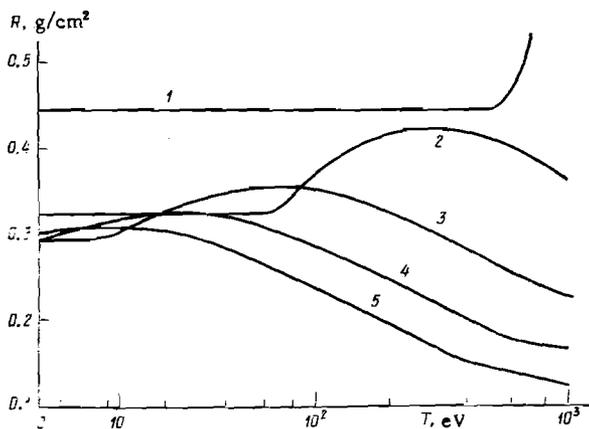


FIG. 4. Range of bismuth ions with an energy of 10 GeV/nucleus in lead versus the temperature of the lead for various densities of the lead: 1) 1134; 2) 113.4; 3) 11.34; 4) 1.134; 5) 0.1134 g/cm³.

5. EFFECTIVE CHARGE

As in most other studies we assume that the effective charge Z_{1ef} of the fast ion does not depend on the chemical composition or density of the medium but is instead determined by some universal function of Z_1 and v_1 . On the other hand, we are obliged to give up the conventional semiempirical expressions for Z_{1ef} , since they were generally derived under the assumption $Z_{1ef}^2 = S(Z_1, v_1)/S(1, v_1)$, which ignores the Z_1 dependence of the Coulomb logarithms. In calculating the stopping power S from the expressions given above, and comparing it with the experimental data of Refs. 27-29 and the tables of Ref. 30, we used the following expression for the effective charge:

$$Z_{1ef}/Z_1 = [1 + (0.62Z_1^{1/2}v_0/v_1)^{1.7}]^{-1/1.7}, \quad (32)$$

where $v_0 = e^2/\hbar$ is the Bohr velocity. The functional dependence in (32) is taken from Ref. 31. For protons we assumed $Z_{1ef} = 1$, regardless of the value of the velocity v_1 (Ref. 26). For the other elements, the values of Z_{1ef} were limited from below by the value of y_1 - the equilibrium degree of ionization of an atom (Z_1, A_1) incorporated in a medium (Z_2, A_2) with a density ρ and an electron temperature T_e .

6. CALCULATED RESULTS

The scheme described above can be compared directly with experimental data only for media with standard temperatures and densities. Figure 2 makes this comparison for protons being stopped in various metals from various parts of the periodic table. Since we have assumed $Z_{1ef} = 1$ for protons, the relative scatter of the theoretical and experimental curves in this figure is a fairly good indication of the error of our scheme in describing the spectrum of energy excitations of the medium. As expected, the errors become large at energies $E_1 \leq 0.1$ MeV/amu, corresponding to Coulomb logarithms $L_{ee} \leq 1$ and $L_{ie} \leq 1$. For other values of ρ and T_e (very different from the standard values), this error should be comparable to that in Fig. 2, if only because the proposed scheme accurately conveys the change in the average excitation energy $\hbar\bar{\omega}$ with increasing degree of ionization (Fig. 1).

Figure 3 shows the error caused in the stopping power of silver under standard conditions by using the universal formula (32) for Z_{1ef} . Since Fig. 3 is basically a comparison with the extrapolation of experimental data in accordance with Ref. 30, the following points must be mentioned. First, the tables of Ref. 30 only show the values of the electron component of the stopping power, whose contribution to S dominates at $E_1 > 0.1$ MeV/amu. Second, as recent experiments^{28,29} have shown, the tables of Ref. 30 systematically underestimate S in the energy range 0.1-10 MeV/amu as can be seen clearly from curves 7 and 4 in Fig. 3. Far from standard conditions, the error caused by using the simple formula (32) may increase slightly, as is indicated by the systematically lower values of Z_{1ef} for gases and for metals.³² Even when we take this circumstance into account, however, the error caused in S by approximation (32) is on the same order of magnitude as the error in the calculations of L_{be} and L_{fe} , and a more detailed description of Z_{1ef} within the framework of this scheme would apparently be unwarranted.

Figure 4 shows the change in the stopping power of fast ions far from standard conditions, for the case of bismuth ions with an energy of 10 GeV (47.85 MeV/amu) being stopped in a lead plasma. The values of Z_1, Z_2 , and E_1 are taken to be the same as in Ref. 7, but in contrast with the results of Ref. 7, our curves of the range versus the temperature and density of the medium are complicated and nonmonotonic. We are first struck by the following effect: At plasma densities $\geq 10^{22}$ atoms/cm³, the range initially increases 10-40% with increasing T , and only after this increase does it begin to fall off. This effect was not noted in Refs. 6 and 7, although its physical interpretation is quite transparent: With increasing temperature, the degree of ionization of the atoms of the medium increases. The transition of electrons from the discrete spectrum to the continuum affects the stopping power in two ways. First, the values of the Coulomb logarithm for free electrons, L_{fe} , are larger than L_{be} , and this difference tends to increase the stopping S . Second, the ionization is accompanied by an increase in the plasma frequency ω_p and in the average excitation frequency from the discrete spectrum, $\bar{\omega}$, i.e., by a decrease in the values of the logarithms L_{fe} and L_{be} themselves; the effect is to reduce S . In a low-density plasma, in which the difference between L_{fe} and L_{be} is large, the first of these factors is dominant, and the range falls off monotonically with increasing temperature. In a dense plasma, with no great difference between L_{fe} and L_{be} , the second factor is dominant in a certain temperature interval, and the range increases with increasing T .

Nonmonotonic $R(T)$ curves in a dense plasma were first noted and explained in Ref. 4. However, since the ionization calculations for a dense plasma in Ref. 4 were carried out by the Saha-Raizer method, which clearly exaggerates the increase in the average excitation frequency ($\bar{\omega}$) from the discrete spectrum, the reality of the hump on the $R(T)$ curves has been suspect until the present calculations. On the whole, however, the use of a physically more correct ionization equation, (1), and a more accurate method for estimating $\bar{\omega}$ in the present paper has had little effect on the qualitative behavior of the $R(T)$ curves.

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