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Iron $K\alpha$ fluorescence due to solar flares

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A relatively simple expression is obtained for the flux of $K\alpha$ photons of weakly ionized iron emitted in the process of reprocessing of hard radiation ($h\nu > 7$ keV) of a solar flare in dense layers of the solar atmosphere. An exact solution of the problem of radiation transfer is used for this. The efficiency of $K\alpha$ fluorescence is compared in detail with the efficiency of excitation by electron impact. The information which observations of $K\alpha$ emission of iron in the 1.93–1.94 Å range can provide is discussed briefly.

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The x-ray spectra of solar flares abound with lines of various elements.[1] Iron lines predominate in the $1.8\,\text{Å} < \lambda < 2$ Å range, with the strongest lines in this interval, as shown by measurements[2] with a resolution $\Delta\lambda \leqslant 10^{-3}$ Å, corresponding to transitions to the K level in ions of very high multiplicity: Fe XXIV–Fe XXVI. Along with these strong lines, in the spectra of some flares a faint emission detail is also present[3,4] at $\lambda = 1.93$–1.94 Å, which is naturally identified with the $K\alpha$ emission of neutral or weakly ionized iron (remember that according to Ref. 5 the wavelengths of the $K\alpha$ lines of Fe I and of all ions up to Fe XII coincide with each other with the accuracy of the natural width).

Two mechanisms have been proposed to explain the excitation of the $K\alpha$ emission of weakly ionized iron: 1) ionization of the K shell by the same energetic electrons which are responsible for the hard radiation ($h\nu \gtrsim 10$ keV) of the flare[6]; 2) photoionization of the K shell in the absorption of hard flare radiation with $h\nu > 7.11$ keV: $K\alpha$ fluorescence.[7] In the present report the second mechanism is investigated in detail, with the $K\alpha$ emission being calculated, in contrast to the approximate estimates of Ref. 7, using a rigorous solution of the transfer equations obtained earlier.[8] The values found for the fluxes of $K\alpha$ photons are compared with the results of the calculations of Phillips and Neupert,[6] who studied the first mechanism. We note that the emitting regions are strongly separated geometrically in these two cases: the fluorescence is concentrated in the photospheric layer at a depth of $\gtrsim 1$ g/cm^2, whereas ionization by electron impact occurs in the rarefied layers where the x-ray flare itself is generated.

1. BASIC EQUATIONS

The calculation of the flux of only those $K\alpha$ photons which did not undergo even one scattering in the solar atmosphere is of practical interest, since a) in this case the line photons are concentrated in a narrow spectral interval (1.933–1.943 Å in the case of the $K\alpha$ line of iron) determined by the natural width of the $K\alpha_1$ and $K\alpha_2$ components and b) at the solar abundance of heavy elements the contribution of these photons alone comprises ~70% of the total flux (for more detail on the $K\alpha$ line profile see Ref. 8). Due to Compton recoil the lines of those $K\alpha$ photons which underwent n ($n \geq 1$) scatterings will be distributed over a wide interval $\Delta\lambda = 2n\lambda_C = n \cdot 0.049$ Å, and their observations does not require calculation for now.

We assume that the hard flare radiation is isotropic and has a spectrum $L(\varepsilon)$ [erg/sec·erg] at photon energies $\varepsilon \geq \varepsilon_{th}$, where $\varepsilon_{th} = 7.11$ keV is the threshold for photoionization of the K shell of neutral iron atoms. Entering the dense layers of the solar atmosphere, photons from this spectral range are partly scattered on electrons (both free and bound in hydrogen atoms), partly lost in photoabsorption in atoms of heavy elements, and partly reprocessed into $K\alpha$ photons of iron. The corresponding problem of transfer theory was analyzed in detail in Ref. 8 under the assumptions that the photon energy ε does not change during scattering and that the scattering cross section is isotropic. The most serious error is introduced by the assumption of monoenergetic scattering, which always reduces the flux of $K\alpha$ photons by an amount $\leqslant 10\%$. The differential energy albedo $\psi(\varepsilon, \mu_0, \mu)$ of a plane atmosphere for that part of the $K\alpha$ line which emerges without scatterings is expressed through the Ambartsumyan function $H(\mu, \lambda)$ (the particular case of the Chandrasekhar H function for isotropic scattering)

$$\psi(\varepsilon, \mu_0, \mu) = \frac{\delta\lambda_K}{4}\frac{\varepsilon_K}{\varepsilon}H(\mu_0, \lambda)\frac{H(\mu\lambda_K/\lambda, \lambda)}{\mu_0 + \mu\lambda_K/\lambda}. \tag{1}$$

