The magnetopause of an accreting neutron star

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The shape of the Alfvén surface separating the magnetosphere of a neutron star from the stream of accreting plasma and the velocity and scattering optical depth of the gas layer flowing along the Alfvén surface are calculated. Calculations are made both for spherically symmetrical fall of the gas and for disk accretion. It is shown that the fall of matter from the Alfvén radius to the surface of the neutron star is possible only with the condition of "freezing" of the plasma into the magnetic field of the star.

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Most galactic x-ray sources evidently consist of neutron stars onto which is falling gas flowing from a neighboring normal star. Judging from the results of observations of radio pulsars,[1] neutron stars can possess a dipole magnetic moment $\mathcal{M} \approx 10^{30}$ G·cm^3, which corresponds to a field $H = \mathcal{M}/R_{ns}^3 \approx 10^{12}$ G at the equator. The interaction of the stream of accreting plasma with the magnetic field of the neutron star is an important and a not fully solved problem in the theory of x-ray sources. In the present article, devoted to one aspect of this problem, the shape of the Alfvén surface separating the magnetosphere of the neutron star from the undisturbed stream of accreting plasma is studied and the main parameters of the gas layer flowing along this surface are calculated. In accordance with geophysical terminology, the thin layer of matter through which the currents which shield the field of the star flow, and which itself forms the Alfvén surface, has come to be called the magnetopause.

The main simplifying assumptions used in the work come down to the following: 1) the neutron star is assumed to be nonrotating; 2) the accreting plasma is an ideal conductor; 3) only the inertia and weight of the falling matter are taken into account (the pressure p = 0, dust approximation); 4) the magnetic field of the neutron star is a purely dipole field and the dipole magnetic moment equals $\mathcal{M}$.

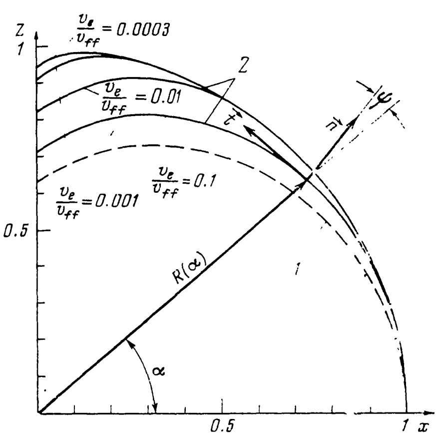

FIG. 1. Geometry of the Alfvén surface. Solid lines: shape of Alfvén surface normalized to a unit equatorial radius in the case of disk accretion with different ratios of meridional velocity v_e at the equator to free-fall velocity v_{ff}. Dashed line: shape of Alfvén surface in the case of radial accretion. 1) Radial accretion; 2) disk accretion.

The assumption 1) limits the region of applicability of the results obtained below to slowly rotating neutron stars. However, the discovery of a whole series of long-period x-ray pulsars indicates that this region includes a rather large number of objects. The assumption 2) imposes no limitations, since the conductance of the plasma under the conditions of interest to us is actually very high. The assumption 3) is a serious simplification and undoubtedly leads to the loss of a considerable part of the information on the parameters of the plasma in the magnetopause; however, such properties as the shape of the Alfven surface, the configuration of the field in the magnetosphere, the scattering optical depth, and the velocity of the matter in the magnetopause depend weakly on it (see Sec. 1d). Remember that knowledge of the optical depth of the gas layer at the Alfvén surface has fundamental importance in a discussion of some mechanisms of pulse formation and the generation of soft radiation[1-3] of x-ray sources.

1. SPHERICALLY SYMMETRIC ACCRETION

In this case the dipole axis OZ (see Figs. 1 and 2) is the axis of symmetry. The spherical radius r, magnetic latitude α (measured from the plane of the magnetic equator, which is perpendicular to the OZ axis), and the azimuth Φ are used below as the independent variables. Outside the Alfvén surface the velocity and density of the accreting matter are, respectively,

$$u=(2GM/r)^{1/2},\quad \rho=\dot{M}/4\pi u r^2. \tag{1}$$

Here M is the mass of the neutron star and $\dot{M}$ is the accretion rate. The luminosity

$$L_x=GM\dot{M}/R_{ns} \tag{2}$$

of the x-ray source, a quantity equivalent to $\dot{M}$, is often used below in place of it (R_{ns} is the radius of the neutron star).

