where $\omega_* = \omega - m\Omega$, $\chi^2 = 2\Omega(2\Omega + r\Omega')$, $\tilde{k}^2 = k^2 + m^2/r^2$, and $c^2 = \gamma p_0/\rho_0$ is the square of the sound velocity.

From Eqs. (6) and (7) we have the following boundary conditions for fitting of the solutions at r = R:

$$[\xi]=\xi(R+0)-\xi(R-0)=0, \tag{8}$$

$$[p]=\xi g[\rho_0]. \tag{9}$$

Solving the system (6)-(9) in the limit $m^2 \ll k^2R^2$, we obtain the dispersion relation

$$\sum_{n=1}^{2}\rho_{0i}(-1)^n\left\{\frac{\left[(-1)^n\alpha_n+\frac{\omega_*+2m\Omega}{\omega_* R}-g/c_n^2\right]}{\left(\frac{k^2}{\omega_*^2}-\frac{1}{c_n^2}\right)}-g\right\}=0, \tag{10}$$

where

$$\alpha_n^2=k^2\left(1+\frac{g^2}{c_n^2\omega_*^2}\right)-\frac{\omega_*^2}{c_n^2}-\frac{g}{c_n^2}\frac{\omega_*+4m\Omega}{R\omega_*}+\frac{2m\Omega(\omega_*+2m\Omega)}{R^2\omega_*^2}. \tag{11}$$

Since we are interested only in the possibility in principle that gutter instability may develop, we shall consider the case of a sufficiently hot medium ($c^2 \to \infty$). Then if $\rho_{01} = \rho_0(r > R) \neq \rho_{02} = \rho_0(r < R)$, Eq. (10) will yield the growth rate

$$\gamma \approx \left\{kgA+\frac{(2-A^2)m^2\Omega^2}{k^2R^2}\right\}^{1/2}, \tag{12}$$

where

$$A=\frac{\rho_{01}-\rho_{02}}{\rho_{01}+\rho_{02}}.$$

As the expression (12) indicates, a necessary condition for instability is

$$gA>0.$$

In other words, gutter instability will set in for $\rho_{01} > \rho_{02}$ if $g = \partial\psi_0/\partial r - \Omega^2 r > 0$, and for $\rho_{01} < \rho_{02}$ if $g < 0$. The second term in the expression (12) is much smaller than the first, and serves as a small correction due to the cylindrical symmetry. Since $|A| < 1$, the effect of curvature exerts a destabilizing influence on gutter perturbations. But the same curvature effect, as Eq. (2) indicates, will exert a stabilizing influence in the case of a tangential velocity discontinuity.

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The role of the Alfvén surface in forming x-ray pulses in accreting pulsars

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A straightforward analysis of the observational evidence shows that the pulsations in the x-ray flux from the sources Her X-1 and Cen X-3 cannot be explained unless one invokes a special mechanism to generate pulses of the shape observed. In the event of disk accretion onto a magnetized neutron star, the plasma layer flowing along the Alfvén surface and spinning together with the star will regularly shield from the observer the bright spots at the magnetic poles where the energy of the infalling gas is released. This effect will produce pulsations in the hard x-ray flux observed. The plasma layer on the Alfvén surface will be a source of soft x rays, which should also pulsate weakly.

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About 10 x-ray pulsars have now been identified. Analyses of the x-ray pulse profile in the pulsar Hercules X-1 have shown[1-3] that it is modulated to considerable depth and has a rather steep edge. The same appears to be true[4] for the pulses of Centaurus X-3. In Sec. 1 of this paper we shall show that special mechanisms must be invoked to explain the shape of the x-ray pulses observed in Her X-1 and Cen X-3. In Sec. 2 we briefly discuss pulse formation mechanisms suggested previously. Calculations indicate that they would be effectively restricted to the luminosity range $L_X \lesssim (0.3\text{-}1)\cdot 10^{37}$ erg/sec (note that the luminosity of the SMC X-1 source, in which pulses also have been detected, definitely exceeds 10^{38} erg/sec). A new mechanism will be proposed, which in the case of disk accretion can yield deep modulation for an x-ray source of high luminosity $L_X > 10^{37}$ erg/sec. The operation of this mechanism will be illustrated by the best studied source, Her X-1.