Here ε is the energy of incident photons of the continuous spectrum, $\theta_0 = \arccos\mu_0$ and $\theta = \arccos\mu$ are the angle of incidence of photons with an energy ε and the angle of emergence of K photons with energy ε_K, respectively,

$$\lambda(\varepsilon) = \sigma_T/[\sigma_T + \sigma_{ph}(\varepsilon)]; \quad \lambda_K = \lambda(\varepsilon_K), \tag{2}$$

$$\delta(\varepsilon) = \omega_K\sigma_{ph,\,K}(\varepsilon)/\sigma_T, \tag{3}$$

where σ_T is the Thomson scattering cross section, $\sigma_{ph}(\varepsilon)$ is the total cross section for photoabsorption of quanta with an energy ε normalized to one electron, ω_K is the probability of $K\alpha$ fluorescence, and $\sigma_{ph,K}$ is the cross section for photoabsorption in the K shell of iron normalized to one electron.

We further assume that the surface of the photosphere represents an infinite plane. Such an assumption raises the flux of $K\alpha$ photons by an amount of $\sim\sqrt{2h/R_\odot} \leqslant 20\%$ when $h \leqslant 10^9$ cm, where h is the height of the flare. Then an x-ray flux of $^1/_2 L(\varepsilon)d\varepsilon \sin\theta_0 d\theta_0$ erg/sec falls on a ring of the plane atmosphere seen from the point of the flare in the interval of angles (θ_0, $\theta_0 + d\theta_0$) and in the interval of photon energies (ε, $\varepsilon + d\varepsilon$). The fraction of the energy of this radiation which is reprocessed into the energy of of $K\alpha$ radiation emerging from the atmosphere at an angle θ is

$$^{1}/_{2}L(\varepsilon)\,d\varepsilon\,\sin\theta_0\,d\theta_0\,\frac{\cos\theta}{\pi}\,\psi(\varepsilon,\cos\theta_0\cos\theta)\ \text{erg/sec}\cdot\text{sr}.$$

Integrating this expression over the angle θ_0 and over the energy ε, and using the integral equation

$$H(\mu,\lambda)=1+\frac{\lambda}{2}\mu H(\mu,\lambda)\int\limits_0^1\frac{H(\mu',\lambda)}{\mu+\mu'}d\mu' \qquad (4)$$

which the function $H(\mu, \lambda)$ satisfies, we obtain a relatively simple expression for the flux of Kα photons at the earth:

$$F_K(\theta)=\int\limits_{\varepsilon_{th}}^{\infty}F(\varepsilon)\delta(\varepsilon)\left[H\left(\frac{\lambda_K}{\lambda}\cos\theta,\lambda\right)-1\right]d\varepsilon\,[\text{photon/cm}^2\cdot\text{sec}]. \qquad (5)$$

Here we introduced the notation

$$F(\varepsilon)\equiv L(\varepsilon)/4\pi D^2\varepsilon\ [\text{photon/cm}^2\cdot\text{sec}\cdot\text{erg}] \qquad (6)$$

for the spectral flux of the number of photons of the continuous x-ray spectrum at the earth ($D = 1$ AU); θ is the angle between the normal to the solar surface at the point of the flare and the direction toward the earth. It is convenient to characterize the intensity of iron Kα radiation in solar flares by the quantity

$$\Gamma(\theta)=F_K(\theta)\bigg/\int\limits_{\varepsilon_{th}}^{\infty}F(\varepsilon)\,d\varepsilon, \qquad (7)$$

which represents the ratio of the flux of Kα photons to the flux of all photons with an energy $\varepsilon \geq \varepsilon_{th}$. The quantity Γ is determined by three main parameters: 1) the form of the spectrum $L(\varepsilon)$ of the x-ray flare, 2) the value of the angle θ, and 3) the abundance of heavy elements (mainly iron). Values of Γ calculated for different values of these parameters are given in the next section. We note that from Eqs. (1)-(7) one can also easily calculate the Kα emission of other heavy elements, such as argon, sulfur, etc., although the corresponding values of the fluxes prove to be far smaller than in the case of iron.

2. RESULTS OF CALCULATIONS

In the numerical calculations we took the same values as in Ref. 9 for the abundance of heavy elements with $Z < 26$. The photoabsorption cross section at $\varepsilon < \varepsilon_{th}$ is well approximated by the expression

$$\sigma_{ph}(\varepsilon)=3.9\cdot10^{-22}\ (1\ \text{keV}/\varepsilon)^3\ \ \text{cm}^2/\text{atom of hydrogen}. \qquad (8)$$

For the cross section for photoabsorption in the K shell of iron atoms we adopted an interpolation equation for the results of the calculations of Rakavy and Ron[10]:

$$\sigma_{ph,K}(\varepsilon)=8.13\cdot10^{-18}\ (1\,\text{keV}/\varepsilon)^{2.78}\ \text{cm}^2/\text{atom of iron}. \qquad (9)$$