a) Basic equations. The problem consists in finding the equation for the surface $r = R(\alpha)$ separating the two regions: the outer region $r > R(\alpha)$, where the magnetic field strength H = 0 and the accreting matter are described by Eq. (1), and the inner region $r < R(\alpha)$, where the plasma density $\rho = 0$ while the magnetic field $H \neq 0$ (see Figs. 1 and 2). In order that the problem be self-consistent one must assume that a layer of matter with a surface density $\sigma(\alpha)$ and a meridional velocity $v(\alpha)$ flows along the Alfvén surface $r = R(\alpha)$. An electric current with a surface density $i(\alpha)$ flows through this layer in the azimuthal direction. One must write four equations to find the four functions $R(\alpha)$, $\sigma(\alpha)$, $v(\alpha)$, and $i(\alpha)$. The first, the simplest one, is the continuity equation

$$4\pi R\sigma v\cos\alpha=\dot{M}\sin\alpha. \tag{3}$$

Two more equations follow from the law of conservation of momentum:

$$\frac{d}{dt}(v\cdot\mathbf{t})=\frac{v}{R}\cos\psi\frac{d}{d\alpha}(v\cdot\mathbf{t})=\mathbf{f}_r+\mathbf{f}_g+\frac{p_n}{\sigma}\mathbf{n}, \tag{4}$$

where **t** and **n** are unit vectors directed along the tangent and along the normal to an element of the Alfvén surface and ψ is the angle between **n** and the radius vector of the surface element (see Fig. 1). Substituting the values of the acceleration of gravity

$$\mathbf{f}_g=\frac{GM}{R^2}(\mathbf{t}\sin\psi-\mathbf{n}\cos\psi) \tag{5}$$

and the pressure of the incoming stream

$$\sigma\mathbf{f}_r=\rho u\cos\psi[(u\sin\psi-v)\mathbf{t}-u\cos\psi\mathbf{n}] \tag{6}$$

into Eq. (4) and using the geometrical relations

$$\operatorname{tg}\psi=-\frac{1}{R}\frac{dR}{d\alpha},\quad \frac{d\mathbf{t}}{d\alpha}=-\mathbf{n}\left(1+\frac{d\psi}{d\alpha}\right), \tag{7}$$

we obtain the differential equation for the velocity

$$v\frac{dv}{d\alpha}=-\frac{GMR_\alpha}{R^2}-v\operatorname{ctg}\alpha\left[v+R_\alpha\left(\frac{2GM}{R^3+RR_\alpha^2}\right)^{1/2}\right] \tag{8}$$

and the expression for the magnetic forces acting on the matter in the magnetopause

$$p_n=H^2/8\pi=2\pi i^2/c^2=\frac{\dot{M}}{4\pi}\left\{\frac{(2GM/R)^{1/2}}{R^2+R_\alpha^2}+\frac{\operatorname{tg}\alpha}{v}\frac{GM}{R^2(R^2+R_\alpha^2)^{1/2}}-v\operatorname{tg}\alpha\frac{R+2R_\alpha^2/R-R_{\alpha\alpha}}{(R^2+R_\alpha^2)^{3/2}}\right\}. \tag{9}$$

Here and below the first and second derivatives of the function $R(\alpha)$ are designated as R_α and $R_{\alpha\alpha}$, respectively. On the right side of (8) the first term describes the acceleration of the matter in the gravitational field of the neutron star while the second and third terms allow for the direct transfer of momentum from the falling gas. On the right side of (9) the first term is the frontal pressure of the incoming stream, the second is the weight of the matter in the magnetopause, and the third is its centrifugal acceleration.

And finally, the fourth and last equation (the equation of magnetostatics) must express the fact that the currents flowing in the magnetopause fully shield the field of the neutron star (the dipole field). Since the problem is axisymmetric, it is convenient to use the Φ-component of the vector potential $A(r, \alpha)$ to describe the magnetic field. The vector potential of the surface of rotation $r = R(\alpha)$ for $r > R(\alpha)$ can be represented in the form

$$A(r,\alpha)=\frac{2\pi}{c}\sum_{n=1}^{\infty}\frac{P_n^1(\sin\alpha)}{n(n+1)r^{n+1}}\times\int_{-\pi/2}^{\pi/2}i(\alpha')R^{n+1}(\alpha')P_n^1(\sin\alpha')\cos\alpha'\sqrt{R^2+R_{\alpha'}^2}\,d\alpha'. \tag{10}$$