1. THE NEED TO INVOKE SPECIAL PULSE FORMATION MECHANISMS

A very simple model endeavoring to account for the presence of the pulsating component in the radiation of an x-ray pulsar would be the following. The magnetic field guides the accreting matter so that it does not arrive uniformly over the whole surface of the neutron star, but only in the vicinity of hot spots. The spinning of the star, not all of whose surface is radiating but only individual bright spots, produces the pulsar phenomenon. It is assumed here that the x rays emitted reach the observer after hardly having been absorbed at all (in Sec. 2 we shall consider in some detail how an allowance for absorption at the Alfvén surface can affect the shape of the pulses received). We now show that this simplified model is incapable of explaining the shape of the x-ray pulses observed in the pulsars Her X-1 and Cen X-3. The pulses observed have sharper peaks than can be attributed to the spinning of bright spots.

To prove this claim we introduce a parameter h to describe quantitatively the modulation of the x-ray signal:

$$h[f(\varphi)]=\left[\int_0^1 f^2\,d\varphi\right]\left[\int_0^1 f\,d\varphi\right]^{-2}-1. \qquad (1)$$

Here $f(\varphi)$ denotes the signal received as a function of the phase of the pulse ($0 \le \varphi < 1$). Some examples will give an idea of the behavior of the functional h:

$$h[\text{const}]=0;\quad h[\sin(\pi\varphi)]=0.234;\quad h[\sin^2(\pi\varphi)]=0.5;\quad h[\delta(\varphi)]=\infty.$$

The value of h[f] is independent of the normalization of f:

$$h[\alpha f(\varphi)]=h[f(\varphi)]. \qquad (2)$$

The functional h has another important property:

$$h[f(\varphi)+g(\varphi)]\leqslant\max\{h[f];\ h[g]\}. \qquad (3)$$

We may infer a fundamental property from Eq. (2) and the condition (3):

The value of the functional h for the light curve of a spinning bright spot cannot exceed the greatest of all possible h values calculated separately for the spinning elementary areas whose radiant flux contributes to the radiation of the bright spot.

It remains now to compare the h values for observed pulse shapes and those predicted theoretically. According to the Her X-1 observations,[1-3] the x-ray pulse from this source has a fairly complicated profile (Figs. 1a and 1b). It may be divided into a main pulse ($0.4 \leqslant \varphi < 1$) and a subpulse ($0 \le \varphi \leqslant 0.4$). We shall subject only the main pulse to analysis, subtracting the nonpulsating component from the x-ray flux F_x and setting

$$f_e(\varphi)=\begin{cases}F_x(\varphi)-F_x(0.4), & 0.4\leqslant\varphi<1,\\ 0, & 0\leqslant\varphi<0.4.\end{cases} \qquad (4)$$

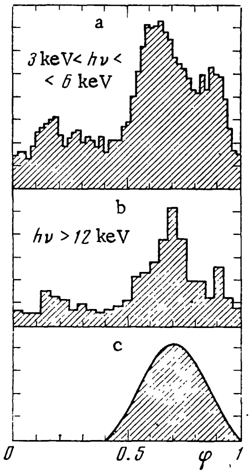

Fig. 1. X-ray flux (arbitrary units) as a function of pulse phase, $0 \le \varphi < 1$. a, b) Observational data[2] for Her X-1; c) theoretical pulse profile $f_t(\varphi)$ used for quantitative analysis of the modulation depth. The values of the function $f_t(\varphi)$ are proportional to the energy flux from a plane area of a purely scattering atmosphere at an angle θ to the normal, where θ depends linearly on φ, with $\theta = 0°, 90°$ at $\varphi = 0.7, 1$.