The iron abundance was represented in the form

$$N_{Fe}=10^{-5}Y_{Fe}N_H. \qquad (10)$$

The probability of K fluorescence is[11] $\omega_K = 0.342$ and the fraction of Kα photons in the total number of K photons is[2] 150:167. In Tables I-IV we present the values of Γ – the efficiency of reprocessing of photons of the hard spectrum with $\varepsilon > \varepsilon_{th}$ into photons of the Kα lines of weakly ionized iron with $\varepsilon_K = 6.40$ keV – at different values of the parameters for two kinds of spectrum:

power-law

$$F(\varepsilon)=A\varepsilon^{-\gamma} \qquad (11)$$

and exponential

$$F(\varepsilon)=A\varepsilon^{-1}\exp\left(-\varepsilon/kT_x\right). \qquad (12)$$

TABLE I. Power-Law Spectrum $F(\varepsilon)\propto\varepsilon^{-\gamma}$, $\theta = 0$, $Y_{Fe} = 3$

γ	2	2.5	3	3.5	4	4.5	5	∞
Γ, %	2.29	2.73	3.09	3.36	3.57	3.74	3.87	4.91

TABLE II. Power-Law Spectrum $F(\varepsilon)\propto\varepsilon^{-3}$, $Y_{Fe} = 3$

θ	0°	10°	20°	30°	40°	50°	60°	70°	80°
Γ, %	3.09	3.07	3.01	2.91	2.77	2.56	2.27	1.86	1.25

From these tables it is seen that Γ depends rather weakly on the parameters γ, kT_x, and θ and is determined mainly by the iron content in the solar atmosphere. The weak dependence on the angle θ is due mainly to the rather strong law of limb brightening (1). Typical values of Γ for solar flares on the disk should evidently be 2-4%.

3. COMPARISON WITH THE MECHANISM OF ELECTRON EXCITATION

Phillips and Neupert[6] calculated the flux of Kα photons of weakly ionized iron excited by electron collisions in solar flares in two extreme cases: 1) the exciting electrons are concentrated in a beam with a nonthermal spectrum of the type $dN_e = BE^{-\beta}dE$; 2) the exciting electrons have a Maxwellian distribution with a temperature $T_e \approx 10^{7}$°K. Here it was assumed that in case 1) the bremsstrahlung of the electrons themselves accounts for the hard radiation $F(\varepsilon) = A\varepsilon^{-\gamma}$ (10 keV $\lessdot \varepsilon \lessdot$ 100 keV) of solar flares. It is interesting to compare the efficiency of this mechanism with the efficiency of fluorescence.

First we consider the case of thermal electrons. In this variant the Kα emission of weakly ionized component is low, since at $T_e \gtrsim 10^{7}$°K iron is present mainly in the form of the ions Fe XX-XXVI, for which the wavelengths of the K transitions lie far outside the limits of the interval of 1.933-1.943 Å. Only the Kα line of Fe XVIII, whose wavelength is 1.927 Å according to Ref. 5, deserves attention. At $T_e = 10^{7}$°K and $Y_{Fe} = 10$ the intensity of this line in the case of excitation by electron impact is $\Gamma_e \approx 3.4\%$ according to Phillips and Neupert [the formal definition (7) is retained for Γ_e, although this quantity is no longer the coefficient of reprocessing of the hard part of the x-ray spectrum $\varepsilon > \varepsilon_{th}$ into Kα quanta], but at $T_e =$

TABLE III. Power-Law Spectrum $F(\varepsilon)\propto\varepsilon^{-3}$, $\theta = 0$

lg Y_{Fe}	–0.4	–0.2	0	0.2	0.4	0.6	0.8	1
Γ, %	0.54	0.84	1.27	1.87	2.69	3.74	4.99	6.38

TABLE IV. Exponential Spectrum $F(\varepsilon) \propto \varepsilon^{-1}\exp(-\varepsilon/kT_X)$, $\theta = 0$, $Y_{Fe} = 3$

kT_x, keV	1	2	3	5	7	10	15	20	30
Γ, %	4.43	4.08	3.81	3.42	3.14	2.84	2.51	2.29	2.00

$1.5 \cdot 10^7$°K this quantity already decreases by two orders of magnitude. Thus, if the x-ray spectrum of a solar flare is due to the bremsstrahlung of thermal electrons then the fluorescence mechanism always dominates in the Kα emission of weakly ionized iron.