Here $P_n^1(x)$ are associated Legendre functions of the first kind. Equating (10) to the expression for the dipole vector potential

$$A_d(r,\alpha)=\frac{\mathscr{M}\cos\alpha}{r^2}, \tag{11}$$

taken with the opposite sign, and allowing for the parity of the functions $i(\alpha)$ and $R(\alpha)$, we find that the sought fourth equation is equivalent to the following system of integral equations:

$$\left\{\begin{aligned}&\frac{2\pi}{c}\int_0^{\pi/2}i(\alpha)R^2\sqrt{R^2+R_\alpha^2}\cos^2\alpha\,d\alpha=-\mathscr{M},\\&\int_0^{\pi/2}i(\alpha)R^{2n+2}\sqrt{R^2+R_\alpha^2}P_{2n+1}(\sin\alpha)\cos\alpha\,d\alpha=0,\qquad n=1,2,\ldots\end{aligned}\right. \tag{12}$$

Thus, to find the four functions $\sigma(\alpha)$, $v(\alpha)$, $i(\alpha)$, and $R(\alpha)$ we have four equations: (3), (8), (9), and (12).

b. Method of solution. Equations (12) were solved by a numerical method analogous to that used in Refs. 4 and 5. The essence of this method consists in the following. The function $R(\alpha)$ is sought in the form

$$R(\alpha)=R_e\left[1-\sum_{k=1}^{N}c_k(2\alpha/\pi)^{2k}\right]; \tag{13}$$

The N coefficients $\{c_k\}$ are chosen in such a way that the values of the N integrals of (12) are reduced to zero. In the process the 2N multipole moments of the magnetopause (beginning with the quadrupole moment) are reduced to zero. The value of R_e is found from the first equation in (12).

One of the peculiarities of the given problem is the singular behavior of the expression (9) near $\alpha = \pi/2$, obligating an infinitely high surface density $\sigma(\alpha)$ of matter at this point. In contrast to Refs. 4 and 5, therefore, the N coefficients $\{c_k\}$ were chosen in such a way as to reduce to zero $N-1$ of the integrals of (12) and liquidate the singularity near $\alpha = \pi/2$ in (9). In other words, an additional condition was imposed on the surface $r = R(\alpha)$ at the point $\alpha = \pi/2$, the physical meaning of which consists in the following: Near $\alpha = \pi/2$ the matter with an infinitely high surface density ceases to "feel" the magnetic field and moves only under the action of the forces of inertia and gravity, i.e., along the arc of an ellipse. Owing to this circumstance the curve $r = R(\alpha)$ (the intersection of the Alfvén surface with the meridional plane) intersects the axis of symmetry at a finite angle and the magnetic field at the point of intersection is reduced to zero. Remember that in the problem with a constant external pressure[5,6]

$R(\alpha)$ is tangent to the axis of symmetry in a power-law fashion with a finite magnetic field strength at the point of tangency. Such a difference allows one to be confident that the value of $R_c \equiv R(\pi/2)$ is known much more accurately in the problem under consideration than in the problem with a constant external pressure.

The system of N transcendental equations for finding the $\{c_k\}$ was solved by a modified Newtonian method; the integrals were calculated using Gaussian quadrature equations with 40 nodes. The differential equation (8) was solved by the Runge-Kutta method.

c) Results of the calculations. In the problem under consideration all the physical quantities can be normalized in such a way that the dependence on the accretion rate and the characteristics of the neutron star enter only into the normalizing constants. As such for the radius, the velocity, and the scattering optical thickness we take

$$r_0=\mathscr{M}^{4/7}[2\dot{M}\sqrt{2GM}]^{-2/7}=2\cdot 10^8 H_{12}^{4/7} L_{37}^{-2/7} R_6^{10/7} (M/M_\odot)^{1/7}\ \text{cm}, \quad (14)$$

$$v_0=\sqrt{2GM/r_0}=1.15\cdot 10^9 L_{37}^{1/7} H_{12}^{-2/7} R_6^{-5/7} (M/M_\odot)^{3/7}\ \text{cm/sec}, \quad (15)$$

$$\tau_0=\varkappa_T \dot{M}/2\pi\sqrt{2GMr_0}=2.1\cdot 10^{-2} L_{37}^{8/7} R_6^{2/7} H_{12}^{-2/7} (M/M_\odot)^{-11/7}. \quad (16)$$

Here L_{37} is the luminosity L_X in units of 10^{37} erg/sec, H_{12} is the magnetic field at the pole of the neutron star in units of 10^{12} G, R_6 is the radius of the neutron star in units of 10^6 cm, and $\varkappa_T = 0.4\ \text{cm}^2/\text{g}$ is the opacity of the matter relative to Thomson scattering. We note that the radius r_0 is chosen so that in the equatorial plane the energy density $\mathscr{M}^2/8\pi r_0^6$ of the undisturbed dipole field at this radius equals the dynamic pressure $\rho(r_0)u^2(r_0)$ of the accreting plasma; τ_0 is none other than the optical depth of the undisturbed stream of accreting plasma from $r = r_0$ to $r = \infty$.