As a theoretical light curve $f_t(\varphi)$, let us take the radiant flux from a plane area of a purely scattering atmosphere emitting at various angles θ to the normal, with $\theta = 90°$ at phases $\varphi = 0.4$ and 1 (Fig. 1c). In a purely scattering atmosphere the sharpest directional pattern[6] will have the form1) $f(\theta) \propto (\cos\theta)\,\psi(1, \cos\theta) \approx (\cos\theta)(1 + 2\cos\theta)$ [for comparison we recall that $f(\theta) \propto \cos\theta$ in the case of a plane area radiating like a black body]. To sufficient accuracy we may approximate $f_t(\varphi)$ by

$$f_t(\varphi)=\begin{cases}\sin[\pi(\varphi-0.4)/0.6]+2\sin^2[\pi(\varphi-0.4)/0.6], & 0.4\leqslant\varphi<1,\\ 0, & 0\leqslant\varphi<0.4.\end{cases} \qquad (5)$$

Performing a numerical calculation, we find

$$\begin{cases}h[f_t]\simeq h[f_e]\simeq 1.3, & 3\,\text{keV}<h\nu<6\,\text{keV}.\\ h[f_t]<h[f_e]\simeq 1.9, & h\nu>12\,\text{keV}.\end{cases} \qquad (6)$$

In terms of the simplified model described above we should have $h[f_t] > h[f_e]$, an inequality that ought to be satisfied with some reserve; hence both the relations (6), especially the second, contradict this model. For Cen X-3 the pulse shape is less well established[4] than for Her X-1. If we regard the main pulse in this source as occupying the whole phase interval $0 \le \varphi < 1$, we obtain

$$h[f_t]\simeq 0.38<h[f_e]\simeq 0.8, \qquad 7\,\text{keV}<h\nu<11\,\text{keV}.$$

Accordingly, in order to explain the pulse shape observed in the pulsars Her X-1 and Cen X-3 we must apply considerations more refined than the mere presence of bright spots on the surface of the neutron star.

2. PARTICULAR PULSE FORMATION MECHANISMS

The various mechanisms that have been proposed to explain the shapes of x-ray pulses may be divided into three main groups.

a. Several particular physical mechanisms proposed theoretically[7-9] entail sharply beamed radiation of bright spots in a magnetic field. The presence of a narrow beam would in principle account for the pulsation of the x-ray flux from Her X-1 and Cen X-3. For example, if in Sec. 1 we replace the beam of a purely scattering atmosphere by the beam we have discussed elsewhere,[9] we would have $h[f_t] = 2.4 > h[f_e]$. However, the hypothesis of directive radiation of bright spots has the disadvantage that all beaming mechanisms cease to operate if the neutron star has a luminosity $L_X \gtrsim (0.3\text{-}1)\cdot 10^{37}\,(M_X/M_\odot)$ erg/sec, because in this event[10] the optical depth across the accretion column would exceed 1, and the diffusion of radiation across the magnetic field would isotropize the x-ray flux. In fact, Her X-1 has a luminosity $L_X \approx 10^{37}$ erg/sec; Cen X-3, $L_X \gtrsim 10^{37}$ erg/sec; and the SMC X-1 pulsar, $L_X \approx 10^{38}\text{-}10^{39}$ erg/sec.

b. Feigelson[11] has suggested an essentially different mechanism for generating x-ray pulses. The rotation of the neutron star would modulate the accretion, and as a result matter would fall onto the surface in the form of individual condensations, producing bright spots with a luminosity variable in time. But the effective operation of this mechanism is also restricted to the range of low x-ray luminosities $L_X \leqslant (0.3\text{-}1)\cdot 10^{37}$ $(M_X/M_\odot)$ erg/sec, since if L_X were higher the infalling matter would be halted by radiation pressure, and in the magnetic duct there would form a column of slowly settling material (as we have discussed in fuller detail[10]) that would completely damp the pulsed character of the radiation. This restriction will hold whatever the ratio between the pulse length and the pulsation period, provided only that the pulsation period is much shorter than the relaxation time of the settling column, which is 10^2-10^4 sec.

c. We have proposed elsewhere,[10] followed by McCray and Lamb,[12] a pulse formation mechanism that can operate efficiently for a neutron star of high luminosity radiating by disk accretion. The principle behind this mechanism is the following. The strong intrinsic magnetic field of the neutron star will disrupt the disk accretion pattern and compel the gas to flow off toward the magnetic funnels along the so-called Alfvén surface (Fig. 2). Since the magnetic field in the plane of the disk will not be axisymmetric, one would expect the plasma flow to cover only part of the Alfvén surface. The layer of matter on the Alfvén surface will spin at the same angular velocity as the neutron star itself, and will periodically shield from the observer the x-ray source on its surface near the magnetic poles. The modulation depth of the x-ray flux received by a distant observer will be determined by the optical thickness of the gas layer at the Alfvén radius, and as we have shown,[10] this thickness may considerably exceed 1.