For the case of nonthermal electron spectrum Phillips and Neupert obtained rather large values for the flux of Kα photons: with $Y_{Fe} = 10$ and $\gamma = 2.8$ they got $\Gamma_e \approx 12\%$, while $\Gamma_e \approx 70\%$ already with $\gamma = 4$. We note that with $\gamma = 2.8$ and the more realistic value of $Y_{Fe} = 3$ the contributions of the fluorescence and electron excitations are almost comparable, since the dependence of Γ on Y_{Fe} at $3 \leqslant Y_{Fe} \leqslant 10$ is weaker than direct proportionality (see Table III). But the fact that the results of Phillips and Neupert for nonthermal electrons may prove to be strongly overstated is far more important. The point is that they solved a nonself-consistent problem: the heating of the surrounding plasma by the very same nonthermal electrons which give the Kα emission and the radiation in the hard x-ray range was now allowed for in their calculations. In order to demonstrate the importance of this effect, let us estimate the rate of plasma heating by an electron stream with an energy spectrum of the type

$$dL_e = (\beta-1) L_e (E_0/E)^\beta dE/E_0 \text{ [electron/sec]}, \tag{13}$$

propagating along magnetic field lines with a beam cross section S:

$$q^+ = -\frac{1}{S}\int_{E_0}^{\infty} \frac{dL_e}{dE}\frac{dE}{dx}\, dE \text{ [erg/cm}^3\cdot\text{sec]}. \tag{14}$$

The rate of energy loss by electrons in Coulomb collisions is

$$\frac{dE}{dx} = -N_e \frac{2\pi e^4 \ln\Lambda}{E}. \tag{15}$$

From (13)-(15) we easily obtain an estimate for the plasma temperature as a function of the time t of electron injection:

$$kT \simeq L_e t \frac{\beta-1}{3\beta}\frac{2\pi e^4 \ln\Lambda}{SE_0} \tag{16}$$

(plasma cooling can be neglected in this stage). Substituting $\ln\Lambda = 20$, $\beta = 2$, $E_0 = 10$ keV, and $S = 10^{18}$ cm^2 into (16) we find that even at $L_e t \geqslant 2 \cdot 10^{37}$ the electron temperature is $T \geqslant 10^7$°K. But at such a temperature the fraction of the lower ions of iron decreases rapidly, and the actual intensity of the Kα line in the vicinity of $\lambda = 1.938$ Å will be far lower than according to Ref. 6. We note that values of $L_e t > 10^{37}$ are rather typical of bright solar flares.[13]

On the basis of the order-of-magnitude estimate (16) (because of the large uncertainty in the value of the parameter S) it is hard to draw a definite conclusion about the ratio of efficiencies of the mechanisms of fluorescence and of excitation by electron impact in the case of a nonthermal electron spectrum. For this one must make self-consistent calculations of the efficiency of the latter mechanism. It is quite possible that fluorescence predominates in some flares whereas these mechanisms give comparable fluxes of Kα photons in others.

4. OBSERVATIONS OF Kα EMISSION OF WEAKLY IONIZED IRON IN SOLAR FLARES

Although the emission detail at a wavelength of 1.93-1.94 Å is clearly seen in the spectra of some solar flares (Refs. 3 and 4), the absolute values of the fluxes of Kα photons are not yet determined with good accuracy. The determination of this quantity would make it possible in principle (if fluorescence predominates in the generation of the Kα line) to give a good (with an error of ~10-20%) independent estimate of the iron content in the solar photosphere. For this one must allow for the following circumstances in addition to the calculations made above:

1) if the height of the flare is $h > 10^8$ cm then one must allow for the spherical shape of the solar surface and calculate the Kα emission not from the simple Eq. (5) but from the more complicated expressions presented in Ref. 8;

2) in determining the flux of Kα photons from observational data one must allow for the contribution of the reflection effect, as a consequence of which a significant fraction of the photons are shifted from the region of intense emission of ionized iron at 1.85-1.88 Å into the region of 1.90-1.93 Å (Ref. 7).

Thus, the following experimental data are needed for a calculation of the iron abundance: 1) the value of the absolute flux of Kα photons of weakly ionized iron in the 1.93-1.94 Å range, 2) the form of the spectrum and the value of the absolute flux of exciting photons of the continuous spectrum with $\varepsilon > 7.11$ keV, and 3) the coordinates of the flare, enabling one to determine the angle θ. Here it is assumed that one knows the content of the most abundant elements with $Z < 26$, which are responsible for the absorption of Kα photons with $\varepsilon = \varepsilon_K = 6.40$ keV. It is best to conduct the observations in the "explosive" hard part of the flare, when the flux is greatest in the range of $\varepsilon > 7$ keV. Flares in the immediate vicinity of the center of the solar disk are naturally the most promising for this. We note that an experimental confirmation of the dependence (5) on the angle θ (also see Table II) could be a strong argument in favor of the fact that the fluorescence mechanism predominates over the mechanism of electron excitation.

If the x-ray spectrometer has a good angular resolution as well as a good spectral resolution, then from the brightness distribution of the Kα line in the vicinity of a flare one can determine the height of the flare above the photosphere, like the way it was proposed to do this in Ref. 14 from observations of the reflected x-ray flux in the continuum.

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