The shape of the Alfvén surface is shown in Fig. 2. The main parameters of this surface with N = 10 are summarized in Table I. A further increase in N evidently makes no sense, since already with a change in N from 6 to 10 the main characteristics of the Alfvén surface vary by an amount on the order of 10^{-4}. In parallel with the problem described above we also calculated the shape of the magnetopause (dashed line in Fig. 2) for a dipole immersed in a perfectly conducting fluid with a constant pressure p_0 [in this case the normalization radius equals $r_0 = \mathscr{M}^{1/3}(8\pi p_0)^{-1/6}$]. This was done for a control, on the one hand, since such a problem was solved long ago,[4,7] and for comparison, on the other. The parameters of the corresponding surface are given in parentheses in Table I. It is seen from Fig. 2 that the two surfaces are rather close to one another: $\Delta R/R < 6\%$. This indicates that the detailed law of distribution of the normal pressure over the Alfvén surface has a weak effect on its shape. The absolute value of the pressure, which determines the characteristic size of the Alfvén surface (such as the equatorial radius R_e), is more important.

As the calculations show, the scattering optical depth $\tau(\alpha) = \varkappa_T \sigma(\alpha)$ of the magnetopause hardly varies in the region of angles $|\alpha| \leqslant 60°$ (see Fig. 3). The optical depth $\alpha = 0$ is easily expressed through the curvature of the Alfvén surface:

$$\tau_e \equiv \tau(0) = 0.5\tau_0\sqrt{r_0/R_e}\,(\sqrt{1+\pi^2/2c_1}-1). \quad (17)$$

This equation also lets one easily estimate what contribution to the total pressure is made by the weight of the matter concentrated in the magnetopause. At $\alpha = 0$ the ratio of the weight force $GM\sigma/R_e^2$ to the dynamic pressure ρu^2 is 1.370; this ratio grows monotonically with an increase in α.

The velocity of the gas flowing along the Alfvén surface is equal to zero at $\alpha = 0$ and grows linearly with greater distance from the equator:

$$v = v_0\sqrt{r_0/R_e}\,[\sqrt{1+\pi^2/2c_1}-1]^{-1}\alpha + O(\alpha^2). \quad (18)$$

A comparison with the purely potential variation in the velocity $v(\alpha) = [2GM/R(\alpha) - 2GM/R_e]^{1/2}$ (dashed line in Fig. 3) shows that the dynamic pressure (6) of the falling matter slows the flow of gas along the Alfvén surface by no more than 6%. The highest velocity $v_c \equiv v(\pi/2)$ is reached at the axis of symmetry $\alpha = \pi/2$, where the magnetopause has the shape of a conical funnel. The values of v_c and of the height $R_c \equiv R(\pi/2)$ of the cusp of the funnel are given in Table I.

The configuration of the magnetic field below the Alfvén surface is of independent interest. In the immediate vicinity of the surface itself the field is parallel to it and can be estimated from the known value of the external pressure from Eq. (9). Thus, at the equator ($\alpha = 0$, $r = R_e$) the field is strengthened by

$$\Gamma \equiv H/H_d = R_e^3\sqrt{8\pi p_n}/\mathscr{M} = 2^{-1/2}(R_e/r_0)^{7/4}(1+\sqrt{1+\pi^2/2c_1})^{1/2} = 2.7778 \quad (19)$$

times. We note that in the problem with a constant external pressure $p_n = p_0$ the quantity $\Gamma = (R_e/r_0)^3 = 2.827$ (also see Table I). At $r < R_e$ one can use a multipole expansion for the field created by the currents in the magnetopause. The expression for the Φ-component of the vector potential $A(r, \alpha)$ in this region has the form

$$A(r,\alpha) = \frac{\mathscr{M}\cos\alpha}{r^2}\left\{1 - \frac{1}{2}\left(\frac{R_e}{r_0}\right)^{7/4}\left(\frac{r}{R_e}\right)^3 \times\left[d_0 + \sum_{i=1}^{\infty} d_i\left(\frac{r}{R_e}\right)^{2i}\frac{P_{2i+1}^1(\sin\alpha)}{\cos\alpha}\right]\right\}. \quad (20)$$