Taking the Her X-1 source as an example, we shall here demonstrate that our proposed model can give a satisfactory explanation of the chief qualitative peculiarities of the pulses observed. We shall regard all the accreting material outside the Alfvén surface as concentrated in the disk, an interpretation supported by the observations[13] in the case of Her X-1. If matter were to fall onto the Alfvén surface from all sides, as appears to happen in detached binary systems with strong stellar wind,[14] then the modulation of the x-ray signal might be much weaker. Substantial difficulties arise if one attempts to outline qualitatively the flow around the Alfvén surface in this case, and we shall not consider it here, even though it is this latter case that probably applies to long-period pulsars.

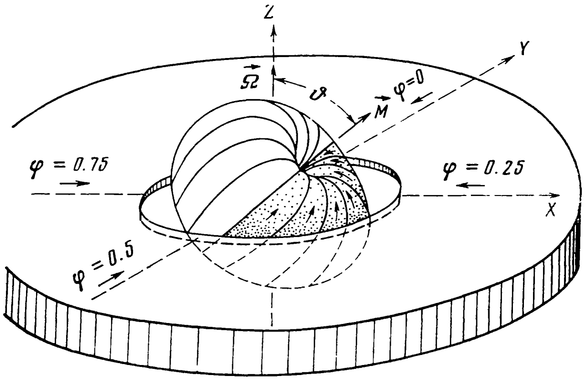

Fig. 2. Qualitative configuration of flow around the Alfvén surface in the case of a magnetic dipole with axis M forming an angle ϑ relative to the rotation axis Ω of the neutron star. The phase φ of observation is to be measured from a point selected in accord with Fig. 1. The little studied transition zone between the disk and the gas stream on the Alfvén surface is not shown in the figure.

The observational evidence is consistent with a model wherein the line of sight would intersect the orbit plane of the Her X-1 (HZ Her) system at an angle of $\approx 5°$-$20°$ (the orbital inclination $i \approx 70°$-$85°$). We shall regard the plane of the disk as coincident with the orbit plane, the rotation axis of the neutron star as normal to the plane of the disk, and the angle ϑ between the magnetic-dipole axis and the rotation axis as different from zero. Since the main pulse lasts about one-half the period $p = 1.24$ sec (see Figs. 1a, 1b), we shall assume that the matter flowing toward the upper magnetic funnel (Fig. 2) shields the neutron star from us during the other half of the period $(0 \leqslant \varphi \leqslant 0.5)$.

A comparison with the observations shows that at phases $0 < \varphi < 0.5$ the hard x-ray flux (emitted by the surface of the neutron star) does not drop to zero but only decreases by a few times. To explain this fact, let us consider two alternative possibilities: a) the shielding layer has an optical depth $\tau_T \gg 1$ with respect to scattering; b) $\tau_T \approx 1$-3. In case (a) the flux of hard radiation at phases $0 < \varphi < 0.5$ might be attributable to reflection from the inner surface of the gas layer flowing toward the lower magnetic pole. Calculations demonstrate[15] that the albedo of the heated reflecting surface may reach values of ≈ 0.3-0.5. Figure 3a schematically illustrates the light curve. Simple estimates show that the most favorable values in this case are $\vartheta \approx 60°$-$70°$, but even then the intermediate maximum (at $\varphi = 0.25$) would be at least 10-20 times less high than the primary maximum $(\varphi = 0.75)$, which poorly fits the observations (Fig. 1b). In case (b) the gaseous envelope would simply emit some of the hard radiation. The secondary maximum at $\varphi = 0.25$ can be explained by a decrease in τ_T in that direction, since at $\varphi = 0.25$ the line of sight intersects the portion of the Alfvén surface closest to the magnetic funnel, where the matter is accelerated by gravity to great velocities. Qualitatively the light curve has the same appearance as in the preceding case (Fig. 3a). In order to establish it quantitatively, one must find the velocity field on the Alfvén surface.