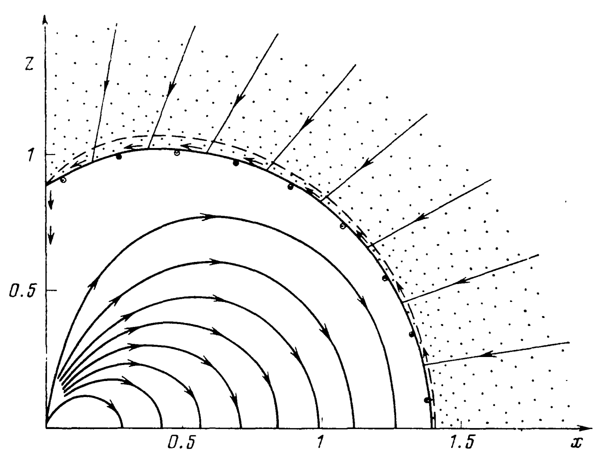

FIG. 2. The magnetopause in the case of radial accretion (solid line) and in the case of a constant external pressure (dashed line). The deformed magnetic field lines are shown. All the distances are normalized to r_0.

TABLE I

R_e/r_0	1.4011 (1.4140)	c_1	0.379957 (0.296186)	c_6	−0.090945 (0.061747)	d_0	1.2924 (0.7963)
R_c/r_0	0.8830 (0.8937)	c_2	−0.013005 (0.025630)	c_7	0.170134 (−0.169834)	d_1	0.1971 (0.0924)
τ_e/τ_0	1.1574	c_3	0.004696 (0.013822)	c_8	−0.193718 (0.305427)	d_2	0.1167 (0.0510)
v_c/v_0	0.6348	c_4	−0.005470 (0.009460)	c_9	0.121170 (−0.288383)	d_3	0.1085 (0.0459)
Γ	2.7778 (2.8270)	c_5	0.029881 (−0.006282)	c_{10}	−0.032927 (0.120174)	d_4	0.1274 (0.0530)

The values of the first several coefficients d_i are presented in Table I. In the problem with a constant external pressure the factor $(R_e/r_0)^3$ should stand in place of $(R_e/r_0)^{7/4}$ in (20). The magnetic field lines calculated in accordance with (20) are shown in Fig. 2.

d) The fall from the Alfvén radius. Up to now it has been tacitly assumed that all the accreting plasma ultimately falls onto the neutron star. Within the framework of the dust approximation we could assume that the matter flows down along the Alfvén surface to the two funnels ($\alpha = \pm\pi/2$), from which it falls to the poles of the star along two infinitely thin filaments. Below we will dwell briefly on how this process can take place with allowance for the pressure of the plasma and other real processes.

One of the most attractive models was constructed in Ref. 5. According to the authors of this report, the flow of plasma along the Alfvén surface is unstable. In the entire region of latitudes $|\alpha| \lesssim 75°$ the matter continuously penetrates into the magnetosphere in the form of separate clusters. These clusters are evidently also unstable; their breakup continues until the plasma is "frozen into" the magnetic field of the magnetosphere, after which the matter falls freely along the field lines toward the magnetic poles. First of all we note that such a complication of the flow pattern (resulting in the magnetopause having a finite geometrical thickness) should not appreciably affect the results of the present calculations, since even after the penetration below the Alfvén surface the centrifugal forces and the weight of the matter continue to have a strong effect on the magnetosphere of the neutron star. From this aspect the present calculations on the determination of the main parameters of the magnetopause are more correct than those made in Ref. 5, where the weight and velocity of the matter in the magnetopause were not taken into account.

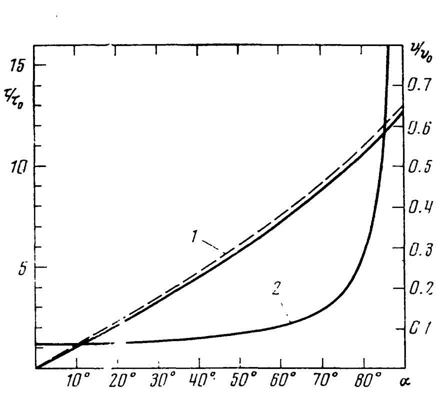

FIG. 3. Scattering optical depth (in units of τ_0) and velocity (in units of v_0) of the matter of the magnetopause in the case of radial accretion. Dashed line: velocity of the matter found with allowance for gravitational forces only; 1) velocity; 2) optical depth.

One of the most important parameters in the theory of x-ray sources is the size of the hot spots at the magnetic poles of the neutron star. By assigning an equatorial radius r_e of the magnetic field lines at which the "freezing in" of the matter takes place, one can calculate the angular size θ_c of the spot "cut out" by these field lines on the surface $r = R_{ns}$ of the neutron star (the radius of the spot equals $R_{ns}\theta_c$). Such calculations were made using the expansion (20), and the results are presented in Fig. 4. The values of θ_c estimated using the undisturbed dipole field of the neutron star [Eq. (25) in Ref. 8] are shown in this figure by dashed lines.

For a small depth of penetration of the plasma into the magnetosphere, $R_e - r_e \ll R_e$, the expansion (20) is inapplicable, and to find θ_c we used the equation

$$\theta_c = \left[\Gamma \frac{R_{ns}}{R_e}\left(1 - \frac{r_e}{R_e}\right)\right]^{1/2}, \qquad (21)$$

which is correct in the limit $\delta = 1 - r_e/R_e \ll 1$. The coefficient Γ is defined in (19). Equation (21) was obtained in the following way. From the equation for the field lines

$$Ar\cos\alpha = \text{const} \qquad (22)$$

it follows that $A(r, \alpha) = 0$ at the Alfvén surface. The first term of the expansion of $rA(r, 0)$ in the equatorial plane by powers of $\delta = 1 - r/R_e$ is expressed through the magnetic field strength $H = +(1/r)\partial(rA)/\partial r$ at the point $r = R_e$, $\alpha = 0$, i.e., through the normal pressure of the matter of the magnetopause. Substituting it into (22) and neglecting the effect of the currents of the magnetopause at $r = R_{ns} \ll R_e$, we obtain (21). Numerical calculations show that for $\delta > 0.15$ one can use the expansion (20) with sufficient accuracy ($\sim 1\%$), while for $\delta < 0.15$ Eq. (21) works well.

One can imagine an alternative possibility which differs fundamentally from the situation analyzed in Ref. 5: A diamagnetic plasma pushes the magnetic field apart and, not being "frozen-in," falls to the magnetic poles along the two narrow channels. The following fact, it would seem, testifies in favor of this possibility: If in the dust approximation one allows for the weight of the matter and neglects the centrifugal force, then the conical funnels at the poles descend almost to the surface of the neutron star, $\lim_{\alpha\to\pi/2} R(\alpha) = 0$; the angular radius $\theta(R) = \pi/2 - \alpha(R)$ of a funnel varies

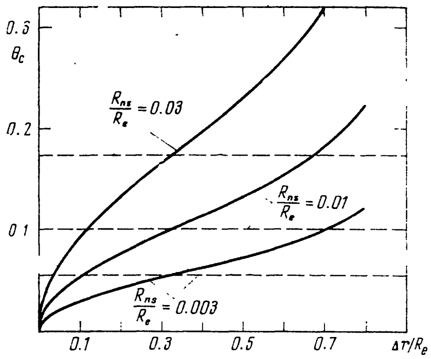

FIG. 4. Angular size of the hot spots at the magnetic poles of a neutron star of radius R_{ns} as a function of the "freezing-in" depth $\Delta r/R_e$ at the equator $r = R_e$ of the Alfvén surface in the case of radial accretion. The values estimated from the purely dipole field are marked by dashed lines.

according to the law

$$\theta=\theta_0 \exp\left[-\frac{16}{7}\left(\frac{r_0}{R}\right)^{7/2}\right]. \tag{23}$$

Such a statement of the problem is incorrect in the dust approximation, however: Allowance for the centrifugal forces leads to the fact that the funnel cusp $R_c \approx 0.6R_e$ (see Table I). Below we will show that such a possibility cannot be realized even with allowance for the gas pressure. For this we assume the opposite, i.e., we assume that at $r < R_c$ the plasma falls along a narrow axisymmetric channel with a cross-sectional radius $a(r) \ll r$. The optical depth across the channel in its upper part ($r \approx R_c$) with a gas velocity $v(R_c) \approx 0.3\sqrt{2GM/R_c}$ (see Figs. 2 and 3) is

$$\tau_t=\varkappa_T \rho a=0.7 L_{37} R_6 \left(\frac{M_\odot}{M}\right)^{3/2}\left(\frac{2\cdot 10^8\ \mathrm{cm}}{R_c}\right)^{1/2}\left(\frac{0.1R_c}{a(R_c)}\right).$$