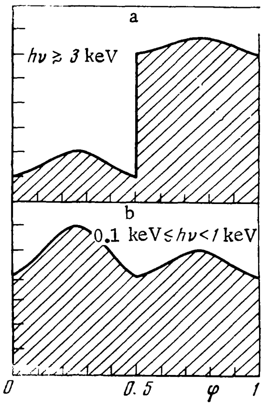

Fig. 3. Qualitative form of the x-ray pulse shape in two energy ranges for models of the type discussed in the text.

As has been noted previously,[10,12] the layer of gas on the Alfvén surface should reemit the energy falling on it in the soft x-ray range $0.1 \text{ keV} < h\nu < 1 \text{ keV}$ (further details are given in another recent communication[16]), and the luminosity of the Alfvén surface in this energy range may considerably exceed the luminosity of the neutron star itself. Moreover, by applying simple geometrical considerations we can readily draw a schematic light curve for the soft range (Fig. 3b). There are several observational implications:

a. The amplitude of the pulsations in the soft range (they would arise solely from the change in the apparent area of the radiating surface as the magnetosphere rotates) should be substantially smaller than in the hard range.

b. The contribution of the second harmonic will increase sharply in the soft range, since the light curve forms a double wave with unequal (for $i < 90°$) maxima (see Fig. 3b).

c. Primary maximum in the soft x-ray range should be shifted by 180° in phase relative to the primary maximum in the hard range.

All these characteristic properties of the soft x-ray pulsations are in good agreement with the observations.[3]

One feature of the model we have discussed is the substantially lower transmissivity and reflectivity of the gas layer on the Alfvén surface in the range $1 \text{ keV} < h\nu < 6 \text{ keV}$ compared to the range $h\nu \gtrsim 6$ keV. This rise in the opacity takes place at a temperature $T \lesssim 2 \cdot 10^6\,°\mathrm{K}$, when the heavy elements are not yet very highly ionized, due to the strong dependence of the photoabsorption cross section on frequency ($\sigma_{\mathrm{ph}} \propto \nu^{-3}$). On the one hand, this circumstance affords a natural explanation of the very steep rising and falling edges of the main pulse[2,3] (see Fig. 1a) observed for $h\nu < 6$ keV, which are difficult to understand in terms of earlier models.[7-9] On the other, it should produce a deeper modulation of the x-ray flux in the range $1 \text{ keV} < h\nu < 6 \text{ keV}$ compared to the hard range, which, however, is not in good accord with the available observational evidence (compare Figs. 1a, 1b), and appears to pose a difficulty for our present model.

The optical depth of the various parts of the gas layer on the Alfvén surface may undergo random variations with time, and this process should be manifested as different features in the pulse shape. Nevertheless, the most general and distinctive properties of the x-ray pulses (Fig. 3) should remain unchanged, as appears to be observed for the Her X-1 source.[1-3]

As an observational test of the proposed model, one might take advantage of the circumstance that the modulation depth in the hard range ($h\nu > 1$-3 keV) should increase with the x-ray luminosity of the neutron star, accompanied by a growth in the rate of disk accretion. It is worth noting that in all the other models discussed above the modulation depth should, on the contrary, decrease toward higher luminosities. We are not able to apply this test to Her X-1, since its total luminosity, despite the presence of the 35-day cycle, evidently does not change.

A matter of considerable interest from the observational point of view is the polarization of the x rays. For our present model the soft x rays (0.1-1 keV) should possess weak (several percent) linear polarization, since the magnetic field on the Alfvén surface, where this radiation originates, is not strong, and the polarization would arise mainly from the dominant contribution of scattering to the opacity. Hard radiation ($h\nu > 2$ keV) should be polarized by processes taking place in the neighborhood of the magnetic poles of the neutron star, since the Alfvén surface merely plays a shielding role in this case. For a low luminosity $L_X \lesssim 3 \cdot 10^{36}\,(M_X/M_\odot)$ erg/sec, the hard x rays should, in accord with the beaming mechanisms that have been proposed,[7-9] be characterized by strong linear and circular polarization (further discussions have been given by Gnedin and one of us[17] and by Rees[18]). But for high luminosity $L_X > 10^{37}\,(M_X/M_\odot)$ erg/sec, the operation of these mechanisms will cease; the polarization will become weaker and should amount to ≈ 1-10%.

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1) Here $\psi(\alpha, \mu)$ is the Ambartsumyan function.[6]

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