The pressure of the plasma in the channel must balance the lateral pressure $H^2/8\pi \approx \mathscr{M}^2/2\pi r^6$ of the magnetic field. From this condition it necessarily follows that τ_T grows with a decrease in r, while the radiation pressure $p_r = \varepsilon/3$ considerably exceeds the gas pressure. The latter fact means that the radiant energy density ε is constant across the channel cross section, and the flux $F_\perp = \varepsilon c/4$ of the radiant energy emitted from a unit of lateral surface of the accreting column is brought to the surface by a turbulent mechanism (the existence of turbulence in the accretion channel is fully admissible since the plasma does not contain a magnetic field). The equations describing the motion of the plasma in such a channel are discussed in detail in Ref. 9 and have the form

$$\left\{\begin{aligned} &\rho v a^2=-\dot{M}/2\pi,\\ &v\frac{dv}{dr}+\frac{1}{3\rho}\frac{d\varepsilon}{dr}+\frac{GM}{r^2}=0,\\ &\frac{4}{3}\varepsilon\frac{d(va^2)}{dr}+(va^2)\frac{d\varepsilon}{dr}+\frac{\varepsilon c a}{2}=0.\end{aligned}\right. \tag{24}$$

Here $v < 0$ is the falling velocity of the matter and c is the velocity of light. In contrast to the situation analyzed in Ref. 9, here the unknown functions are $\rho(r)$, $v(r)$, and $a(r)$, whereas the radiant energy density is assigned:

$$\varepsilon=\frac{3H^2}{8\pi}=\frac{3}{2\pi}\frac{\mathscr{M}^2}{r^6}. \tag{25}$$

After the introduction of the dimensionless quantities

$$\xi=r/r_0,\ y=v^2/v_0^2,\quad z=(a/r_0)\,y^{1/4}\xi^{-9/4}$$

the system (24) is converted to the form

$$\left\{\begin{aligned} &dy/d\xi=24z^2\xi^{-5/2}-\xi^{-2},\\ &dz/d\xi=\beta y^{-1/4}\xi^{-9/4},\end{aligned}\right. \tag{26}$$

where the constant $\beta = 3c/16v_0$. Taking $R_c = r_0$ for simplicity (actually, $R_c \approx 0.9r_0$), we assign the boundary conditions in the form

$$y(1)=y_0<1,\quad z(1)=z_0<1. \tag{27}$$

Since the square of the gas velocity is $y < 1/\xi$ in the entire accretion channel ($\xi < 1$), from the second equation of (26) we at once conclude that the function $z(\xi)$ is reduced to zero at $\xi = \xi_0 > \beta/(\beta + z_0) \gtrsim 0.3$. This means that a diamagnetic plasma cannot fall onto a gravitating magnetic dipole. A more detailed analysis of the system (26) with the boundary conditions (27) shows that accretion onto a neutron star without penetration of the plasma into the magnetosphere (i.e., without "freezing-in") is possible only when the radius R_c of the Alfvén surface essentially coincides with the radius R_{ns} of the neutron star, i.e., an extended magnetosphere is simply absent. We note that this conclusion directly contradicts that form of the magnetopause which was obtained in Ref. 10. The error com-

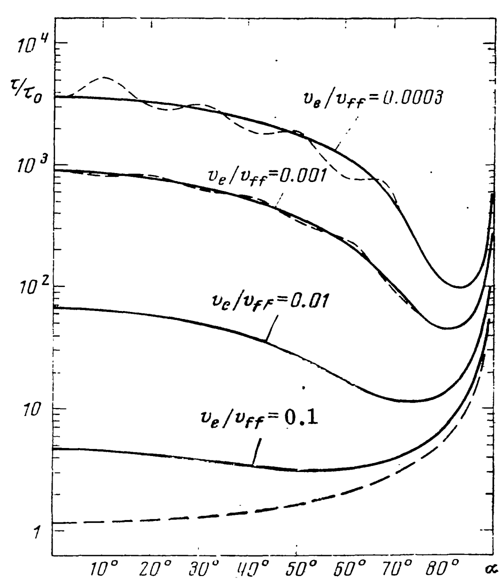

FIG. 5. Scattering optical depth of the magnetopause (in unts of τ_0) in the case of disk accretion with different ratios of the meridional velocity v_e at the equator to the free-fall velocity v_{ff}. The optical depth of the magnetopause in the case of radial accretion is shown by a dashed line. The dashed lines for $v_e/v_{ff} \le 10^{-3}$ are the results of calculations when the series (13) is cut off at N = 8.

TABLE II

$v_e/\sqrt{2GM/R_e}$	0.1	0.01	0.001	0.0003
R_e/r_0	1.1772	0.6124	0.32064	0.22345
R_c/r_0	0.8383	0.5034	0.29081	0.20971
τ_e/τ_0	4.6083	63.89	883.0	3526
v_c/v_0	0.5932	0.5946	0.5656	0.5416
Γ	2.9433	2.9975	3.0551	2.9648
c_1	0.069808	$6.846\cdot10^{-4}$	$3.647\cdot10^{-5}$	$-8.955\cdot10^{-6}$
c_2	0.222479	0.006816	−0.001529	$5.385\cdot10^{-4}$
c_3	0.430252	−0.076172	0.029952	−0.010536
c_4	−1.198975	0.882279	−0.257123	0.097523
c_5	1.329068	−4.140367	1.107327	−0.470276
c_6	−0.703890	10.015245	−2.400530	1.210515
c_7	0.133338	−9.835165	2.409091	−1.567519
c_8	0.005779	3.324584	−0.794186	0.801272

mitted by the authors of Ref. 10 consists in the fact that the shape of the surface $r = R(\alpha)$ was sought in a form which assumes in advance that $R_c = R(\pi/2) = 0$.

2. DISK ACCRETION

Besides the case of radial accretion analyzed above, the case of disk accretion is of enormous interest for the theory of x-ray stars. But in this case even the very statement of the problem of the shape of the magnetopause and its parameters encounters serious difficulties.

a) Basic assumptions. In addition to the simplifications made earlier, we assume below that 1) the plane of the infinitely thin accreting disk coincides with the equatorial plane $\alpha = 0$ of the magnetic dipole; 2) the Alfvén surface is closed and the magnetic field is absent outside it; 3) the Φ-component of the velocity of the matter flowing along the Alfvén surface is equal to zero. After this the shape and the main characteristics of the magnetopause depend on only one parameter — the ratio of the initial meridional velocity $v_e = v(\alpha = 0)$ at the Alfvén radius $r = R_e$ to the free-fall velocity $v_{ff} = \sqrt{2GM/R_e}$ at this radius. One cannot make any reliable estimate of v_e from theoretical considerations. Evidently, this quantity is close to the radial velocity of the motion of matter through the disk,[11] and the ratio v_e/v_{ff} comprises 10^{-5}–10^{-3}.

We note that the assumptions 1-3) are arbitrary to a certain extent, and they may considerably limit the region of applicability of the calculations made. The condition (3), for example, assumes that the "freezing" of matter into the magnetic field occurs at the boundary between the disk and the magnetosphere (in a region with a size on the order of the thickness of the disk). The condition (2) may not be satisfied, since the dipole field lines may close at the disk (in this case one can expect that the accretion channel has just the shape which is discussed in detail in Ref. 9 and which leads to supercritical x-ray luminosity. The condition (1) is also evidently not satisfied in the general case. All the same, the calculations performed can be useful in a discussion of one or another model of x-ray sources.

b) Results of the calculations. The shape of the Alfvén surface was calculated by the same scheme which was used above. In the case of disk accretion Eqs. (3), (8), and (9) take the form

$$\begin{cases} 2\pi R\sigma v\cos\alpha=\dot M/2,\\ v^2=v_e^2+2GM(R^{-1}-R_e^{-1}),\\ p_n=\dfrac{\dot M}{4\pi vR\cos\alpha\,(R^2+R_\alpha^2)^{1/2}}\left[\dfrac{GM}{R}-v^2\dfrac{R^2+2R_\alpha^2-RR_{\alpha\alpha}}{R^2+R_\alpha^2}\right].\end{cases} \tag{28}$$

The results of calculations for $N = 8$ and a set of values of v_e/v_{ff} from $3\cdot10^{-4}$ to 10^{-1} are presented in Table II and Figs. 1 and 5. In Fig. 2 the shape of the Alfvén surface is normalized to a unit equatorial radius: The normalization constants R_e/r_0 are given in Table II. For small values of v_e/v_{ff} the Alfvén surface is very close to spherical except for a narrow region near the dipole axis. A curious feature of the calculated models is the presence of a sharp minimum (see Fig. 5) in the optical depth $\tau(\alpha)$ of the magnetopause at a latitude $|\alpha| \approx 70°$–$80°$ (under certain conditions such a minimum can lead to pulsations of the x-ray radiation[1],[2]). We note that with a decrease in v_e/v_{ff} and the approach of the Alfvén surface to a sphere the error in the determination of the velocity $v(\alpha)$ and the optical depth $\tau(\alpha)$ of the matter in the magnetopause grows rapidly (for a fixed N). Thus, with $N = 8$ and $v_e/v_{ff} \lesssim 10^{-3}$ the present calculations have only a very approximate character, which is indicated by the wavy form of the dashed curves in Fig. 5 and the negative value of c_1 for $v_e/v_{ff} = 3\cdot10^{-4}$.

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1)The available observational data on accreting x-ray pulsars do not yet permit a reliable estimate of the magnetic fields of these objects.

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